IF5110 Teori Komputasi

Teori Kompleksitas
(Bagian 3)

Oleh: Rinaldi Munir

Program Studi Magister Informatika STEI-ITB

Sumber: *Complexity Theory, based on* Garey M., Johnson D.S., *Computers and Intractability A guide to the Theory of NP-Completeness,* Freeman and Company - New York - 2000

SAT

- SAT = *Satisfiability Problem*

Up to now we never encountered *NP*-complete problems

The first example of NP-complete problem was found by Cook in 1971

(before this date, the concept of *NP*-completeness did not even exist)

Given $X = \{x_1, x_2, ..., x_n\}$ a set of Boolean variable, that can assume value $\{0,1\}$, and a *clause* over X, that is a set containing variables or negation of variables, a collection C of clauses is *satisfiable* if and only if there exists some truth assignment for X that simultaneously satifies all the clauses.

SATISFIABILITY PROBLEM (SAT) in Boolean algebra

Instance: a set X of variables and a collection C of clauses
Question: is there a satisfying truth assignment for C?

Example *Yes*

$X = \{x_1, x_2, x_3\}$ $\quad\quad$ $C = (\bar{x}_1 \vee x_2 \vee \bar{x}_3) \wedge (x_2) \wedge (x_1 \vee \bar{x}_2)$

The truth assignment $x_1 = 1, x_2 = 1, x_3 = 1$ satisfies C.

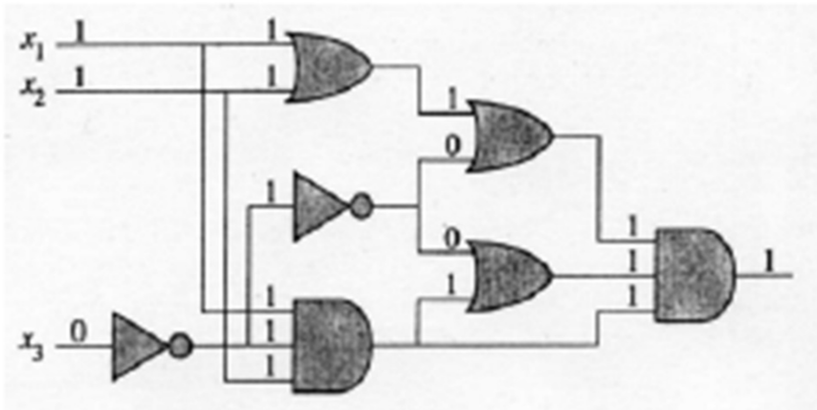
The answer is *Yes*

Example *No*

$X = \{x_1, x_2, x_3\}$ $C' = (x_1 \vee x_2) \wedge (x_1 \vee \bar{x}_2) \wedge \bar{x}_1 \wedge (x_1 \vee x_3)$

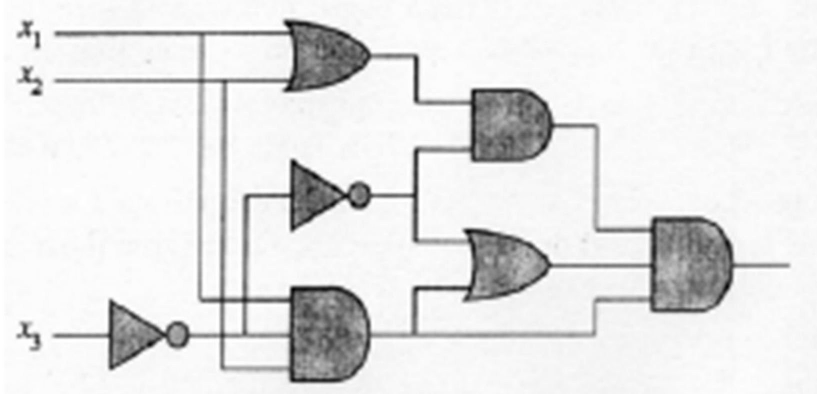
There are no truth assignments that satisfies C'

The answer is *No*



(a)

satisfiable



not satisfiable

Cook's Theorem (1971)

SATISFIABILITY PROBLEM (SAT) in Boolean algebra
is *NP*-complete

proof: very complex, since there are infinitely many languages in *NP*, and we cannot prove directly that, for each $L \in NP$ we have $L \propto L_{SAT}$, showing a transformation for each language. We prove the theorem in two steps:

1) SAT is in NP because any assignment of Boolean values to Boolean variables that is claimed to satisfy the given expression can be *verified* in polynomial time by a deterministic Turing machine.

2) Now suppose that a given problem in NP can be solved by the nondeterministic Turing machine NDTM. Suppose further that NDTM accepts or rejects an instance I of the problem in time $p(n)$.
For each input, I, we specify a Boolean expression which is satisfiable if and only if the machine NDTM accepts I.

Six examples of NP-complete problems

3-SATISFIABILITY (3-SAT)
***Instance*: a set X of variables and a collection C of clauses that contains exactly 3 literals**
***Question*: is there a satisfying truth assignment for C?**

Example

$$X = \{x_1, x_2, x_3, x_4\}$$

$$C = (x_1 \vee \bar{x}_2 \vee x_3) \wedge (\bar{x}_1 \vee x_2 \vee x_4) \wedge (x_1 \vee \bar{x}_2 \vee \bar{x}_3)$$

3-DIMENSIONAL MATCHING (3-DM)
***Instance*: a set $M \subseteq W \times X \times Y$ where W, X, Y are disjoint sets having the same number q of elements.**
***Question*: does M contain a matching, that is, a subset $M' \subseteq M$ such that $|M'| = q$ and no two elements of M' agree in any coordinate?**

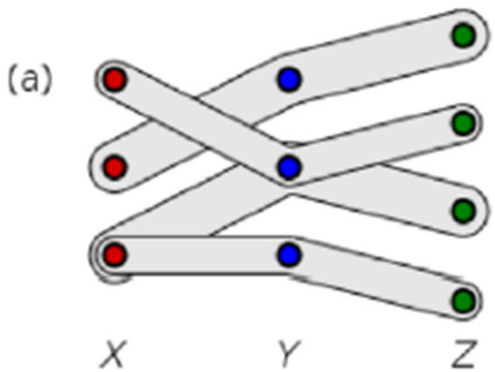
It is a generalization of the "marriage problem":

Given n unmarried men and n unmarried women, along with a list of all male-female pairs who would be willing to marry one another, is it possible to arrange n marriages so that polygamy is avoided and everyone receives an acceptable spouse? (best algorithm $O(n^5)$)

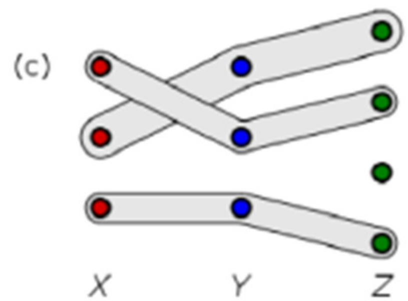
Example

$\boldsymbol{M} \subseteq W \times X \times Y = \{axm, axn, axo, axp, aym, ayn, ayo, ayp, ..., dko, dkp\}$

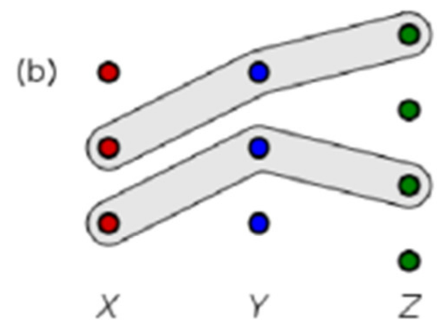
$\boldsymbol{M'} = \{axm, byn, czo, dkp\}$



Instance



Solution 1



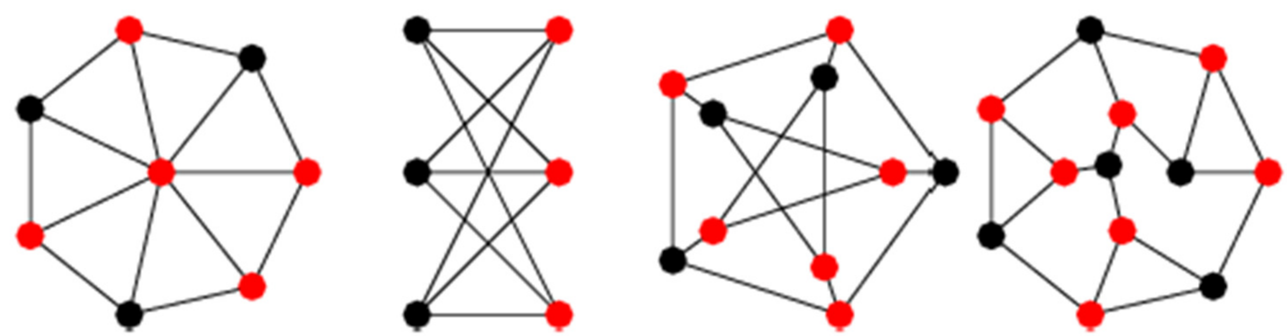
Solution 2

VERTEX COVER (VC)

***Instance*: a graph $G = (V, E)$ and a positive integer $K \leq |V|$**

***Question*: is there a vertex cover of size K or less for G, that is a subset $V' \subseteq V$ such that $|V'| \leq K$ and, for each edge $\{u,v\} \in E$ at least one of u and v belongs to V'?**

Example

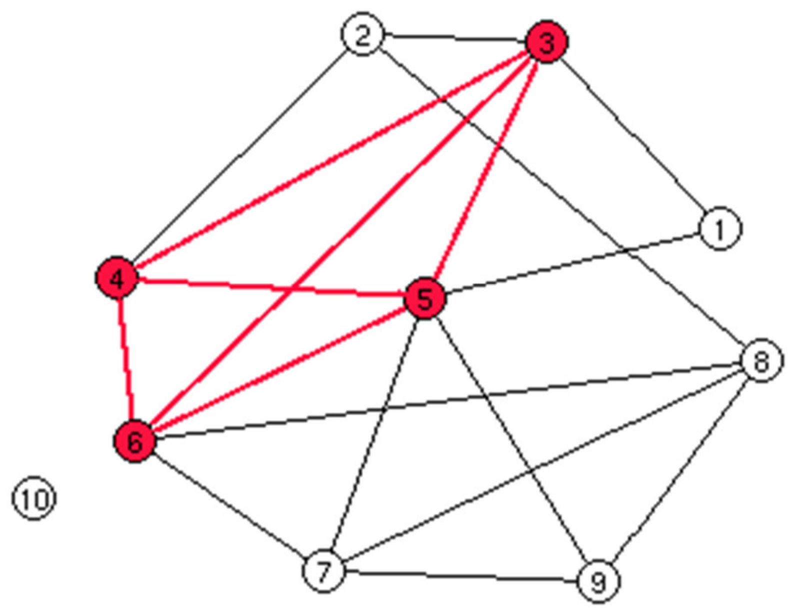


CLIQUE

Instance: **a graph $G = (V, E)$ and a positive integer $K \leq |V|$**

Question: **does G contain a clique of size K or more, that is a subset $V' \subseteq V$ such that $|V'| \leq K$ and every two vertices in V' are joined by an edge in E?**

Example

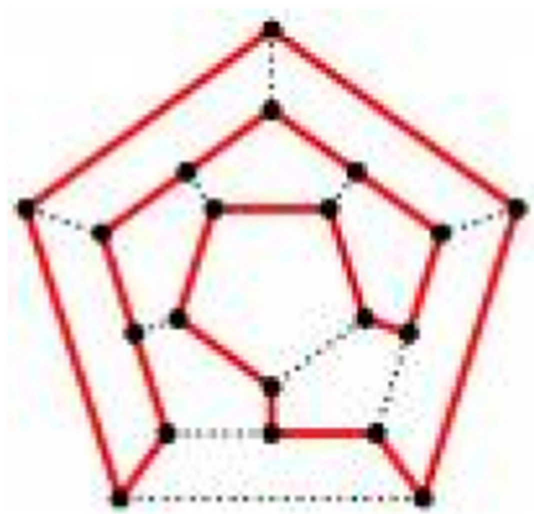


HAMILTONIAN CIRCUIT (HC)

Instance: **a graph $G = (V, E)$**

Question: **does G contain a Hamiltonian circuit, that is an ordering $<v_1, v_2, \ldots, v_n>$ of the vertices of G, where $n = |V|$, such that $\{v_n, v_1\} \in E$ and $\{v_i, v_{i+1}\} \in E$ for all i, $1 \leq i < n$?**

Example



PARTITION

Instance: **a finite set A and a "size" $s(a) \in Z^+$ for each $a \in A$**

Question: **is there a subset $A' \subseteq A$ such that**

$$\sum_{a \in A} s(a) = \sum_{a \in A'-A} s(a)$$

Example

$$A = \{14, 1, 23, 3, 5, 32, 11, 21\}$$

$$11+23+21 = 1+3+5+32$$