Bayesian Network as Bayes Theorem's Application

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***Abstract*—Bayes Theorem is one of basic rules in probability. It shows the relationship between the probability of hypothesis event's occurrence and effect event's occurrence. As years passed, Bayes Theorem developed and new applications of the rule were found. One of them is Bayesian Network. Bayesian Network is an effective way to model the causal dependencies between each variable in a system through directed acyclic graph (DAG). The effectivity of Bayesian Network makes it a popular approach for modelling knowledge in various sectors of science. In this report we will discuss the advantages and disadvantages of Bayesian Network along with its application in diverse fields.**

***Index Terms*—Bayes Theorem, Bayesian Network, conditional probabilities, Node Probability Table (NPT).**

I. Introduction

In everyday life we deal with events that have cause and effect relationship between them. To analyze such cases, we use Bayes Theorem: a derivation of conditional probability tools. However, when working with complex system with complex causal dependencies, we can model our system in a more convenient way – Bayesian Network.

Bayesian Network is a probabilitic graphical model that represents random variables and their conditional dependencies through directed acyclic graph. Bayesian Network basically uses Bayes Theorem for obtaining probability properties but uses graphical model to make it easier to do analysis on the data.

In this report we mainly discuss the Bayesian Networks and it's ability to model real-life phenomenon as the application of Bayes Theorem.

In addition to that, we will also discuss the advantages and disadvantages of using Bayesian Networks as models for various problems.

II. Bayes Theorem

Bayes Theorem was the work by Thomas Bayes which was first published in 1763 by his friend Richard Price after his death on 1761. The paper contains a description of a theorem derived from probability theory. In the later years, as hypothesis testing and confidence intervals became important aspects of statistic, Bayes Theorem along with other 'classical statistics' quickly applied to diverse areas of human life.

In principle, Bayes Theorem shows the relationship between conditionally dependent variables but also states that the probabilities of the variables do not depend on the order of their occurrence.

A. Definition

The probability of H conditional on E is defined as :

$$P(H|E) = \frac{P(E|H).P(H)}{P(E)}$$

The above expression means that the probability of H knowing that E occurs is equal to the probability of both things occuring together divided by the probability of E's occurrence.

In Bayes' Theorem, we have different names for each element of the equation. Here:

- H represents hypothesis and E evidence.
- P(H) is called 'prior probability'. It is the probability of H event occurs before we analyze any other event related to event H.
- P(H|E) is called the 'posterior probability'. It is the probability of H event (our hypothesis) occurs given that E event (our evidence) occurs.
- P(E|H) is called the 'likelihood function'. It indicates the probability E event (evidence event) occurs given the hypothesis event occurs.
- P(E) is called 'marginal probability' of E. It is the probability of E event's occurrence.

The Bayes Theorem is actually a simple derivation of conditional probability rule. The derivation is as follows:

The probability of event A occurs given event B occurs:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

The probability of event B occurs given event A occurs:

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

From above statements we can conclude:

$$P(A|B)P(B) = P(A \cap B) = P(B|A)P(A)$$

Or in another term:

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

The above expression is the one we use to call Bayes' Theorem.

B. Special forms and Extension of Bayes Theorem

- Odd ratio rule

$$O(H|E) = \frac{P(H|E)}{P(H|\overline{E})}$$

- Likelihood ratio rule

$$\Lambda(\mathrm{H|E}) = \frac{P(H|E)}{P(H|\overline{E})}$$

- Extension of Bayes' Theorem

$$P(H|E1 \cap E2 = \frac{P(H)P(E|H)P(E2|H \cap E1)}{P(E1)P(E2|E1)}$$

III. BAYESIAN NETWORK

Bayesian Network is a probabilistic graphical model that represents a set of random variables and their conditional dependencies through a directed acyclic graph (DAG). It is also called belief network or directed acyclic graphical model. In Bayesian Network, each node represents a variable and the edge represents the connection or 'conditional dependencies' between two variables or more.

The probability values of Bayesian Networks' variables are calculated using the Bayes theorem.

Because Bayesian Network is a graphical model, it is easier to understand and explain the cause-effect relationship between two variables in the network. In addition to that, Bayesian Network provides us with tables consisted of the probability properties of the variables and their dependencies to each other. All these features are some reasons for Bayesian Network's popularity for modelling world's phenomenons where the probability of one event conditionally depends on the probability of other event.

An example of Bayesian Network can be seen in Fig. 1. It's a causal model of lung diseases. The directions of the arrows show the cause-effect relationship between two nodes (variables).

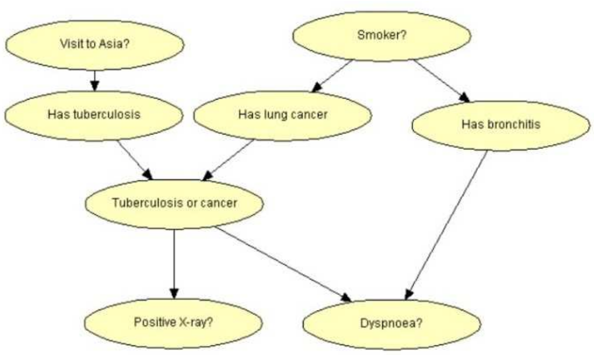

Fig. 1 - Bayesian Network example

In Fig. 1 we can easily see the relationship and probability properties of each variable given. The table beside each node presents the probability of the cause (hypothesis) event occurs (true) given the effect (evidence) occurs. Bayesian Network is still convenient to use even when we deal with huge numbers of variables or complex relationship between them.

A. Some Benefits of Bayesian Network Modelling

As stated before, Bayesian Network's popularity is the effect of its many beneficial qualities. Some of them are:

- *Explicit display of causal factors*. The problem with other modelling tools (i.e. regression model) is their difficulty in expressing cause-effect relationship between variables. On the other hand, Bayesian Network model solves this problem, giving the cause-effect relationship between variables in simple graph. It allows the Bayesian Network model to describe not only the parent node's (causes) based on the evidence, but also predict the impact to the children (effect).
- *Easiness in updating probability of variables based on new data*. In conditional probabilities, the probability of each variable changes along with the data. Using Bayesian Network, we can easily update the probability of the 'cause' nodes and 'effect' nodes when we have new data without manually compute all the variables' probabilities' changes.
- *Make prediction with uncomplete data*. Unlike traditional modelling techniques where complete data are required to make prediction, Bayesian Network model doesn't need all the observations to be entered to all 'inputs'. Bayesian Network can also work with subjective probabilities (i.e. expert judgement).

For better understanding on this benefits of Bayesian Networks we will take a closer look at Fig. 1. Fig. 1 only represents the variables and the relationship between them. On the other hand, Fig. 2 shows the probability aspects of the variables. In Fig. 2, the probability properties of the root nodes (nodes with no parent or no 'cause') are based on subjective estimation or empirical data.

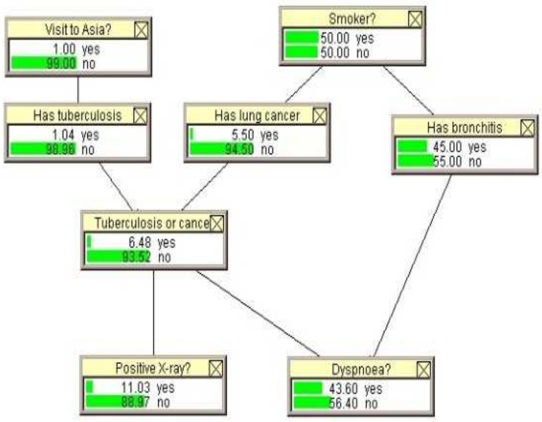

Fig. 2 - Bayesian Network with Probability Properties

When one of the variables' probability value changes, the values of all nodes connecting to it change as well, as shown in Fig. 3. In Fig. 3, we know that event 'smoker' occurs (patient is a smoker) and event 'dyspnoea' occurs (patient has dyspnoea).

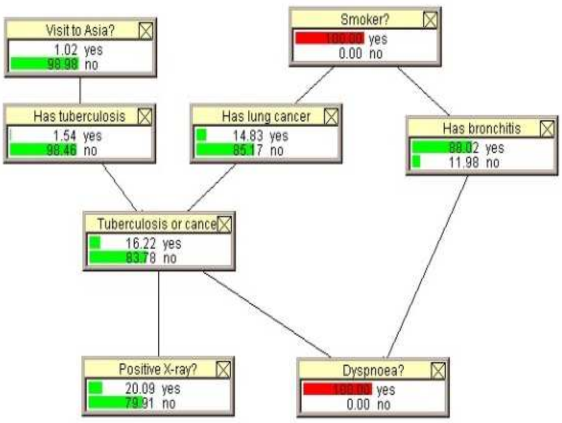

Fig. 3 – Bayesian Network's change on probability value

As seen in Fig. 3, the change of information in a node creates change in all nodes corresponding to that node. In Fig. 3 for example, given the certainty that the patient is smoker and that he has dyspnoea, the probabilities of all other nodes increases.

One other interesting feature of the network which we can observe is how the change in probability value 'effect' changes the probability value of 'cause'.

All these benefits of Bayesian Network make it a powerful tool for modelling cause-effect phenomenon in various aspect of science and technology.

B. Constructing Bayesian Network

Constructing a Bayesian Network involves three main steps as shown in Fig. 4.

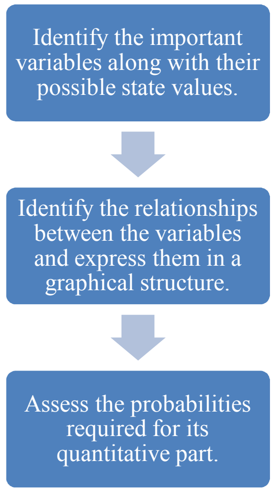

Fig. 4 – Constructing a Bayesian Network

In practice, however, the making of a Bayesian Network is more an iteration of these three steps until the desired network is achieved.

Identifying the variables which are of important and connecting them in DAG according to their relationship is not always easy. In fact, in some cases this step is the most difficult part of building a Bayesian Network. Fortunately, Bayesian Network allows variables to be conveniently added or removed without disturbing the rest of the network.

The next step to do after the variables have been defined is constructing the graph to represent the network. It includes representing the probabilistic dependencies between variables through directed edges (arcs). The cause-effect relationship between variables must be carefully defined.

Afterwards we need to measure the probability values and assign them to the node probability table (NPT). The node probability table represents the strenght of the dependency between connected nodes. In prior node (root node) the node's probability table is the prior prabability based on subjective estimation or empirical data.

C. Software for Building Bayesian Network

Bayesian Network, as stated before, can be used to deal with huge numbers of events, but the calculation of probability values takes more time and becomes more complex as the number of nodes in the graph increases. To solve that problem several softwares have been made to help people build Bayesian Network. Here are some of these softwares:

- AgenaRisk,
- Netica,
- Hugin,
- Microsoft MSBNX,
- BayesiaLab
- Riscue, and
- OpenBayes.

These softwares are great help for building and maintaining Bayesian Network models for large numbers of events. Some of these softwares have graphical tools and runtime module to analyze the data along with their probability properties. These features and the propagation algorithm used by the softwares make buliding a Bayesian Network less complicated.

There are more features developed and integrated to the softwares in order to simplify the making of Bayesian Network or to solve problems regarding the network structure or node probability table management.

- *Object oriented Bayesian Network.* The idea is combining object oriented design and programming with Bayesian Network to describe complex problems.
- *Efficient Node Probability Table (NPT) elicitation.* Building large NPT manually is a frustrating task. Over the years different techniques have been proposed to minimizethe effort for a range of special cases. However, this technique only applies to binary nodes.
- *Dynamic Bayesian Network.* In real world problems, we want to model the change of uncertain variables' values. Dynamic Bayesian Network supports this kind of modelling and widely used for image tracking and condition monitoring.
- *Hybrid Bayesian Network.* Hybrid Bayesian Networks contain both discrete and continous variables. The idea is to make Bayesian Network modelling applicable for both discrete and continous variables. However, most Bayesian Network tools cannot deal with continous variables accurately.

IV. Some Applications of Bayesian Network

Due to its flexibilities and its beneficial aspects Bayesian Network is widely used nowadays. Bayesian Networks is applied in diverse fields of science through various ways.

In informatics Bayesian Network is used for:

- Project scheduling
- Predicting software defect
- Predicting software maintainability
- Desicion support system
- Document classification
- Image processing

Other applications of Bayesian Networks can be found in biomedical field. Some of them are:

- Modelling diseases, for instance breast cancer.
- Analyzing detection performance in surveillance system.
- Modelling DNA sequence information
- Protein structure analysis.

In other sections of science, for example:

- Air combat simulation
- Plant diseases modelling
- Reconstructing traffic accident
- Shopping assistance
- Ecosystem modelling
- Meteorology

V. Limits of Bayesian Network

Beneficial and convenient as it is, Bayesian Network has the following weaknesses:

- Each Bayesian Network is 'specially made' for unique purpose. Therefore different system requires different network. The existing Bayesian Network cannot be applied to other industry or other type of system.
- The reliability of Bayesian Network depends on the reliability of root nodes (nodes with no parent) that use probability values based on subjective judgement or estimation. Any excessive difference of these values from their real values will distort all probability values in the network.
- Though good at modelling discrete variables, Bayesian Network cannot model the countinous variables accurately.

VI. Conclusion

Bayesian Network is based on Bayes Theorem. It gives powerful assistance in modelling real-world phenomenons. The beneficial aspects of Bayesian Networks are its graphical presentation, its flexibility and ability to process new data, and its capacity to work with uncomplete data.

Due to this advantages, Bayesian Networks are widely used in diverse fields such as bioinformatics, medicine, documents classifications, risk management, etc.

However Bayesian Network has its own weaknesses. Its specifically-built functions along with its incapability in modelling continous variables are among them.

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PERNYATAAN

Dengan ini saya menyatakan bahwa makalah yang saya tulis ini adalah tulisan saya sendiri, bukan saduran, atau terjemahan dari makalah orang lain, dan bukan plagiasi.

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