

ORBITAL TRAJECTORY SIMULATION ON TWIN STARS SYSTEM IN ITERATED FUNCTION SYSTEMS FRACTAL MODEL BASED ON HYBRID ANIMATION METHOD

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Abstract

One way to model fractal object is by IFS (Iterated Function Systems) model based on affine transformation functions. Rotational effect of star-like object in IFS fractal model can be exhibited by means of metamorphical animation method instead of by the affine rotational operation of non metamorphical animation method. The metamorphical version has an advantage over the non metamorphical version in the independency of object's relative position to the fixed point as an absolute centroid. Therefore, the rotational effect can be exhibited at any positions. In addition, it can be combined with rotational effect of the local centroid itself around the absolute centroid as a fixed point by the primitive rotational operation to form an interesting behaviour of orbital trajectory. So based on the hybrid of both animation methods, the animation simulation of orbital trajectory on a twin stars rotating to each other as a system can be done. Both objects are rotated in the same angular direction, but started in the opposite position around two closely different fixed points. So, the orbital trajectory yielded forms an elliptical path. The two similar objects can be created efficiently by cloning-scaling technique. In general, the animation method can be modeled as an animation framework.

Keywords: *IFS fractal, orbital trajectory, hybrid animation method, cloning-scaling technique*

Abstrak

Salah satu cara memodelkan objek fraktal adalah dengan model IFS (*Iterated Function Systems*) berdasarkan transformasi affine. Efek rotasi objek yang menyerupai bintang dari fraktal IFS dapat dijalankan melalui metoda animasi metamorfik sebagai pengganti operasi rotasi affine pada metoda animasi non metamorfik. Kelebihan metoda animasi yang metamorfik dibandingkan metoda animasi yang non metamorfik adalah dalam hal ketidakbergantungan posisi objek terhadap titik tetap sebagai titik pusat (*centroid*) absolut, sehingga efek rotasi dapat terjadi di manapun objek berada dan selain itu memungkinkan dikombinasikan dengan efek rotasi di seputar *centroid* lokal dengan efek rotasi operasi rotasi affine objek tersebut terhadap *centroid* absolut, sehingga menghasilkan hasil animasi berbentuk jejak orbit objek yang menarik untuk diobservasi. Jadi berdasarkan hibrida kedua metoda animasi, maka memungkinkan simulasi animasi yang menghasilkan jejak orbit sistem bintang ganda dapat dilakukan. Kedua objek bintang berotasi dengan arah rotasi yang sama tetapi berada pada posisi berlawanan seputar dua titik tetap berbeda, maka jejak orbit kedua objek tersebut berbentuk lonjong. Kedua objek dapat dikonstruksi secara efisien dengan teknik *cloning-scaling*. Secara umum hibrida metoda animasi dapat dimodelkan sebagai *framework* animasi.

Kata Kunci: *Model fraktal IFS, jejak orbit, hibrida metoda animasi, teknik cloning-scaling*

1. Introduction

In this paper, the discussion is related to and as the extension version of the previous research dealing with the orbital trajectory animation of two celestial bodies in IFS fractal model [1]. The animation simulation of the orbital trajectory of celestial bodies is important in Astronomy field in predicting and calculating the periodicity movement of celestial bodies. In the previous research version there is no metamorphical animation method used, so all the rotational effects of the satellite and the planet as the first and second celestial

bodies is generated by rotational operation based on affine transformation. In order to have the satellite rotated around the planet, its centroid should be shifted to and coincide with the centroid of the planet, so the shifting centroid technique is suitable to be implemented for simulating animation of the orbital trajectory of satellite around its planet, while the planet is rotating around a fixed point as an absolute centroid. In this paper the animation simulation of the first and second star-like objects in a twin stars system which has a unique rotational effect, the hybrid of metamorphical and non-

metamorphical animation methods is used and is suitable to be implemented for simulating animation of the orbital trajectory of two celestial objects which are rotating to each other. The mechanism of animation method will be explained later in the next paragraph and in the next corresponding section.

The twin stars which is also called as binary star or double stars as a system has an interesting behavior rotating to each other. The simulation of the twin stars rotating to each other as a system is interesting to be observed as a unique natural phenomena in the cosmos especially as a special two rigid bodies motion. The orbital trajectory of a twin stars system which has two formations as the closest and the farthest distances in certain direction, so forming an elliptical form [2-3]. To simulate the orbital trajectory in elliptical form, one star of the couple has a centroid shifted in some distance to a certain direction where the closest and farthest distances condition occurs periodically. To differentiate the stars in a system, the second star size of the twin stars is downsized into 75 percent of the first star by cloning-scaling technique [4]. There are two spiral objects of IFS fractal blended as a single object as a star representation. The second spiral object is 45 degrees rotated version of the first spiral object, so there is no empty space radially in the star body, but there is still small empty space in the center of body.

A spiral object as a component of star body is modelled by 4 small collage in North, South, West and East directions and one big collage in the middle rotated around 20 degrees clockwise or encountered clockwise of the others. To fill the small hole in the middle of star-like object, the last small collage is located in it, so the optimal model of star-like object consists of 10 collages. The first 8 small collages are located around the edge of object in 8 border directions, the biggest one is located in the middle but rotated 20 degrees encountered clockwise and the last one also located in the middle with the size just enough to cover the hole. To have pseudo-rotational effect of metamorphical animation between the source and target star-like objects, the target object is the 45 degrees rotated version of the source object. To have the rotation rate for example rotated in 3 degrees at a single transition event, then the number of transition image can be set in 15 transitions to cover 45 degrees range of the orientation difference between the source and target objects.

IFS Fractal

The IFS fractal is introduced for the first time by Michael Barnsley [5,6], after the term fractal itself coined for the first time by Benoit Mandelbrot as

the father of fractals as a new scientific field [7]. The main property of fractals is self-similarity which is revealed by Hutchinson [8] and inspired Mandelbrot to discover fractals. The self-similarity property of fractal can be generated by means of the collage theorem and affine transformation function in two or three dimension, so the function itself is called as self-affine [9]. The IFS code as the representation of IFS fractal is the collection of the affine coefficient functions [6,10]. To encode the IFS code to be a fractal object as an image, there are two types of IFS fractal generator algorithm, the deterministic and non-deterministic versions. The non-deterministic algorithm is called random iteration algorithm [5,11], because the iteration mechanism governed by random number that depends on the probability factor of each affine function. In this paper the non-deterministic version is used instead of the deterministic version.

Affine Transformation

The affine transformation as a formula can be used as a mapping function interrelate between a point positions from the past to the new current position iteratively and can be expressed in polar or Cartesian coordinate system. The conversion from the first to the second coordinate system is known as the inverse problem process of IFS fractal model or encoding phase from the design model of collage collection to the IFS code, which will be discussed in the two next sections. The expression of both affine functions form mathematically are expressed as equation(1) and equation (2) below. The **a** and **c** affine coefficients are calculated based on the dimension factor of collage in horizontal direction (**r**) and the angle deviation of collage from horizontal axis (**t**). The **b** and **d** affine coefficients are calculated based on the dimension factor of collage in vertical direction (**s**) and the angle deviation of collage (**u**) from vertical axis. The conversion between the two systems is expressed as equation(3-6) below.

$$w \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} * \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} e \\ f \end{bmatrix} \quad (1)$$

$$w \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} r * \cos t & -s * \sin u \\ r * \sin t & s * \cos u \end{bmatrix} * \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} e \\ f \end{bmatrix} \quad (2)$$

$$a = r * \cos t \quad (3)$$

$$b = -s * \sin u \quad (4)$$

$$c = r * \sin t \quad (5)$$

$$d = s * \cos u \quad (6)$$

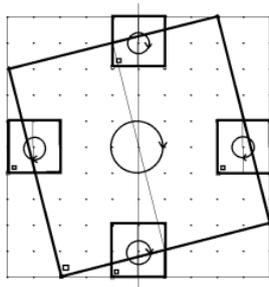


Figure 1. Four small collages in four borders around the big and -20 degree rotated collage in the middle as the example of collages composition of the spiral-like object in IFS fractal model.

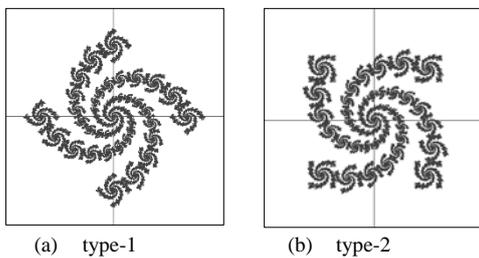


Figure 2. Spiral-like of IFS fractal objects as the result of decoding IFS code in Table I & II.

The **a** and **d** affine coefficients are the representation factors influencing the position values of object component each in horizontal and vertical directions respectively based on the previous position values in each directions. The **b** affine coefficient is the representation factor from the previous vertical position of object component influencing its current horizontal position. Contrarily the **c** affine coefficient is the representation factor from the previous horizontal position of object component influencing its current vertical position. The **e** and **f** affine coefficients are the representation factors of scalar in horizontal and vertical directions from the centroid of object respectively. All of affine coefficients as a self-affine function is the representation of affine transformation of object component which determines the form, position and orientation of the object component and the number of self-affine function is describing the number of object components.

Collage Theorem

According to the collage theorem, each collage in the collection of collages influences the others in such a way iteratively, so the appearance of component as part of object and as the representation of a collage is similar to the appearance of the whole object as the representation or the result of the composing collages, but in small form fashion. As an example of composing collages which has the

TABLE 1
AFFINE COEFFICIENTS OF SPIRAL-LIKE FRACTAL OBJECTS
TYPE-1

a	b	c	d	e	f	p
0.200	0.0	0.0	0.200	0.385	0.0	0.12
0.200	0.0	0.0	0.200	-0.385	0.0	0.12
0.200	0.0	0.0	0.200	0.0	0.385	0.12
0.200	0.0	0.0	0.200	0.0	-0.385	0.12
0.799	0.29	-0.29	0.799	0.0	0.0	0.52

TABLE 2
AFFINE COEFFICIENTS OF SPIRAL-LIKE FRACTAL OBJECTS
TYPE-2

a	b	c	d	e	f	p
0.200	0.0	0.0	0.200	0.275	-0.275	0.12
0.200	0.0	0.0	0.200	-0.275	0.275	0.12
0.200	0.0	0.0	0.200	0.275	0.275	0.12
0.200	0.0	0.0	0.200	-0.275	-0.275	0.12
0.799	0.29	-0.29	0.799	0.0	0.0	0.52

correlation with the fractal object used and has already discussed in the previous section, there are four small collages in the left, right, top and bottom positions of the collage composition and another big collage rotated encounter clockwise in the middle as illustrated in Figure 1. All collage is designed by positioning 4 points clockwise start from left-bottom corner.

IFS Code

As already explained in the previous section the affine coefficients as a single self-affine function as the object component representation is coded as one row of IFS code, so the IFS code is the representation of the IFS fractal object. Table I shows the IFS code example correlated to and as the representation of fractal object as the result of composing collage illustrated in Figure 1 above. The **a** to **f** factors in the first row down to the fourth row in the table are the representation of collage-1 to 4 respectively and the last row is the representation of the big collage in the Figure 1 above. The affine coefficients in Table II is the -45 degrees rotated version of the affine coefficients in Table I. The **p** factor in each row of self-affine function is the probability factor calculated from the size area of each collage relative to the size area of collages composition as a whole.

Algorithm of Fractal Object Construction

The fractal object can be constructed by means of non-deterministic algorithm of fractal object construction based on random numbers, so the algorithm is called as random iteration algorithm. The three IFS code examples in the Table 1 and 2 can be encoded to be the fractal object images as illustrated in Figure 2 (a) and (b).

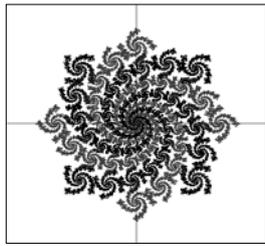


Figure 3. Two spiral-like of IFS fractal objects (type-1 and 2) as a multi-object of star-like (blended version of the two spiral-like objects).

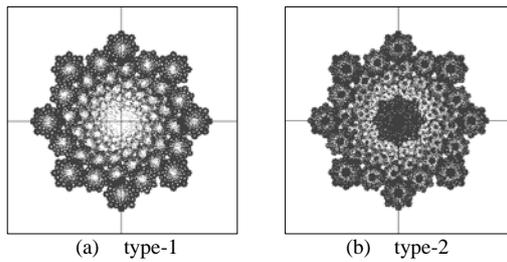


Figure 4. Star-like of IFS fractal objectst as the result of decoding IFS code in Table III & IV.

Multi-object of Fractal

The composition of objects of fractal can be generated by the modified version of the construction algorithm of fractal object based on the partitioned-random number generation method. Each fractal object construction is build based on each partitioned-random numbers. So the modified algorithm is called as the partitioned-random iteration algorithm [12]. The range of partitioned random numbers is set relatively based on the size area of each object relative to the size of multi-object as a whole area. As an example Figure 3 below illustrate the appearance of the multi-object which consists of the two spiral-like objects (type-1 and 2 in Figure 2 (a) and (b)) blended as the representation of a single star-like fractal image.

IFS code Modification Approach

There is another way to build the IFS code rather than encoded from the collage composition, such as just merging two IFS code sets to a new IFS code set. For example the IFS code of spiral-like object type-2 (Table 2) is merged into IFS code of spiral-like type-1 (Table 1) becoming the new IFS code of star-like object as described in Table 3 below, except the last row is the representation of both of the last rows in Table 1 and 2. Table 4 consists of affine coefficients as the modification of affine coefficients in Table 3 by adding one row on top as a new affine function derived from the last row by increasing the value of **a** and **d**

TABLE 3
AFFINE COEFFICIENTS OF STAR-LIKE FRACTAL OBJECTS
TYPE-1

a	b	c	d	e	f	p
0.200	0.0	0.0	0.200	0.385	0.0	0.06
0.200	0.0	0.0	0.200	-.385	0.0	0.06
0.200	0.0	0.0	0.200	0.0	0.385	0.06
0.200	0.0	0.0	0.200	0.0	-.385	0.06
0.200	0.0	0.0	0.200	0.275	-.275	0.06
0.200	0.0	0.0	0.200	-.275	0.275	0.06
0.200	0.0	0.0	0.200	0.275	0.275	0.06
0.200	0.0	0.0	0.200	-.275	-.275	0.06
0.799	0.291	-.29	0.799	0.0	0.0	0.52

TABLE 4
AFFINE COEFFICIENTS OF STAR-LIKE FRACTAL OBJECTS
TYPE-2

a	b	c	d	e	f	p
0.300	0.0	0.0	0.300	0.0	0.0	0.08
0.200	0.0	0.0	0.200	0.385	0.0	0.05
0.200	0.0	0.0	0.200	-.385	0.0	0.05
0.200	0.0	0.0	0.200	0.0	0.385	0.05
0.200	0.0	0.0	0.200	0.0	-.385	0.05
0.200	0.0	0.0	0.200	0.275	-.275	0.05
0.200	0.0	0.0	0.200	-.275	0.275	0.05
0.200	0.0	0.0	0.200	0.275	0.275	0.05
0.200	0.0	0.0	0.200	-.275	-.275	0.05
0.799	0.291	-.29	0.799	0.0	0.0	0.52

coefficients 50% and normalizing **e** coefficient to zero, so the top row is becoming the representation of a new component located in the middle of object which has the dimension size 150% of the other 8 small components. All of **p** factors values is normalized so its total value is still 1.0.

The appearance of objects as the decoded result of the two star-like object which have the IFS code as described in Table 3 and 4 is illustrated in Figure 4 (a) and (b). The first object image in Figure 4 (a) still has a hole in the middle of body object, but there is no hole in the middle of object in Figure 4 (b). The second object is used as the star-like objects in the simulation of orbital trajectory of stars in the twin stars system discussed in this paper.

2. Methods

There are two major animation methods, either the non-metamorphical or the metamorphical animation method which can be applied to fractal objects or multi-objects [13]. In this paper animation discussion is limited to the rotational animation in hybrid of both types of animation method [12] and in general an animation frame-work can be modelled as the general solution of fractal animation model including both types of animation method.

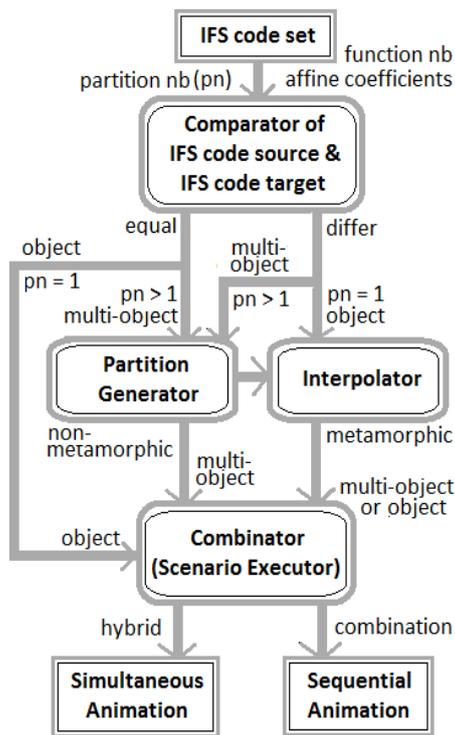


Figure 5. Animation framework in IFS fractal model.

Non-metamorphical Animation Method

In non-metamorphical animation method, there are many primitive affine transformation which can be used in procedures of animation scenario. One as the most correlated kind of affine transformation to the topic of this paper is the rotational operation of affine function to be used in rotating the star-like objects clockwise or encountered clockwise around a fix point [6]. The rotational procedure in non metamorphical animation affecting all of the affine coefficients as described in 6 equations(7-11) which depend on the small angle de-iation dt .

Metamorphical Animation Method

In metamorphical animation method on IFS fractal model, there are at least two different objects as the source and as the target objects. Ideally both objects is in the same family with the same number of affine function, so there is no need to insert a dummy affine function to accomplish interpolation process between the pairs of the same affine coefficients of the two objects. To have the natural effect of the transition images in between the two objects such as in growing and weaving effects of tree-like objects, usually the target object is the modification version of the source obje-

ct in such a way. A special rotational effect of metamorphical animation version which is relevant to be discussed in this paper can be obtained with a special preparation and it will be explained soon in the next section.

Rotational Animation

As it has already been explained in the two previous sections, there are two kinds of rotational animation. The rotational animation of non-metamorphical method can be used in rotating each star-like object of the twin stars system around its fixed point. While both star-like objects are rotating around its fixed point, each star-like object is also rotating around its local centroid in the middle of each object. To accomplish rotational animation which is not depended to the fixed point as an absolute centroid, metamorphical animation method can be used [4]. As an example which is used in this paper, the star-like objects in Figure 4 has similarity if it is rotated by 45 degrees clockwise or encountered clockwise, so the target object should be as the modification version of the source object as rotated by 45 or -45 degrees. As it has already been explained in the introduction section to have the angular rate of rotation in 3 degrees between two transition images, so the number of transition should be in 15 to reach 45 degrees cycle rate.

$$a' = a * \cos(dt) * \cos(dt) - (b + c) * \cos(dt) * \sin(dt) + d * \sin(dt) * \sin(dt) \quad (7)$$

$$b' = (a - d) * \cos(dt) * \sin(dt) + b * \cos(dt) * \cos(dt) - c * \sin(dt) * \sin(dt) \quad (8)$$

$$c' = (a - d) * \cos(dt) * \sin(dt) - b * \sin(dt) * \sin(dt) + c * \cos(dt) * \cos(dt) \quad (9)$$

$$d' = a * \sin(dt) * \sin(dt) + (b + c) * \cos(dt) * \sin(dt) + d * \cos(dt) * \cos(dt) \quad (4.d)$$

$$e' = e * \cos(dt) - f * \sin(dt) \quad (10)$$

$$f' = e * \sin(dt) + f * \cos(dt) \quad (11)$$

Animation Framework

To generalize the animation in IFS fractal model, an animation framework as a model can be modeled in which both non metamorphical and metamorphical animation methods on objects or multi-objects of fractal are accommodated. The anima-

tion framework can differentiate object type between object or multi-object of fractal based on number of partition to be generated by partition generator. The animation framework can differentiate animation method type between non metamorphical or metamorphical animation method based on the equality of source and target of IFS code set. The animation framework can differentiate the timing of animation scenario between non metamorphical and metamorphical animations accomplished sequentially or simultaneously based on the animation method approach.

There are two approaches, the combination or hybrid of non metamorphical and metamorphical animations. In this paper the hybrid animation approach is chosen, so the pseudo-rotational effect of metamorphical animation on star-like object is occurred around its local centroid and the real rotational effect of non metamorphical animation on the same star-like object is also occurred around a fixed point as the absolute centroid simultaneously. The animation framework model itself can be seen in the Figure 5.

3. Results and Analysis

In this paper there are two kinds of animation techniques discussed, the last one is the modification of the first one. The first animation technique is cloning technique. By this kind of animation technique, any cloning objects can be redrawn by shifting each local centroid to anywhere. The second animation technique combining cloning technique and scaling technique, so the size of the cloning object can be bigger or smaller than the original one [4]. The animation technique used in this paper is the last animation technique. The second star of the twin stars system is located somewhere to the South of the first star location and the size is scaled down to 75 percent of the size of the original one.

There are two major scenarios of animation can be delivered by the animation framework that leads to the simultaneous animation or sequential animation as discussed in the previous section. In this paper, there are two kinds of animation on twin stars system in which each individual star is orbiting to each other can be animated resulting spin and rotational effects simultaneously by means of the hybrid animation approach. Each star always is spinning in clockwise direction and rotating around each absolute centroid either in clockwise direction or in encountered clockwise direction. Additionally there are two other animations, the first animation is using metamorphical animation method and the other is using only non metamorphical animation method. All of animations are simulated and the results are displayed in

the next section. In this section there are two animation simulations as the introductory and there are two other animation simulations as the main simulation.

Pseudo-rotational Effect

In the first introductory simulation of animation, each star is only rotating around or spinning in each its local centroid by means of metamorphical animation method as can be seen in Figure 6 (appendix). Because of both stars as a system and the second star as a cloning version of the first one is animated by the same animation method, so both stars has an identical behavior as can be observed and indicated by its arrow as the indicator of rotational effect.

Stationary Mode

In the second introductory simulation of animation, each star as rigid body is only rotating around each its fixed point in stationary mode by means of non metamorphical animation method as can be seen in Figure 7 below (appendix). The arrows indicate the half surface of each star is always facing to its fixed point as the absolute centroid differently, so the orbital trajectory forms an elliptical path, which has two conditions of the farthest and closest distances between the two stars. The farthest distance condition occurs when the first as the biggest star is at the Northern position, and the closest distance condition occurs when the second as the smallest star is at the Northern position in turn periodically every 180 degrees rotated encountered clockwise.

Non-stationary Mode

In the first simulation of the main simulation as the simulation of orbital trajectory animation on the twin stars system is in non-stationary mode. The angular direction on rotational effect of both metamorphical and non metamorphical animations are the same (in clockwise direction), as it can be seen in Figure 8 below (appendix). Similar to the previous simulation discussed in the previous section, an elliptical path of orbital trajectory of the twin stars system is formed, so the distance between the two stars is varied in periodical situation. In this simulation the farthest distance condition occurs when the first as the biggest star is at the Northern position, and the closest distance condition occurs when the second as the smallest star is at the Northern position in turn periodically every 180 degrees rotated in clock-wise direction. Because of the angular direction of the spin and the rotation around fixed point is in the sa-

me directional condition, so the angular rate of pseudo-rotational effect of metamorphical animation is affected and as a whole the effect is looked like in the fasting up fashion in clockwise direction.

In the second simulation of the main simulation as the simulation of orbital trajectory animation on the twin stars system is in non-stationary mode. The angular direction on rotational effect of metamorphical and non-metamorphical animations are different. The angular direction on rotational effect of metamorphical animation is in clockwise direction, but the angular direction on rotational effect of non metamorphical animation is in encountered clock-wise direction, as it can be seen in Figure 9 below (appendix). Similar to the previous simulation discussed above, an elliptical path of orbital trajectory of the twin stars system is formed, so the distance between the two stars is also varied in periodical situation. In this simulation the farthest distance condition occurs when the first as the biggest star is at the Northern position, and the closest distance condition occurs when the second as the smallest star is at the Northern position in turn periodically every 180 degrees but rotated in encountered clockwise direction. Because of the angular direction of the spin and the rotation around fixed point is in the opposite directional condition, so the angular rate of pseudo-rotational effect of metamorphical animation is affected and as a whole the effect is looked like in the slowing down fashion compared to the first simulation (Figure 8 in appendix) and still rotated in clockwise direction.

4. Conclusion

The first conclusion is the orbital trajectory simulation on twin stars system in IFS fractal model can be realized by hybrid animation method and an interesting elliptical path with the farthest and closest distances between the two stars which is changed periodically can be exhibited.

The second conclusion is the spin effect of the stars in the twin stars system as a whole can be made apparently in slowing down or fasting up depends on the angular direction of the spin and rotation is in the same or in the opposite direction.

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Appendix

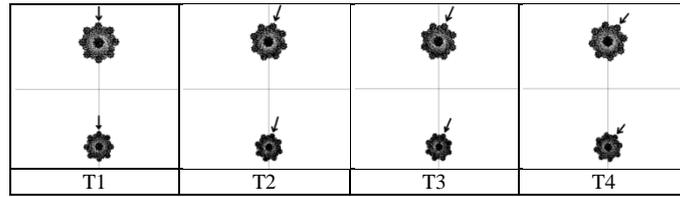


Figure 6. Pseudo-rotational effect exhibited by metamorphical animation on twin stars as rotated around its local centroid synchronously (see arrows).

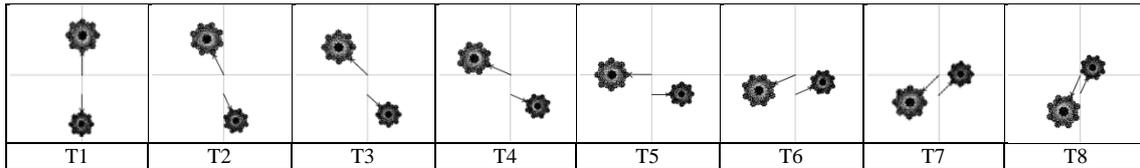


Figure 7. Elliptical path of orbital trajectory simulation on twin stars as two rigid bodies each rotated around different absolute centroid in stationary mode based on rotational operation of non metamorphical animation method.

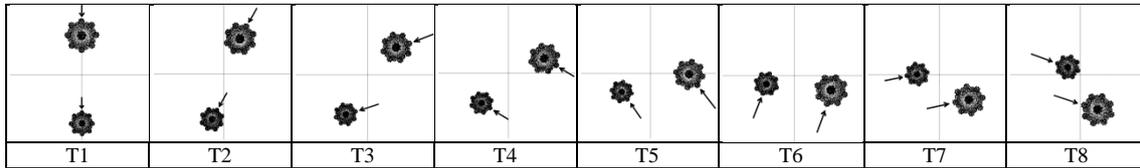


Figure 8. Elliptical path of orbital trajectory simulation on twin stars system rotating clockwise based on hybrid of non metamorphical and metamorphical animation methods.

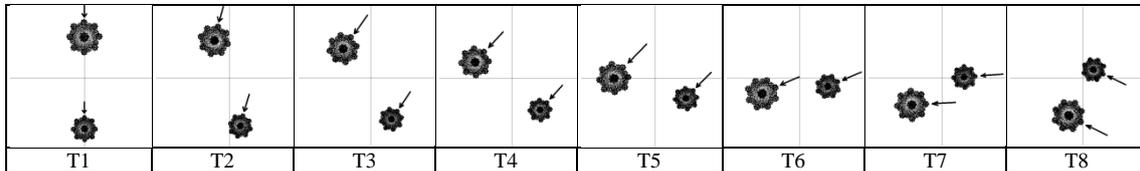


Figure 9. Elliptical path of orbital trajectory simulation on twin stars system rotating encountered clockwise based on hybrid of non metamorphical and metamorphical animation methods.