Evaluation of Trajectory Model for Differential Steering Mobile Robot Using Numerical Method

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Conversion of position and movement of Differential steering Mobile robots into the field of cartesian coordinates are one of the problems in the field of robotics, Given in the reality that there is no coordinate anywhere to determine a position Mobile robots are in motion. Trajectory model need to be develop in order to convert the velocity of right and left wheels to a cartesian coordinate. The model was driven from basic equation differential drive and using a forward kinematics. The model then evaluated using a numerical method (Simpson's Rule). The result shows that the model gives the best result when converting the θ.

Keywords—Differential Driven Mobile Robot; Kinematic; Simpson's Rules; Trajectory;

I. Introduction

Various types of robot have been developed especially in the field of mobile robot [1]. Mobile robot is a moving robot, which is generally using legs or wheels for the movement system [2]. Based on its driving types mobile robot divided into various types which are Differential Drive, Tricycle Steering, Omnidirectional, Synchro Drive, Ackerman steering, and Skid Steering [3]. One of mobile robot types is differentially Mobile Robot Drive, a mobile robot that uses two independent wheels, so that the translation movement and rotation is resulted from a movement combination of two actuators. In order to stabilize the robot, an Omnidirectional wheel (castor wheel) is added [4].

In its movement, mobile robots require a control system so that it can move to the desired goal. A differential mobile robot only use the speed of its right and left wheel. The movements are able to go forward and turn, but unable to move in any direction. This type of robot called Nonholonomic robot [5].

Many researches on nonholonomic control system has been done and its refer to a kinematic and dynamic control model. Kinematic control model assumed that mass and inertia from robot and wheels can be ignored. Typically, input for kinematic model control is speed.

Numerical methods are techniques used to formulated a mathematical problem so that it can be solved with an arithmetics operators [6]. Numerical method also used for generating an approximation solution. This paper will develop a kinematic model for differential mobile robot. The model will be tested using numerical method.

II. Literature Rivew

A. Mobile Robot

Mobile robot is a robot that can move itself from one place to another. This robot is the most popular robot in the field of robotics research. In terms of benefits, the robot is expected to help humans in transport automatization, platform movement for industrial robots, unmanned exploration, and many more. Some examples of mobile robot include, the Robot Line Tracker, Flying Robot, Under Water Robot [3].

B. Differential Drive

Based on its movements, mobile robot is divided into some types and one of them is differential drive. Differential drive is a steering type which consist of two fixed wheels mounted into left side and right side of the robot platform. Both wheels are driven independently. One or two passive castor wheels are used for balance and stability.

Differential drives are a simple mechanical controller because they do not require axis rotation. If the two wheels are spinning at the same speed, the robot moves straight forward or backward. If one of the wheels runs faster than the other, the robot follows a curved path along a momentous circular arc. If the two wheels are spinning at the same speed but in the opposite direction, the robot turns around the point middle of two drive wheels.

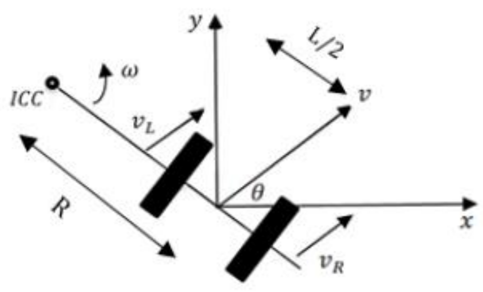


Fig. 1. Differential drive kinematic

Instantaneous Center of Curvature (ICC) from differential drive mobile robot is located at the cross point of all wheel axis. ICC is the center of circle with radius (R) depending on the speed of the two wheels (Fig.1). Radius (R) determined by the folling equation (1) and (2).

$V_R(t) = \omega(t)R + \omega(t)\ L/2$ (1)

$V_L(t) = \omega(t)R - \omega(t)\ L/2$ (2)

By adding the two equations then radius (R) is obtained (equation 3).

$R = L(V_R(t) + V_L(t))\ /\ 2(V_R(t) - V_L(t))$ (3)

C. *Nonholonomic Movement*

Based on kinematic model, robot is divided into two groups namely holonomic and nonholonomic movement. Nonholonomic model is a type of robot that that each of its wheel can move freely. Example of this is wheel chair. Mobile robot with nonholonomic systems cannot move into right or left position without doing a maneuver (forward or backward with a turn). Example of nonholonomic movement in robotic is a robot with two movement wheels and one castor as a stabilizer. Figure 2 is an illustration of such robot.

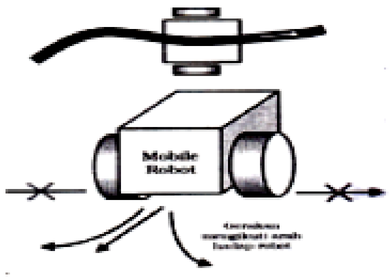


Fig. 2. Nonholonomic mobile robot.

D. *Kinematics*

Kinematics is the study of robot movement regardless of style or other factors that affect the movement of the robot. In kinematics analysis, position, speed and acceleration and all links are calculated regardless of the force that caused the movement. Robotic kinematics is generally divided into two, i.e. forward kinematic and inverse kinematic.

1) Forward Kineamtic

Forward kinematic is a kinematic analysis to obtain the position coordinates (*x, y, z*) if radius r and θ from robot structure is known. If θ is a time-based function, θ(t), then position and orientation of P (t) can be calculated as well. Forward kinematic analysis is relatively simple and easy to implement.

2) Inverse Kinematic

Inverse kinematic is a kinematic analysis to get angle of each joint if coordinate position of data (*x, y, z*) is known. If Position and orientation P (t), a time-based function, is known, it can be calculated as well.

III. MODELING

A. *Mobile Robot Kinematic Modeling*

In this paper will discuss a differential mobile robot which has a right and left wheel. Each wheel driven individually and assumed it moves in a horizontal field. On fig. 2 is illustrated a robot that moves in 2D area in cartesian coordinate (x, y).

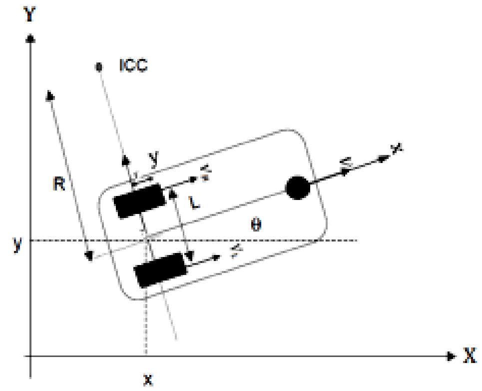


Fig. 3. Mobile Robot wih a Differential Driver in a cartesian field

Parameters in fig. 3 are:

$V_R(t)$: Right wheel linier velocity

$V_L(t)$: Left wheel linier velocity

$\omega_R(t)$: Right wheel angular velocity

$\omega_L(t)$: Left wheel angular velocity

r : Radius of each wheels

L : Distance between wheels

R : Instantaneous Curvature Radius relative from robot trajectory

ICC : Instantaneous Center of Curvature

R+(1/2) : Curvature Radius in right wheel trajectory

R-(1/2) : Curvature Radius in left wheel trajectory

Kinematic general equation for mobile robot differential drive is shwon in eq. 3.

$$\begin{pmatrix} \dot{x}(t) \\ \dot{y}(t) \\ \dot{\theta}(t) \end{pmatrix} = T_{NH}(q)\, u(t) \tag{3}$$

Where ($\dot{x}$, $\dot{y}$, $\ddot{o}$)^T is the first derivation of robot position. T_{NH} is a matrix transformation forn nonholonomic robot. *u*(t) is matrix forward kinematic to approximate position a velocity of robot in cartesian. q is a general coordinate robot.

From eq. 3 we can get inverse kinematic as follows.

$$u(t) = {T_{NH}}^{-1}(q)\dot{q}(t) \tag{4}$$

From fig.3 it is assumed that robot moves in cartesian coordinate (*x,y*). Then we get the linear velocity for each wheels, shown at eq 5.a and 5.b.

$$v_R(t) = \omega(t)\left(R + \frac{L}{2}\right)$$
$$v_R(t) = \omega(t)R + \omega(t)\frac{L}{2} \qquad (5.a)$$

$$v_L(t) = \omega(t)\left(R - \frac{L}{2}\right)$$
$$v_L(t) = \omega(t)R - \omega(t)\frac{L}{2} \qquad (5.b)$$

If eq. 5.a and 5.b is subtracted, then anagular velocity robot is obtained ω(t) (eq. 6).

$$v_R(t) = \omega(t)R + \omega(t)\frac{L}{2}$$
$$v_L(t) = \omega(t)R - \omega(t)\frac{L}{2} \quad -$$
$$v_R(t) - v_L(t) = \frac{2\omega(t)L}{2}$$
$$\omega(t) = \frac{1}{L}(v_R(t) - v_L(t)) \qquad (6)$$

If eq. 5.a and 5.b is added and subtituted into eq. 6 then instantaneous curvature radius relative from robot trajectory is obtained R (eq. 7).

$$v_R(t) = \omega(t)R + \omega(t)\frac{L}{2}$$
$$v_L(t) = \omega(t)R - \omega(t)\frac{L}{2} \quad +$$
$$v_R(t) + v_L(t) = 2\omega(t)R$$
$$R = \frac{v_R(t)+v_L(t)}{2\omega(t)}$$
$$R = \frac{v_R(t)+v_L(t)}{2\frac{v_R(t)-v_L(t)}{L}}$$
$$R = \frac{L(v_R(t)+v_L(t))}{2(v_R(t)-v_L(t))} \qquad (7)$$

And if eq. 6 and eq 7 is multiply then linear velocity is obtained (eq. 8)

$$v(t) = R.\omega(t)$$
$$v(t) = \frac{L(v_L(t)+v_R(t))}{2((v_L(t)-v_R(t))}.\frac{1}{L}(v_R(t) - v_L(t))$$
$$v(t) = \frac{1}{2}(v_R(t) + v_L(t)) \qquad (8)$$

Whereas, to obtain velocity equation which seen from its axle, with assuming no slip, The encounter between the wheel and the floor is considered as point P in the fig. 4.

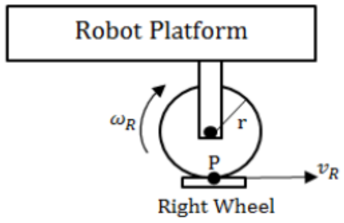


Fig. 4. Wheel velocity from its axle.

S, velocity of right and left wheel which seen from its axis is obtaiend from transformation of each velocity (eq. 9.a and 9.b).

$$v_R(t) = r\omega_R(t) \qquad (9.a)$$

$$v_L(t) = r\omega_L(t) \qquad (9.b)$$

B. *Nonholomonic Constraint*

Mobile robot as shown in Figure 3.1 consists of two driver wheels mounted equally on the rear axle and the free wheels in front, the robot is not able to move in any direction or commonly called a nonholonomic robot. The classical problem in the mobile kinematics control of a differential-driven robot is that it has two actuators but more than two control parameters. Characteristics of nonholonomic constraint (pfaffian constraints) that must be met are:

$$A(q)\,\dot{q} = 0 \qquad (10)$$

Regardless of the analysis of free castor wheels, based on Figure 3.1 can be described as three general variables of the robot's initial position as eq. 12

$$q(t) = [x(t), y(t), \theta(t)]^T \qquad (12)$$

To find the constraint equations we have to observe the starting position of the wheel motion where the wheel is assumed not to slip as shown in Fig.5 below:

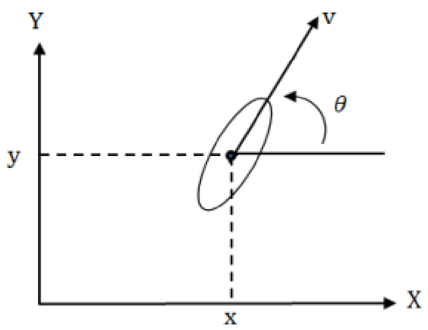


Fig. 5. Wheel's Kinematic Structure in Horizontal Field

From fig.5 the initial position of wheel in cartesian *x,y* are obtained with these equation (eq. 13) .

$$x(t) = v(t)\sin\theta(t)$$
$$y(t) = v(t)\cos\theta(t)$$
$$\theta(t) = v_\theta(t) \qquad (13)$$

The first derivation of initial position fro eq 13 is shown in eq 14.

$$\dot{x}(t) = v(t)cos\theta(t)$$
$$\dot{y}(t) = v(t)sin\theta(t)$$
$$\dot{\theta}(t) = \omega(t) \qquad (14)$$

The Cartesian coordinate velocity of axis field and can be calculated that is by looking at equation 15.

For,

$$\dot{x}(t) = v(t)cos\theta(t)$$
$$v(t) = \frac{\dot{x}(t)}{cos\theta(t)}$$

For,

$$\dot{y}(t) = v(t)sin\theta(t)$$
$$v(t) = \frac{\dot{y}(t)}{sin\theta(t)}$$

Then,

$$\frac{\dot{x}(t)}{cos\theta(t)} = \frac{\dot{y}(t)}{sin\theta(t)}$$
$$\dot{y}(t)\ cos\theta(t) = \dot{x}(t)\ sin\theta(t)$$
$$\dot{x}(t)\ sin\theta(t) - \dot{y}(t)\ cos\theta(t) = 0 \qquad (15.a)$$

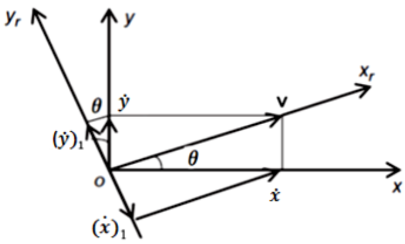


Fig. 6. Diagram illustration nonholonomic constraints[3]

Fig. 6 expresses the fact that the midpoint of the wheels moves along and the velocity along is zero. So, we get a speed equation at the center of the two wheels as follows,

$$v(t) = \dot{x}(t)\cos\theta(t) + \dot{y}(t)\sin\theta(t) \qquad (15.b)$$

Using equations of right wheel velocity (5.a) and left (5.b) then substituting equations (9.a), (9.b) and (15.b) into the equations of right and left wheel velocities, we get the constraint equation of right wheel (15.c) and left wheel (15.d).

$$\dot{x}(t)\cos\theta(t) + \dot{y}(t)\sin\theta(t) + \omega(t)\frac{L}{2} - r\omega_R(t) = 0 \qquad (15.c)$$

$$\dot{x}(t)\cos\theta(t) + \dot{y}(t)\sin\theta(t) - \omega(t)\frac{L}{2} - r\omega_L(t) = 0 \qquad (15.d)$$

From the constraint equation found in equation (15.a), (15.c) and (15.d) then can be rewritten in equation 16.

$$A(q) = \begin{bmatrix} \sin\theta(t) & -\cos\theta(t) & 0 & 0 & 0 \\ \cos\theta(t) & \sin\theta(t) & \frac{1}{2}L & -r & 0 \\ \cos\theta(t) & \sin\theta(t) & -\frac{1}{2}L & 0 & -r \end{bmatrix} \qquad (16)$$

The nonholonomic transformation of the starting position of the differential-driven mobile robot can be determined by substitution of equation 8, the robot velocity equation, into equation (13), the first derivative equation of the starting position of the wheel. Thus the nonholonomic transformation of the starting position of the differential-driven mobile robot is represented by the following equations (17.a) to (17.e).

$$\dot{q}(t) = (\dot{x}(t), \dot{y}(t), \dot{\theta}(t))^T \qquad (17.a)$$

$$\dot{x}(t) = v(t)\cos\theta(t)$$
$$\dot{y}(t) = v(t)\sin\theta(t)$$
$$\dot{\theta}(t) = \frac{v_R(t) - v_L(t)}{L} \qquad (17.b)$$

Then we obtained the equation of nonholonomic transformation for the starting position of differential drive mobile robot as follows

$$\dot{x}(t) = v(t)\cos\theta(t)$$
$$\dot{x}(t) = \frac{1}{2}(v_R(t) + v_L(t))\cos\theta(t)$$
$$\dot{x}(t) = \frac{v_R(t)\cos\theta(t) + v_L(t)\cos\theta(t)}{2}$$
$$\dot{x}(t) = \frac{r\omega_R(t)\cos\theta(t) + r\omega_L(t)\cos\theta(t)}{2}$$
$$\dot{x}(t) = \frac{r}{2}\omega_R(t)\cos\theta(t) + \frac{r}{2}\omega_L(t)\cos\theta(t) \qquad (17.c)$$

$$\dot{y}(t) = v(t)\sin\theta(t)$$
$$\dot{y}(t) = \frac{1}{2}(v_R(t) + v_L(t))\sin\theta(t)$$
$$\dot{y}(t) = \frac{v_R(t)\sin\theta(t) + v_L(t)\sin\theta(t)}{2}$$
$$\dot{y}(t) = \frac{r\omega_R(t)\sin\theta(t) + r\omega_L(t)\sin\theta(t)}{2}$$
$$\dot{y}(t) = \frac{r}{2}\omega_R(t)\sin\theta(t) + \frac{r}{2}\omega_L(t)\sin\theta(t) \qquad (17.d)$$

$$\dot{\theta}(t) = \frac{v_R(t) - v_L(t)}{L}$$
$$\dot{\theta}(t) = \frac{r\omega_R(t) - r\omega_L(t)}{L}$$
$$\dot{\theta}(t) = \frac{r}{L}\omega_R(t) - \frac{r}{L}\omega_L(t) \qquad (17.e)$$

The nonholonomic transforms contained in equations (17.c) to (17.e) can be rewritten in the following matrix form.

$$T_{NH}(q) = \begin{bmatrix} \frac{r}{2}\cos\theta(t) & \frac{r}{2}\cos\theta(t) \\ \frac{r}{2}\sin\theta(t) & \frac{r}{2}\sin\theta(t) \\ \frac{r}{L} & -\frac{r}{L} \end{bmatrix} \qquad (18)$$

If we assume the angular velocity in the mobile kinematic equations of a differential-driven robot as the robot input as follows,

$$u_1(t) = \omega_R(t)$$
$$u_2(t) = \omega_L(t) \qquad (19.a)$$

Then we get the kinematic model of differential drive mobile robot as follows,

$$\begin{bmatrix} \dot{x}(t) \\ \dot{y}(t) \\ \dot{\theta}(t) \end{bmatrix} = \begin{bmatrix} \frac{r}{2}\cos\theta(t) & \frac{r}{2}\cos\theta(t) \\ \frac{r}{2}\sin\theta(t) & \frac{r}{2}\sin\theta(t) \\ \frac{r}{L} & -\frac{r}{L} \end{bmatrix} \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix} \qquad (19.b)$$

To obtain the robot input equation, we substitute equations (9.a) and (9.b) into the equations of the right and left wheels contained in equations (5.a) and (5.b). The input equation is show in equations 19.c and 19.d

$$u_1(t) = \frac{1}{r}v(t) + \omega(t)\frac{L}{2r} \qquad (19.c)$$

$$u_2(t) = \frac{1}{r}v(t) - \omega(t)\frac{L}{2r} \qquad (19.d)$$

The robotic input equation in equation (19.c) and (19.d) if rewritten in matrix form is as follows,

$$\begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix} = \begin{bmatrix} \frac{1}{r} & \frac{L}{2r} \\ \frac{1}{r} & -\frac{L}{2r} \end{bmatrix} \begin{bmatrix} v(t) \\ \omega(t) \end{bmatrix} \qquad (20)$$

If we substitute the robot input in equation (20) into equation (19.b) we find eq. (21) as follows,

$$\begin{bmatrix} \dot{x}(t) \\ \dot{y}(t) \\ \dot{\theta}(t) \end{bmatrix} = \begin{bmatrix} \cos\theta(t) & 0 \\ \sin\theta(t) & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v(t) \\ \omega(t) \end{bmatrix} \qquad (21)$$

Equation (21) can be rewritten in the following equation,

$$\begin{bmatrix} \dot{x}(t) \\ \dot{y}(t) \\ \dot{\theta}(t) \end{bmatrix} = \begin{bmatrix} v(t)\cos\theta(t) \\ v(t)\sin\theta(t) \\ \omega(t) \end{bmatrix} \qquad (22)$$

By substituting equation (8) and equation (6) to equation (22), we obtained the differential kinematic mobile robot equation used to construct the model equation (23).

$$\begin{bmatrix} \dot{x}(t) \\ \dot{y}(t) \\ \dot{\theta}(t) \end{bmatrix} = \begin{bmatrix} \frac{1}{2}\cos\theta(t) & \frac{1}{2}\cos\theta(t) \\ \frac{1}{2}\sin\theta(t) & \frac{1}{2}\sin\theta(t) \\ \frac{r}{L} & -\frac{r}{L} \end{bmatrix} \begin{bmatrix} v_R(t) \\ v_L(t) \end{bmatrix} \qquad (23)$$

The model equatin then will be evaluated with a numerical method. Numerical method thas used to evaluate is simpson's rule (1/3 Simpson's Rule).

IV. Evaluation

A. *Evaluation Procedure*

The evaluation procedure as follows:

- Enter the specification of robot (r and L).
- Enter the initial position of robot (x, y).
- Enter the velocity of right and left wheels.
- Enter the simulation duration (in second).

B. Result

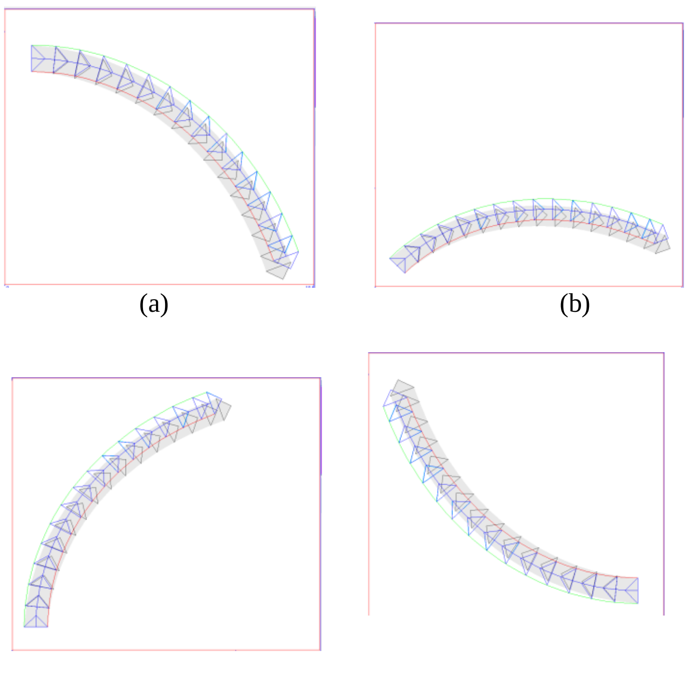
Specifications of Differential Drive Mobile Robot as follows:

r = 1 cm

L = 0.8

TABLE I. EVALUATION RESULT (INITIAL X = 0 AND Y = 0 H = 0.5)

θ	Var	Actual	Simpson's	Error
0.0	x	9.56	9.28	-0.29
	y	-7.42	-7.82	-0.40
	θ	–69.8	–69.8	0.0
45	x	12.01	12.09	0.08
	y	1.52	1.03	-0.49
	θ	-24.8	-24.8	0.0
90	x	7.42	7.82	0.40
	y	9.56	9.28	0.29
	θ	20.2	20.2	0.0
180	X	-9.56	9.28	-0.29
	y	7.42	7.82	0.40
	θ	110.2	110.2	0.0



(a) (b) (c) (d)

Fig. 7. Simulation Result (a) 0.0; (b) 45, (c) 90, (d) 180

V. CONCLUSION

The Trajectory model for Differential Drive Mobile Robot has been developed. The model was evaluate using Simpson Rule. The evaluation result shows that the θ model gives the best result with 0 error. Meanwhile model for x and y still need to be develop.

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PERNYATAAN

Dengan ini saya menyatakan bahwa makalah yang saya tulis ini adalah tulisan saya sendiri, bukan saduran, atau terjemahan dari makalah orang lain, dan bukan plagiasi.

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