A Number Theory Analysis on Variations of The Navigation Compass Puzzle in Honkai: Star Rail

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***Abstract*— Honkai: Star Rail is a free-to-play turn-based space fantasy RPG (Role-Playing Game) developed and published by Hoyoverse on April 26, 2023. Some of the main features of Honkai: Star Rail include collecting and upgrading characters and their equipment to successfully engage and win in a fight with some enemies. To get new characters, players must go through a gacha system called Warp that requires an in-game currency called Stellar Jades. Similarly, players need some other types of resources to upgrade their characters and equipment. One way of obtaining said resources is to solve puzzles that are available on many parts of the map. One of the available puzzles in Honkai: Star Rail is The Navigation Compass Puzzle. This paper investigates the mathematical model of variations of the navigation compass puzzle using number theory, develops some simple solvability checks, and develops a solver program for a variation of the puzzle. The findings reveal that certain variations of the puzzle are always solvable, while others are unsolvable or not yet determined.**

***Keywords*—Honkai: Star Rail, Modular arithmetic, Navigation compass puzzle, Number theory**

I. Introduction

Video games are a type of entertainment media that can be accessed and enjoyed by people of any age group, whether children, teenagers, or even adults. The act of playing video games, often called gaming, allows players to immerse themselves in a new world, adapt to different realities, strategize, and build their skills at their own pace. A healthy amount of playing video games might improve the player's motor skill, focus, multitasking and problem-solving skills, etc. Moreover, video games provide a space to improve the player's social skills by interacting with other players whether in-game or through the game's community [1].

Honkai: Star Rail is a free-to-play turn-based space fantasy RPG (Role-Playing Game) developed and published by Hoyoverse on multiple platforms, which are PC, PS5, and iOS/Android based platforms [2]. Belongs to the same company that produces other globally successful video games such as Honkai Impact 3rd, Genshin Impact, and Zenless Zone Zero, since its initial release on April 26, 2023, Honkai: Star Rail has 10M+ download count on Google Play Store and has won many awards, including Mobile Game of The Year at The Game Awards (TGA) 2023 [3] and Best Ongoing Awards in Google Play's best of 2024 [4].

Some of the main features of Honkai: Star Rail include collecting and upgrading characters and their equipment to successfully engage and win in a fight with some enemies. To get new characters, players must go through a gacha system called Warp that requires an in-game currency called Stellar Jades. Similarly, players need some other specific types of resources to upgrade their characters and equipment. Those resources are mainly obtained from going through the main story of the game, engaging in time-limited events, defeating enemies in a certain area of the map, and so on.

Another way to obtain said resources in-game is to solve puzzles available on many parts of the map. The navigation compass puzzle is a type of puzzle in Honkai: Star Rail that can be found on an area of the map called The Xianzhou Luofu [5]. The puzzle consists of some discs that can be rotated in some way so that the discs are all aligned in a way specified by the puzzle.

This paper aims to model variations of the navigation compass puzzle with number theory, mainly modular arithmetic, to then check whether said variations are always solvable or not, and whether some method of checking the solvability (often called the solvability check) of the puzzle can be developed. Furthermore, this paper will develop a navigation solver with a brute-force approach using the mathematical model.

II. Theoretical Framework

A. Number Theory and Modular Arithmetic

Number theory is a branch of pure mathematics that deals with the study of integers and functions of integers. One subdivision of number theory is modular arithmetic. Modular arithmetic, in its basic, is arithmetic done with a count that resets itself to zero for every time a certain whole number m greater than one, called the modulus, has been reached [6], [7].

B. The Modulo Operator

Given two integers a and m, $m > 0$, the operation a modulo m, written as $a \ mod \ m$, returns the remainder of a divided by m. It is formally written as

$$\begin{gathered} a \ mod \ m = r \\ a = mq + r \\ 0 \leq r < m \end{gathered} \tag{1}$$

where m is called the modulus. Notice that the result of the operation r will always be an element of the set of numbers from 0 to $m-1$ ($\{0,1,2,\ldots,m-1\}$) [6].

Another unique property of the modulo operator is that it can be used to represent odd and even integers. An odd integer a follows $a\ mod\ 2 = 1 \leftrightarrow a = 2q + 1$ while an even integer b follows $b\ mod\ 2 = 0 \leftrightarrow b = 2q, q \in Z$.

C. Congruence

Congruence of two integers is denoted by $a \equiv b\ (mod\ m)$, which read as a congruent to b in $mod\ m$. Given integers a, b, and m, $m > 0$, the congruence is defined as

$$a \equiv b\ (mod\ m)\ \leftrightarrow m|(a - b)$$
$$(m \text{ is divisible by } a - b) \quad (2)$$

The left-side equation can also written as follows

$$a = b + km, k \in Z \quad (3)$$

Suppose c is an integer and p is a positive integer, (x) has some properties as shown below.

$$(a + c) \equiv (b + c)\ (mod\ m) \ldots (4)$$
$$ac \equiv bc\ (mod\ m) \ldots (5)$$
$$a^p \equiv b^p\ (mod\ m) \ldots (6)\ [6]$$

A system of equations that is built around the conguence operator is called systems of linear congruence. It can also be written as follows

$$\begin{cases} x \equiv a_1\ (mod\ m_1) \\ x \equiv a_2 (mod\ m_2) \\ \vdots \\ x \equiv a_3\ (mod\ m_n)\ [8] \end{cases} \quad (7)$$

According to the Chinese Remainder Theorem, if every modulus in (x) is relatively prime such that $\gcd(m_i, m_j) = 1$ for $i \neq j$, the equation has unique solution in modulus $m = m_1 \times m_2 \times \ldots \times m_n$ [8].

D. Honkai: Star Rail

Honkai: Star Rail is a free-to-play, turn-based space fantasy RPG developed and published by Hoyoverse for PC, PS5, and iOS/Android platforms [2]. The game's core features revolve around collecting and upgrading characters and their equipment to successfully battle various enemies. Players acquire new characters through a gacha system called Warp, which uses an in-game currency known as Stellar Jades. Upgrading characters and equipment requires specific resources, which can be obtained by progressing through the main story, participating in time-limited events, defeating enemies in designated areas of the map, and more.

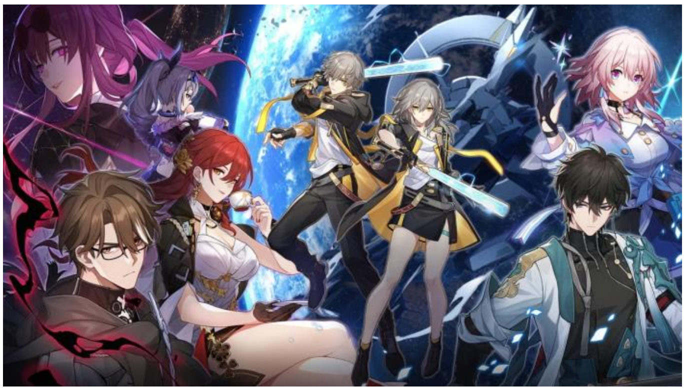

Figure 1. Honkai: Star Rail
source: overclockingid.com

An alternative way to obtain resources in the game is through solving puzzles available on various parts of the map. Every map has its own types of puzzles, such as the Unearthy Marvel puzzle in Herta Space Station, Magnetorheological Fluid Threshold puzzle in Jarilo-VI, Abacus Circuitry in The Xianzhou Luofu, and Dream Ticker in Penacony. Solving each puzzle may reward the players with resources such as equipments, experience points, Stellar Jades, local currency, and some level-up resources [5].

E. Navigation Compass Puzzle

The Navigation compass puzzle is a type of puzzle in Honkai: Star Rail that can be found on several locations on Xianzhou Luofu, an in-game location. Solving a navigation compass will reward the player with equipments from Musketeer of Wild Wheat or Thief of Shooting Meteor equipment sets, Trailblaze EXP (the game's experience points), Stellar Jades, Traveler's Guide (characters' level-up material), Lost Gold Fragment (equipment's level-up material), Strale (currency for shops in The Xianzhou Luofu), and Credit (a global in-game currency) [5].

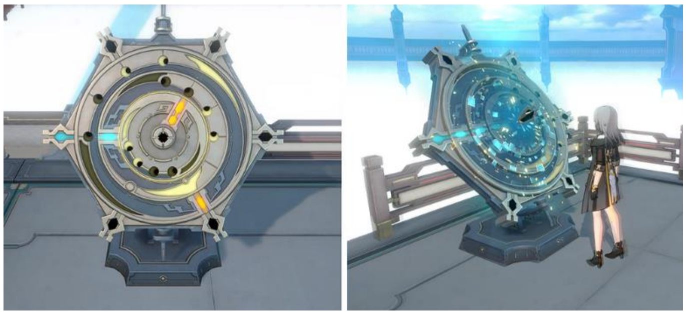

Figure 2. Unsolved (left) and Solved (right) Navigation Compass Puzzle
source: https://honkai-star-rail.fandom.com/wiki/Navigation_ Compass

The navigation compass puzzle consists of three discs that can be rotated with some rules and constraints. Each disc can only be rotated to certain degrees for each command given by the player. For example, the first disc might rotate 180°/command, the second disc might rotate 120°/command, and the third disc might rotate 60°/command. Each disc might rotate clockwise or counterclockwise for a given navigation compass. The navigation compass puzzle is solved if all orange arrows on each disc are aligned to the blue point on the outer part of the compass as shown above. When a disc is aligned with the blue point, the orange line will turn blue. A solved navigation compass will have three blue lines that are aligned with the blue point as shown in the right part of Fig. 2.

There are three types of navigation compass, which is easy, medium, and hard. In the easy mode, players can move each disc independently. In the medium mode, players can move the disc in pairs, with a single disc that can also move independently. In the hard mode, players can only move the disc in pairs [9].

Players in the community have tried to develop solutions to the navigation compass puzzle. The most common method is to provide solutions for every possible combination of the puzzle that is available in-game [10]. This is possible due to the puzzle being the same for every player and not generated randomly. A general idea to solve any solvable navigation compass puzzle has also been developed, though not written formally in formal mathematical notation [9].

III. Navigation Compass Terminologies

This section provides some relevant navigation compass' terminologies that are commonly used in this paper and their meaning. Note that the mentioned terminologies are not official nor universally used either by Hoyoverse or the Honkai: Star Rail community. The terminologies and their meaning are as follows:

1. Ring

   A ring refers to the rotating disc of the navigation compass.
2. M-Cycle Ring

   An m-cycle ring denotes how many moves a ring needs to go back to its initial state. For example, a 3-cycle ring which will rotate $120°$ for each movement will go back to its initial state in 3 moves since it will rotate $3 \times 120° = 360°$ (a full rotation).
3. N-Ring navigation compass

   An N-Ring navigation compass, or a navigation compass with N-Ring, denotes how many rings the mentioned navigation compass has. For example, a 3-ring navigation compass refers to a navigation compass that has three rings.
4. Move

   A move refers to a command made by the player to rotate one or several rings depending on the number of rings a navigation compass has and the current context of analysis. For example, in a 3-ring navigation compass, a move might mean to rotate a single ring or to rotate two rings together. When talking about rings, a move refers to the act of rotating said ring. Further clarification on this terminology, if needed, will be mentioned in each analysis.
5. Position and Intended Position

   The position of a ring in a navigation compass refers to how many moves away a ring is from its intended position. In an n-cycle ring, the intended position p of the ring is the position where the line on the ring is aligned with the outer blue point of the navigation compass. It follows the equation

   $$p \equiv 0 \ (mod\ n) \tag{8}$$

   For example, if a 6-cycle ring is in position 4, it means that it is 4 moves away from its intended position and it needs to move twice to go back to its intended position since $4 + 2 \equiv 0 \ (mod\ 6)$. This also means that a move will increase the position of the moved ring(s) by 1.

IV. Scope of Analysis

This paper will analyze the mathematical model for the navigation compass puzzle in Honkai: Star Rail, the properties of 1-ring navigation compass, 2-ring navigation compass, 3-ring navigation compass, and a general n-ring navigation compass. For navigation compasses with 2-ring and above, the analysis will be divided into several cases as follows:

1. 2-Ring Navigation Compass
   - Case 1: Each move will increase the position of a single ring in the navigation compass by 1 position
   - Case 2: Each move will increase the position of both rings in the navigation compass by 1 position
2. 3-Ring Navigation Compass
   - Case 1: Each move will increase the position of a single ring in the navigation compass by 1 position
   - Case 2: A move will either increase the position of a pair of rings by 1 position or increase the position of a single independent ring by 1 position
   - Case 3: A move will increase the position of a pair of rings by 1 position
3. N-Ring Navigation Compass
   - Case 1: Each move will increase the position of a single ring in the navigation compass by 1 position
   - Case 2: A move will either increase the position of a pair of rings by 1 position or increase the position of a single independent ring by 1 position. The navigation compass will have d number of independent rings, where $d < n$.
   - Case 3: A move will increase the position of a pair of rings by 1 position.

V. Navigation Compass Mathematical Model

A mathematical model is a mathematical representation of reality. Essentially, anything inside our reality is subject to analysis by mathematical models if it can be described by utilizing mathematical expressions [11]. The navigation compass puzzle can be modeled mathematically as shown below.

Suppose an m-cycle ring in position p has moved x times to intended position p'. The resulting position can be denoted as follows:

$$\begin{gathered} p' \equiv 0 \ (mod\ m) \\ p + x \equiv 0 \ (mod\ m) \end{gathered} \tag{9}$$

Furthermore, an n-ring navigation compass can be modeled as a system of equation in modular arithmetic. Suppose the i-th ring of an n-ring navigation compass is an m_i-cycle ring at position p_i and has moved x_i times to intended position $p_i{}'$. The resulting state of the navigation compass can be denoted as a system of equation as follows:

$$\begin{cases} p_1' \equiv 0 \ (mod\ m_1) \\ p_2' \equiv 0 \ (mod\ m_2) \\ \quad \vdots \\ p_n' \equiv 0 \ (mod\ m_n) \end{cases}$$

Or can also be written as

$$\begin{cases} p_1 + x_1 \equiv 0 \ (mod\ m_1) \\ p_2 + x_2 \equiv 0 \ (mod\ m_2) \\ \quad \vdots \\ p_n + x_n \equiv 0 \ (mod\ m_n) \end{cases}$$

(10)

For dealing with more advanced navigation compass, by utilizing (3), the system of equation can be broken down as shown below.

$$\begin{cases} p_1 + x_1 = m_1 k_1 \\ p_2 + x_2 = m_2 k_2 \\ \vdots \\ p_n + x_n = m_n k_n \end{cases} \tag{11}$$

For every navigation compass, $0 \le p_i < m_i, x_i \ge 0, m_i > 0$, and $p_i, x_i, m_i \in Z$.

Note that the system of equation in (10) or (11) might not be a complete equation for an n-ring navigation compass since there might be some extra equations based on the navigation compass' properties.

VI. 1-Ring Navigation Compass

A 1-ring navigation compass is the simplest form of the navigation compass puzzle. In this type of navigation compass, a move refers to the act of increasing the position of the compass' ring by 1 position. A 1-ring navigation compass with an m-cycle ring in position p and has moved x times can be modeled according to (10) without any extra addition. Which means, it can be denoted as

$$p + x \equiv 0 \ (mod\ m) \tag{12}$$

Notice that the solution for (12) is trivial. For (12) to follow (8), we can choose $x = m - p$. This will have results as shown in (13) below.

$$\begin{gathered} p + (m - p) \equiv 0 \ (mod\ m) \\ m \equiv 0 \ (mod\ m) \\ 0 \equiv 0 \ \ (mod\ m) \end{gathered} \tag{13}$$

Though the solution is trivial, this type of navigation compass will provide a foundational basis for more advanced types of navigation compasses.

VII. 2-Ring Navigation Compass

According to (10), a 2-ring navigation compass where its i-th ring is an m_i-cycle ring at position p_i and has moved x_i times can be modeled as follows:

$$\begin{cases} p_1 + x_1 \equiv 0 \ (mod\ m_1) \\ p_2 + x_2 \equiv 0 \ (mod\ m_2) \end{cases} \tag{14}$$

A. Case 1

In this first case, each move will increase the position of a single ring by 1 position. It means that the equations shown in (14) don't correlate with each other. Furthermore, this also means that we can view this problem as solving two 1-ring navigation compasses. Thus, this case of the 2-ring navigation compass can be solved by choosing $x_i = m_i - p_i$ and will have results similar to the one shown in (8).

B. Case 2

For the second case, each move will increase the position of both rings by 1 position. It means that, unlike the first case, there are some relationships of equations shown in (14). The said relationship can be found by noticing that each move will contribute to both x_1 and x_2, and the two said variables will hold the same value ($x_1 = x_2$). This means that for the puzzle to have a solution, there should exist x_r that follows:

$$\begin{gathered} x_r = x_1 + x_2 \\ x_r = x_1 + x_1 \\ x_r = 2x_1 \\ x_r \equiv 0 \ (mod\ 2) \end{gathered} \tag{15}$$

which means that x_r should be a positive even integer. Furthermore, for further analysis, we can modify (14) according to (11) as shown below.

$$\begin{cases} p_1 + x_1 = m_1 k_1 \\ p_2 + x_2 = m_2 k_2 \end{cases} \tag{16}$$

Adding both equations will result in

$$\begin{gathered} p_1 + p_2 + x_1 + x_2 = m_1 k_1 + m_2 k_2 \\ x_r = m_1 k_1 + m_2 k_2 - p_1 - p_2 \\ m_1 k_1 + m_2 k_2 - p_1 - p_2 \equiv 0 \ (mod\ 2) \end{gathered} \tag{17}$$

Furthermore, we can derive another equation from (16) as shown below.

$$\begin{gathered} \begin{cases} x_1 = m_1 k_1 - p_1 \\ x_2 = m_2 k_2 - p_2 \end{cases} \\ x_1 = x_2 \end{gathered}$$

Substitute x_1 and x_2.

$$m_1 k_1 - p_1 = m_2 k_2 - p_2 \tag{18}$$

Thus, for the second case, the solvability of a navigation compass can be determined by solving for positive integers k_1 and k_2 that satisfies (17) and (18) or solving for x_1 and x_2 that satisfies (14) and (15). A simple case where this case of navigation compass has no solution is a 2-ring navigation compass where both rings are a 2-cycle ring, the first ring is in position 1 while the second ring is in position 0. Based on (17), this puzzle can be modeled as follows:

$$\begin{gathered} 2k_1 + 2k_2 - 1 - 0 \equiv 0 \ (mod\ 2) \\ 2(k_1 + k_2) - 1 \equiv 0 \ (mod\ 2) \end{gathered} \tag{19}$$

Notice that the left-side equation of (19) will always be an odd integer, which means that the equation has no solution and so is the puzzle. We can extend this further for m_1 and m_2 is any random positive even integers, the first ring at an even position, and the second ring at an odd position as follows.

$$\begin{gathered} 2m_1{'}k_1 + 2m_2{'}k_2 - 2p_1{'} - (2p_2{'} + 1) \equiv 0 \ (mod\ 2) \\ 2(m_1{'}k_1 + m_2{'}k_2 - p_1{'} - p_2{'}) - 1 \equiv 0 \ (mod\ 2) \end{gathered} \tag{20}$$

which also has no solution.

In contrast, there exists a special type of 2-ring navigation compass that is solvable, which is when the cycle of the rings is relatively prime with each other. Notice that (14) can be written as

$$\begin{cases} x_1 \equiv -p_1 \ (mod \ m_1) \\ x_2 \equiv -p_2 \ (mod \ m_2) \end{cases}$$

since $x_1 = x_2$, we can also write the equation above as

$$\begin{cases} x_1 \equiv -p_1 \ (mod \ m_1) \\ x_1 \equiv -p_2 \ (mod \ m_2) \end{cases} \tag{21}$$

Notice that (21) is a similar equation as (7), which means that according to the Chinese Remainder Theorem, there should exist x_1 as the solution to the systems of linear congruence, hence the solution of the puzzle.

VIII. 3-Ring Navigation Compass

Based on (10), a 3-ring navigation compass where its i-th ring is an m_i-cycle ring at position p_i and has moved x_i times can be modeled as follows:

$$\begin{cases} p_1 + x_1 \equiv 0 \ (mod \ m_1) \\ p_2 + x_2 \equiv 0 \ (mod \ m_2) \\ p_3 + x_3 \equiv 0 \ (mod \ m_3) \end{cases} \tag{22}$$

A. Case 1

In this first case, each move will increase the position of a single ring by 1 position. Just like the first case of the 2-ring navigation compass, it means that the equations shown in (22) don't correlate with each other and that we can view this problem as solving three 1-ring navigation compasses. Thus, this case of the 3-ring navigation compass can be solved by choosing $x_i = m_i - p_i$ and will have results similar to the one shown in (8).

B. Case 2

For the second case, each move will either increase the position of a pair of rings by 1 position or increase the position of a special independent ring by 1 position. This case of the navigation compass can be solved by utilizing the independent ring. Suppose that the third ring is the independent ring of the navigation compass, we can solve the puzzle with the following steps:

1. Move the first and third ring $(m_1 - p_1)$ times. According to (13), the first ring is now in its intended position while the third ring is in position $p_3 + (m_1 - p_1)$.
2. Similarly, Move the second and third ring $(m_2 - p_2)$ times. The second ring is now in its intended position while the third ring is in position $p_3 + (m_1 - p_1) + (m_2 - p_2)$
3. Move the independent third ring to its intended state. This is always possible as shown in (13)

C. Case 3

In the third case, each move will increase the position of a pair of rings by 1 position. Unlike the second case, there is no independent ring available, making the third case a much more complex puzzle to solve. Furthermore, similar with the second case of the 2-ring navigation compass, each equation on (22) is intertwined with each other. Recall that each move will increase the position of a pair of rings by 1 position. This will result in an interesting property where there will always be one ring that has the highest move count. Suppose said ring is the third ring, this means that the number of moves done on each ring, $x_1, x_2, \ x_3$, follows the equation shown below

$$x_3 = x_1 + x_2 \tag{23}$$

Furthermore, the sum of the moves, x_r, should follow this equation

$$\begin{gathered} x_r = x_1 + x_2 + x_3 \\ x_r = 2(x_1 + x_2) \\ x_r \equiv 0 \ (mod \ 2) \end{gathered} \tag{24}$$

Which means that for the puzzle to be solvable, the sum of the moves should be a positive even integer. Moreover, from (22), we can derive

$$\begin{cases} p_1 + x_1 = m_1 k_1 \\ p_2 + x_2 = m_2 k_2 \\ p_3 + x_3 = m_3 k_3 \end{cases}$$

Adding all three equations will have a result shown below

$$\begin{gathered} p_1 + p_2 + p_3 + x_1 + x_2 + x_3 = m_1 k_1 + m_2 k_2 + m_3 k_3 \\ m_1 k_1 + m_2 k_2 + m_3 k_3 - p_1 - p_2 - p_3 = x_1 + x_2 + x_3 \\ m_1 k_1 + m_2 k_2 + m_3 k_3 - p_1 - p_2 - p_3 = x_r \\ m_1 k_1 + m_2 k_2 + m_3 k_3 - p_1 - p_2 - p_3 \equiv 0 \ (mod \ 2) \end{gathered} \tag{25}$$

Thus, we can conclude that the solvability of the third case of a 3-ring navigation compass can be determined by solving for positive integers k_1, k_2, and k_3 that satisfies (25) and solving for x_1, x_2, and x_3 that satistifes (22), (23), and (24). Notice that if all the rings are an even cycle ring ($m_1, m_2,$ and m_3 are a positive even integer), the puzzle will have no solution if one or three rings are at odd position. This can be shown by utilizing (25) as follows

$$m_1 k_1 + m_2 k_2 + m_3 k_3 - p_1 - p_2 - p_3 \equiv 0 \ (mod \ 2)$$

Since m_1, m_2, and m_3 are an even integer, we can write it as

$$2m_1{}'k_1 + 2m_2{}'k_2 + 2m_3{}'k_3 - p_1 - p_2 - p_3 \equiv 0 \ (mod \ 2) \tag{26}$$

Suppose the first ring is at an odd position while the other rings are at an even position

$$\begin{gathered} 2m_1{}'k_1 + 2m_2{}'k_2 + 2m_3{}'k_3 - (2p_1{}' + 1) - 2p_2{}' - 2p_3{}' \\ \equiv 0 \ (mod \ 2) \end{gathered}$$

$$2(m_1' k_1 + m_2' k_2 + m_3' k_3 - p_1{}' - p_2{}' - p_3{}') - 1 \equiv 0 \ (mod \ 2) \tag{27}$$

The left-side equation of (27) will always be an odd number, making the equation, and thus the puzzle, unsolvable. The same thing holds true if all rings are at an odd position. From (25), we can derive

$$2m_1'k_1 + 2m_2'k_2 + 2m_3'k_3 - (2p_1' + 1) - (2p_2' + 1) - (2p_3' + 1) \equiv 0 \ (mod\ 2)$$

$$2(m_1'k_1 + m_2'k_2 + m_3'k_3 - p_1' - p_2' - p_3' - 1) - 1 \equiv 0 \ (mod\ 2) \tag{28}$$

In which left-side equation is an odd number.

IX. N-Ring Navigation Compass

An n-ring navigation compass can be modeled according to (10) as shown below

$$\begin{cases} p_1 + x_1 \equiv 0 \ (mod\ m_1) \\ p_2 + x_2 \equiv 0 \ (mod\ m_2) \\ \vdots \\ p_n + x_n \equiv 0 \ (mod\ m_n) \end{cases}$$

A. Case 1

From the first case of both 1-ring and 2-ring navigation compass, we can see that when the move of each ring is independent with each other, the puzzle will always have a trivial solution. We can observe the problem as solving n number of 1-ring navigation compasses, which can be done by choosing $x_i = m_i - p_i$ for every ring or every 1-ring navigation compass.

B. Case 2

From the second case of the 3-ring navigation compass, an n-ring navigation compass that moves rings pairwise with some independent moving rings will aways have a solution. Suppose ring $1, 2, \dots, a$ are independent rings while ring $a + 1, a + 2, \dots, n$ are not. We can solve the puzzle by moving each non-independent ring pairly with an independent ring to the former's intended position, in a similar way shown in the second case of the 3-ring navigation compass. When all non-independent rings are in their intended position, we can move ring $1, 2, \dots, a$ independently, treating it as solving an a number of 1-ring navigation compasses.

C. Case 3

Unlike the first and second case, the third case of an n-ring navigation compass might not always have a solution. Just like the third case of the 3-ring navigation compass, the absence of the independent rings makes the puzzle more difficult to solve. Despite that, there is a trick to check whether a navigation compass with this case is solvable or not, that is by viewing the problem as solving some number of 2-ring and 3-ring navigation compasses.

Suppose an n-ring navigation compass can be divided into a number of 2-ring navigation compasses and b number of 3-ring navigation compasses. The value of a and b must follow

$$2a + 3b = n \tag{29}$$

The naive yet effective way to check the solvability of the navigation compass is by brute-forcing every possible combination of the number of 2-ring and 3-ring navigation compasses and the combination of the rings' cycle for each ring in each navigation compasses. The puzzle has a solution if there exists at least one combination that can solve the puzzle.

We can identify some subsets of an n-ring navigation compass that are always solvable. First, suppose that the navigation compass has an even number of rings. The puzzle is solvable if we can make $n/2$ pairs of rings such that every ring paired are relatively prime to each other. This utilizes the properties of 2-ring navigation compass shown in (21). Another unique subset of an n-ring navigation compass is where it has a 1-cycle ring, a ring that is always in its intended position, even when moved. We can solve this puzzle by moving every other ring in pair with the 1-cycle ring.

X. 3-Ring Navigation Compass Case 3 Solver

The third case of the 3-ring navigation compass is the most complicated navigation compass variations available in-game. A solver program for said navigation compass can be developed by utilizing (22), (23), and (24). Suppose the number of moves the player can make is limited and the input of the program is always valid, a brute force solver for the navigation compass puzzle, written in Python3, is as follows. The solution is returned as a tuple which elements show how many times each ring will move.

```
def max3(a, b, c):
    if a >= b and a >= c:
        return a
    elif b >= a and b >= c:
        return b
    else:
        return c

def solve(num_of_moves, p1, p2, p3, m1, m2, m3):
    for x1 in range(num_of_moves):
        for x2 in range(num_of_moves):
            for x3 in range(num_of_moves):
                max = max3(x1, x2, x3)
                if 2*max == x1 + x2 + x3:
                    if (x1 + x2 + x3) % 2 == 0:
                        if (p1 + x1) % m1 == 0:
                            if (p2 + x2) % m2 == 0:
                                if (p3 + x3) % m3 == 0:
                                    count = round((x1+x2+x3)/2)
                                    print(f"Solved in {count} moves")
                                    return (x1,x2,x3)

    print("Solution not found")
    return (-1, -1, -1)
```

The solver shown above is developed by modifying (22), (23). And (24). Each relevant equation and their respective representation in the code block are shown in Table 1 below. The full source code of the solver program can be accessed in Appendix A.

Table 1. Code Representation for Each Relevant Equation

Equation	Corresponding Block of Code
(23)	2*max == x1 + x2 + x3
(24)	(x1 + x2 + x3) % 2 == 0
(22)	(p1 + x1) % m1 == 0 (p2 + x2) % m2 == 0 (p3 + x3) % m3 == 0

XI. Conclusion

In this study, we explored the application of number theory, mainly modular arithmetic, in modeling the navigation compass puzzle from Honkai: Star Rail. Mathematical models analyzed in this paper include the 1-ring navigation compass puzzle, two cases of the 2-ring navigation compass puzzle, three cases of the 3-ring navigation compass puzzle that is also the available in-game version of the said puzzle, and three cases of the general n-ring navigation compass puzzle. Furthermore, this paper also develops a solver program for the third case of the navigation compass puzzle, the hardest version of the puzzle available inside the game, based on the mathematical model that is developed.

Based on the mathematical model, there are some variations of the navigation compass that are always solvable, some that are always unsolvable, and some that are still undetermined. Some variations of the navigation compass puzzle that are always solvable include the 1-ring navigation compass, the first and second case of the n-ring navigation compass, the third case of the n-ring navigation compass with an even number of rings that all have a relatively prime cycle pair, and the third case of the n-ring navigation compass where it has a 1-cycle ring.

In contrast, variations of the navigation compass that are always unsolvable are the second case of the 2-ring navigation compass where all rings have an even cycle, with the first ring in an odd position while the second is in an even position, and the third case of the 3-ring navigation compass where all rings have an even cycle but there's exactly one or three rings at an odd position.

This study doesn't cover all possible variations of the navigation compass puzzle nor provides a general solvability check for the analyzed variations. Further research on this topic might include analyzing other variations of the n-ring navigation compass puzzle, developing a general solvability check for any given variations of the navigation compass puzzle, and even developing a solver for variations of the n-ring navigation compass puzzle.

XII. Appendix

A. Appendix A

The full source code of the navigation compass puzzle solver can be found on this github repository:

https://github.com/Nuetaari/Navigation-Compass-Puzzle-Solver

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