Viewing Cases of Music Plagiarism with Probability Theory

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***Abstract*—Music copyright is perhaps a very complicated topic. Detecting plagiarism in music involves various methods. In this paper, the use of enumerations is used to quantify the amount of possible musial pieces, using music theory as a basis. Then, based on that number, basic probability theory is used to approximate the probability that an instance of music plagiarism was really just a case of two musicians coming up with the same melody. This is done to put in perspective just how likely (or unlikely) such an event happening is, and may or may not provide a compelling argument based on the specific case and results.**

***Keywords*—Music, Plagiarism, Probability Theory**

I. INTRODUCTION

A very famous case of music plagiarism happened in 1990. The artist Vanilla Ice released his debut album *The Extreme*, and on it was a track called *Ice Ice Baby*. [1] In an article by *diggersfactory.com*, published on the 29th of March 2023:

"*The single climbed to the top of the charts in 10 countries including the UK Singles chart and the US Billboard 100. However, not everybody felt over the moon with the new rap single and its explosion in popularity. A small rock English rock band by the name of Queen saw in the bass line a very obvious rip off of their own song, co written and co performed with The singer David Bowie,* ***Under Pressure***."

The two songs, *Under Pressure* by Queen and David Bowie, and Vanilla Ice's *Ice Ice Baby*, had an opening bass melody that was extremely similar. So similar, in fact, that they only differed by one note. You can find the audio of the songs and listen to the first 15 to 20 seconds of each to see for yourself, but the following image illustrates the sheet music difference for the melodies in question.

Image 1: Sheet music for the two similar songs, courtesy of *diggersfactory.com*

Even to people who have absolutely zero knowledge of basic music theory, they will realize the glaring similarities of these two melodies. Vanilla Ice admitted later on that the audio for *Ice Ice Baby* was sampled (or a better term in this case, stolen) directly from *Under Pressure* without giving credit.

Consider if Vanilla Ice did not admit to his act, and instead insisted that he came up with the same melody. This brings up an interesting question: how can we actually prove that the music was, in fact, stolen? That is, the matching melodies are not just a coincidence. This leads to the discussion of music possibilities: what exactly is the probability that two identical melodies can be made by two different people? It's important to keep in mind though, that an arbitrary probability number does not provide a definitive proof of whether a particular case was just a coincidence or not. It will, however, provide a very good insight of just how unlikely such a thing happening is, which can act as a good compelling argument depending on the case.

Music can be thought of as just a combination of sounds [2]. Meanwhile, sounds are just mechanical waves. Though those two sentences may seem to oversimplify the concept of music, it really is that simple. Then comes the interesting question: just how many combinations of music is possible? Is it some ridiculously large number? Or is it, perhaps, infinite?

Logically, the answer to this question is dependent on how music itself is defined: what do we consider as two different pieces of music? Consider the frequency range that is audible to humans, which is 20hz to 20khz. Frequency is not a discrete unit (there's 80 Hz, 80.5 Hz, 80.33 Hz, and so on). Therefore, if two sounds that have different frequency is defined to be different, then we have already ended up with an infinite amount of possible sounds.

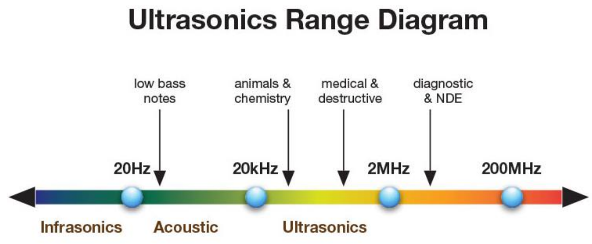

Image 2: The audio spectrum, courtesy of www.realhd-audio.com

Of course, the above is an exaggeration. The difference between, say, 80 Hz and 80.001 Hz is negligible, and no human that has ever lived would notice the difference between the two. The point is, since music possibilities will be the topic of this paper, any quantification methods, resulting conclusions, and implications will be purely within the context of the specific rules, conditions, and definitions (which will be defined) that are written in this paper, and should not be taken out of that context.

The aim of this paper is to demonstrate how simple enumerations and basic probability theories can be used in a specific context, that is obtaining a very rough estimate of the chance that an identical music piece is created by two different people.

II. THEORETICAL BASIS

A. Finite Probability

The theories written here is directly taken from [3], which is Kenneth H. Rosen's book, *Discrete Mathematics and It's Applications 7th Edition*.

An experiment is a procedure that yields one of a given set of possible outcomes. The sample space of the experiment is the set of possible outcomes. An event is a subset of the sample space. Laplace's definition of the probability of an event with finitely many possible outcomes will now be stated.

If S is a finite nonempty sample space of equally likely outcomes, and E is an event, that is, a subset of S, then the *probability* of E is $p(E) = \frac{|E|}{|S|}$.

Image 3: finite probability theorem, directly taken from *Discrete Mathematics and It's Applications*

According to Laplace's definition, the probability of an event is between 0 and 1. To see this, note that if E is an event from a finite sample space S, then $0 \leq |E| \leq |S|$. This is because $E \subseteq S$. Thus, $0 \leq (p(E) = |E| / |S|) \leq 1$. This theorem also asserts that $p(E^c) = 1 - p(E)$, where E^c is the complement of p(E).

B. The Product Rule and The Sum Rule

The product rule and the sum rule are two fundamental counting principles. Below are the definitions of both.

THE PRODUCT RULE Suppose that a procedure can be broken down into a sequence of two tasks. If there are n_1 ways to do the first task and for each of these ways of doing the first task, there are n_2 ways to do the second task, then there are $n_1 n_2$ ways to do the procedure.

THE SUM RULE If a task can be done either in one of n_1 ways or in one of n_2 ways, where none of the set of n_1 ways is the same as any of the set of n_2 ways, then there are $n_1 + n_2$ ways to do the task.

Images 4 and 5: product and sum rule, directly taken from *Discrete Mathematics and It's Applications*

C. Combinations and Permutations

An **r-combination** of elements of a set is an unordered selection of r elements from the set. Thus, an r-combination is simply a subset of the set with r elements.

The number of r-combinations of a set with n elements, where n is a nonnegative integer and r is an integer with $0 \leq r \leq n$, equals

$$C(n, r) = \frac{n!}{r!\,(n-r)!}.$$

Image 6: r-combination formula, directly taken from *Discrete Mathematics and It's Applications*

A permutation of a set of distinct objects is an ordered arrangement of these objects. We also are interested in ordered arrangements of some of the elements of a set. An ordered arrangement of r elements of a set is called an **r-permutation.**

If n and r are integers with $0 \leq r \leq n$, then $P(n, r) = \frac{n!}{(n-r)!}$.

Image 7: r-permutation formula, taken from *Discrete Mathematics and It's Applications*

D. Music Theory: Notes and Scales

The current and following music theory subsections are a very simplified summary of a few chapters from [4], which is a book by Michael Pilhofer and Holly Day called *Music Theory for Dummies*. Only the relevant information will be covered so as to provide the reader with a sufficient understanding of music theory.

Music comprises of notes that represent specific pitches. Notes are represented by letters (A, B, C, etc.). These letters carry with them the information of the pitch of the note. Pitch is what defines the low-ness or high-ness of a note, essentially it is a note's identity. Two different letters defines two different pitches, and thus two different notes. Therefore, the usage of the word "note" from hereon will also carry the implication of the "pitch" of the note in question.

There is no lowest note and there is no highest note. In music notation, G4 is used to represent a note that is higher than G3, but both have the same pitch, which is G. The numbers are called the octaves of the note, which in this case are the 4th and 3rd octave respectively. Thus we do not define G3 to be different than G4. This is a crucial restriction, as without it, the same exact melody that exists in two different octaves will be interpreted as different, which is simply untrue. Below is an illustration to the prior statement.

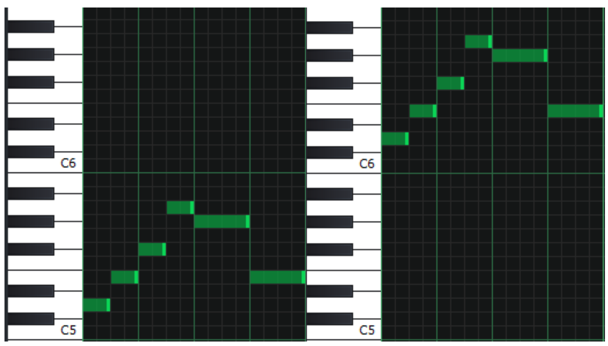

Image 8: illustration of two short melodies, one on the 5th octave (left), and one on the 6th (right), image created by author using LMMS

Each octave starts with a C note (e.g. C5 is the first note of the fifth octave). There exists twelve distinct notes in a single octave. Though there are only seven letters (C, D, E, F, G, A, and, B), an additional five notes exist, each located in between any consecutive two notes, except for E and F. These can be reffered to by going down or up a semi-tone of a note, for example C# is the note in between C and D. It may be easy to view these notes as being represented by the black keys on a piano, while the normal notes are represented by the white keys.

When analyzing the notes of a song, we now have a clear rule in place that restricts the amout of possible notes to a total of 12 values. We shall restrict these further with something called a scale.

Scales are any unordered selection of the twelve notes in an octave, and they often imply the genre of a certain musical piece. Of the 12 notes in an octave, it may be surprising that most modern songs will use only 7 notes, sometimes even less. This is because the songwriters pick a certain scale to choose what notes to create their song with, and they generally don't use notes outside of that scale. Generally, modern songs (including those in the case study of this paper), uses either the major, minor, pentatonic, or blues scale. These have 7, 7, 5, and 6 notes respectively. This will be the only relevant information used in calculations, and thus no further explanation of scales is required.

E. Music Theory: Note Durations and Terminologies

Previously, we have defined notes to represent pitches, but in [4], the authors state that the more complete definition of a note is actually one that also encompasses it's duration; how long the note lasts. A note duration can be infinitely small (or large), thus, it is imperative that we place restrictions for it as well. The following diagram explains note durations using the term beat, which is a unit of measure for how long a note lasts.

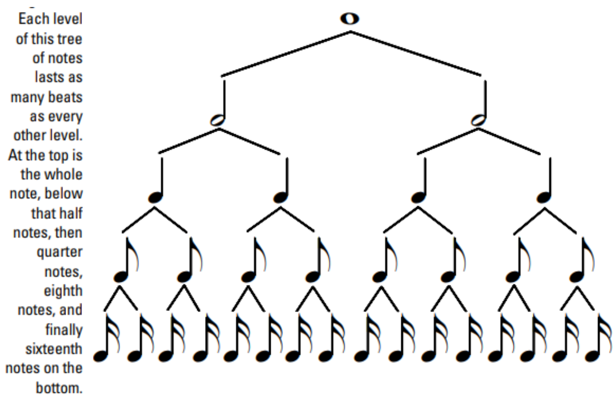

Image 9: diagram of note durations, taken directly from *Music Theory for Dummies*

We shall restrict the range of note duration to be from full notes to the eigth note. This is an arbitrary example, and will be subject to change based on the case in chapter III. In contrast to notes, there are rests, which are periods where there are no notes played. They can also be thought of as silent notes. Rests also have durations.

Finally, it is also useful to define a few musical terminologies to make it easier when referring to specific things later on.

1. A **bar** is a small subsection of the entire melody of a song, and it contains only a certain amount of beats.
2. The **time signature** of a song determines the amount of beats in a bar. If the time signature is 3/4, it means a single bar contains 3 beats, and 4/4 means a bar contains 4 beats.
3. The **tempo** of a song is it's speed. Tempo is often described using BPM or beats per minute.
4. The **key** of a song is the pitch that acts as the base note in a song.

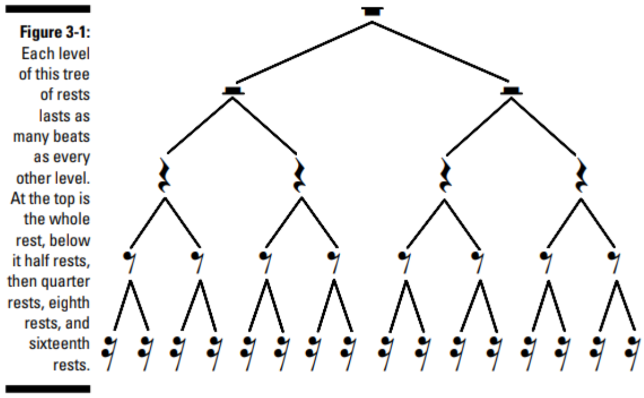

Image 10: diagram of rest durations, taken directly from *Music Theory for Dummies*

As an example, consider a song which lasts roughly 3 minutes, has 160 BPM, and has a time signature of 4/4. This song would have $3 \times 160 = 480$ beats, as well as $480 \div 4 = 120$ bars. These calculations are used to approximate the amount of bars of a song. This is done since a song may have many bars, and any contiguous subset of those bars will have a chance to have a matching melody. When dealing with plagiarism, it's important to give the benefit of the doubt and be as lenient as possible restricting our definitions as mush as we can.

F. Enumerating Melodies and Rhythm

As notes have been properly defined, we now have a basis for building an understanding of song possibilities. In [5], the author has accumulated insight from many different sources regarding the different aspects of a musical piece that can be looked at. They state that most sources that discuss music copyright simplistically define musical pieces as a combination of melody, harmony, and rhythm.

A melody is a series of notes. In a scale with *s* notes, if a melody has *n* consecutive notes with equal note durations, since each note has *s* possibilities, then by the product rule, we get the amount of possible melodies *P* to be the following:

$$\mathrm{P} = s^n$$

It's important to note that many of these melodies may be nonsensical, such as a melody whose notes are all the same. However, placing restrictions on the type of melody would require a significantly more thorough assessment from the perspective of music theory. We simply must keep in mind of this fact when viewing the results. Therefore, we have defined a formula to calculate just the amount of possible melodies.

As mentioned previously, melody alone does not represent the identity of a musical piece. The author of [5] again has mentioned that most musical copyright discussions will also take into account the rhythm of a song. Meanwhile, harmony is not considered as it is not a defining feature of music (i.e. it functions merely as a supporting feature of a song, thus does not contribute to the song's identity). The author also mentions the idea (proposed by Professor Benjamin Boretz) of a musical piece's **percieved structure** as it's defining trait as opposed to it's melody, harmony, and rhythm. However, structure is used when viewing music as a continuous medium rather than discrete, so it is not brought up here.

Rhythm is the second defining feature of a song. Two identical melodies can sound vastly different if their rhythms are extremely distinct [n]. Rhythm is the combinations of time intervals between silent notes and nonsilent notes. Consider the previous formula, P = s^n. Any of those notes can have an arbitrary duration, and be abitrarily seperated by silent notes instead of being equidistant from each other. Stated in the previous subsection, we will be defining only a small possible range of note durations based on the case, but for now, we assume that number to be *b*. Then, by the product rule again, the amount of possible melodies become:

$$\mathrm{P} = (sb)^n$$

The logic behind this method of rhythm calculation is similar to what the author of [6] has used, but has been significantly simplified. This is because they have taken into consideration of various other factors, such as asymmetric rhythms and rhythm cycles, which we do not discuss here as we want to keep the restrictions as tight as possible.

As an example calculation, consider the melodies that follow these restrictions:

1. The melody may not have any silent notes (i.e. no silence between notes).
2. The melody must contain more than 2, but no more than 6 notes.
3. The melody may only have notes that range from half beats to eighth beats, namely 3 possible note durations.
4. The melody uses only notes from the pentatonic scale.
5. The melody lasts for 2 bars.
6. The melody has a time signature of 3/4.
7. Finally, assume the exact same melody exists in two songs, one that lasts 2 minutes with 156 BPM, and the other 3 minutes with 172 BPM.

Here, $s = 5$ (from the pentatonic scale), $b = 3$, and $3 \leq n \leq 6$. Note that we must calculate for each possibility of n. Thus the resulting amount of possible melodies is:

$$P = (5 \times 3)^3 + (5 \times 3)^4 + (5 \times 3)^5 + (5 \times 3)^6 = 12204000$$

Thus, we calculate the probability of two songs having at least one identical melody within them. Song *A* lasts 2 minutes with a tempo 156 BPM, which makes the total amount of bars (2×156) / 3 = 104 bars (rounded up). Song *B* lasts 3 minutes with a tempo of 172 BPM, making it have (3×172) / 3 = 172 bars. Each contiguous set of 2 bars from each song has the potential to be the matching melody. Song *A* would have 104 – 1 of these contiguous sets of 2 bars, while song *B* would have 172 – 1 sets. by the product rule, there are 103×171 ways to choose a pair of melodies that consists of 2 bars from these two songs. This makes the probability:

$$p(E) = \frac{103 \times 171}{1220400} = 0.0014$$

The implication becomes: there would be a (1 – 0.0014) chance that the author of song *A* was plagiarising song *B*. This equates to 0.9986 or 99.86%. At first glance, this may seem like a big percentage. However 0.14% is not such a low probability that it will never happen. The actual threshold for determining whether or not a probability is feasible is perhaps outside the scope of this paper. Instead, beyond the facts and obvious implications, I shall leave it to the reader to interpret the results of the calculations.

III. A CASE STUDY

In Indonesia, there was a decently popular case of music plagiarism allegations in 2017. The case was covered by multiple news articles, one of which have reported on the following information. Based on the article [7], Armada's song, *Asalkan Kau Bahagia* has been alleged by many to be plagiarising the song *Liú Xīng Yǔ* (original text: 流星雨) by a significantly older band called F4. In that same article, it was reported that Armada's main vocalist, Rizal Armada, has denied the allegations and claimed that they did not, in fact, plagiarise anything. Nothing came out of the whole ordeal and no legal consequences associated with the situation has been reported.

This is a prime example for a case study for the topic of this paper. The claim is that the song has not been plagiarised, which implies that the songwriters have both come up with the same melody on their own independence. We shall calculate the probability of this occuring. First, we shall outline the song features in a table.

Feature	*Asalkan Kau Bahagia*	*Liú Xīng Yǔ*
Key	E	E
Tempo	135 BPM	136 BPM
Duration	4 m 5 s	4 m 37 s
Time Signature	4/4	4/4
Scale	Major	Major
Range of Note Durations	Full note – eighth note	Full note – eighth note

Table 1: case study song feature comparison

Next, we calculate the amount of bars for each song. *Asalkan Kau Bahagia* has (4.083 × 135) / 4 = 138 bars (rounded up), and *Liú Xīng Yǔ* has (4.616 × 136) / 4 = 157 bars (rounded up).

I have transcribed the core melody of both songs into MIDI format using LMMS (Linux Multi-Media Studio, a digital audio workspace). The reader may verify the accuracy of these transcriptions by listening to the MP3 files provided in the external link located in the appendix. Both the audio files for the two transcriptions, as well as the audio files for the original songs[1] have been provided. All audio files describe only the matching melody of the two songs.

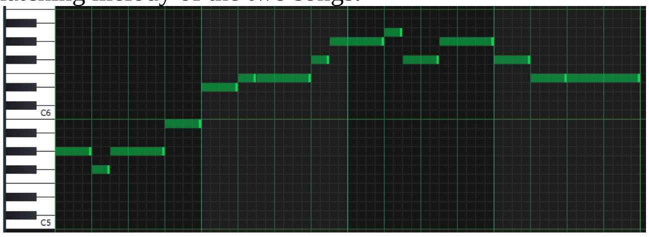

Image 11: MIDI file of the first song, *Asalkan Kau Bahagia*, created by Author using LMMS

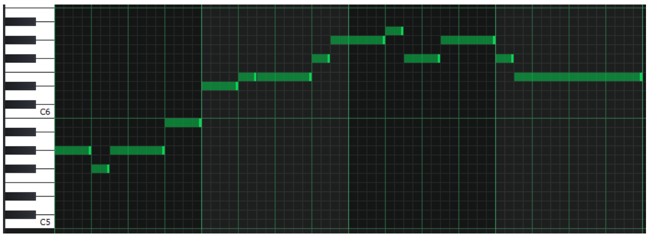

Image 12: MIDI file of the second song, *Liu Xing Yu*, created by Author using LMMS

The melody in question consists of four bars. However, the last bar has slightly different notes. To be lenient, let us consider only the first three bars, which has identical notes. The amount of sets of 3 contiguous bars is 138 – 2 for *Asalkan Kau Bahagia*, and 157 – 2 for *Liu Xing Yu*.

Next, we define the *s*, *b*, and *n* values for the formula stated in chapter II. Here, both songs have used the major scale, thus *s* is 7. The value for *b* is 4, since there are 4 possible note durations (full note, half note, quarter note, and eighth note). Lastly, the melody uses 12 notes, which makes *n* equal to 12. We can now calculate the amount of possible melodies in the context of the song features of these two songs.

$$\mathrm{P} = (7 \times 4)^{12} = 2.32 \times 10^{17}$$

Thus the probability that a pair of subsets from both songs have the same melody is:

$$p(E) = \frac{136 \times 155}{2.32 \times 10^{17}} = 9.07 \times 10^{-14}$$

And this is our final result. The complement probability (i.e. the probability that the song was, in fact, plagiarised), is therefore 1 – p(E). This value is significantly lower than the example calculation in chapter II, and thus this case can be considered as significantly more likely to be a result of plagiarism than that example. Judgements beyond this statement are left for the reader.

As a final reiteration, the methods used in this paper heavily restricts the number of possibilities using only the most basic principles of music. Additionally, there may be further restrictions that were not placed in order to preserve simplicity. Thus the number of possibilities may not yet accurately describe the set of music that is "normal", and the final result should only be judged within that context.

IV. Conclusion

This paper has demonstrated the use of simple enumerations and probability theory to determine the probability that a matching melody between two songs was just a coincidence. Incidentally, the probability that the occurance was a result of plagiarism is the complement of that probability. Finally, the implications of this final result is not to definitively prove whether or not it was just a coincidence, rather it is just to put in perspective the rough estimate of the likeliness of such an event happening, and to be one of many factors to consider when creating a final judgement.

V. Acknowledgment

I would like to express my gratitude to the lecturers of ITB Discrete Mathematics IF2120, Mrs. Ulfa Nur Maulidevi, Mrs. Harlili, Mrs. Fariska Zakhralativa, and Mr. Rinaldi Munir for sharing their knowledges and guiding the students throughout the learning process in the class. And of course, I would also like to thank my friends and family who have provided me their support and accompanied me day to day throughout the entire semester. They have made my days a lot more meaningful.

VI. Appendix

Here is the external link to the aforementiond MP3 files: https://drive.google.com/drive/folders/1XnDGz-aO7BL5dnfr7kIrlpe-UoXLMvNf?usp=drive_link

VII. References

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[1] (This paper is made for the purposes of research and education. Thus, fair use permits the limited use of the copyrighted material without permission.)

STATEMENT

I hereby declare that the contents of the paper I have written is my own writing, not an adaptation or translation of another author's paper, and not plagiarised.

Bandung, 11 Desember 2023

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