Graphical Approach for Task Assignment in Professional Kitchen Environment

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***Abstract*—In a professional kitchen environment, high quality dishes often require a certain amount of preparation. Said process is done by multiple cooks with multiple ingredients that follow a certain recipe. Theory of graph in discrete mathematics can be the solution to achieving the most efficient way possible to produce a dish. The main idea being to represent a recipe in a graphical form and assign each task using topological sort. This paper will elaborate the theory use within the application and analyze real world cases. Furthermore, in professional kitchen, time is one of the aspects that costumer value the most. To serve the best dish possible in the least amount of time is crucial to a culinary establishment reputation. Therefore, the method examined in this paper can be directly applied to the existing workflow of a kitchen restaurant.**

***Keywords*—Graph, Discrete Math, Recipe, Kitchen, Cook.**

I. Introduction

A chef making food for their costumer can simply be referred to as cooking. But in the world of high-quality Food and Beverage (FnB) industry, the complexity of the food, technique, and recipe escalate to a much higher degree. Therefore, the term "Culinary Arts" often be used to describe an act of cooking, serving, and presenting food with a such complex method that it can be called art. Culinary Arts involved not only the taste of the food itself, but the experience in presentation, smell, aesthetic, and even texture that make the costumer satisfied. To maximize the satisfaction, is the main job of a professional cook to factor in all human senses that triggered by a dish and utilized it to give the costumer a satisfactory experience. A chef takes time to prepare and serve dish the best way possible and is often require complex process [1].

A dish is a result of the work of a chef that has been experimented and tested. Dish that already perfected by the chef is often recorded in a recipe. Recipe contain extensive instruction, ingredients, and measurements necessary to construct a dish. Typically, it includes the proportions of each ingredient that should be used, as well as temperature and cooking for the dish. Making a recipe require a deep knowledge of culinary arts and years of experience in the industry. Recipe is written in the form of list of ingredients followed by a set of instruction of what to do to said ingredients. This set of instruction can be summarized to a list of tasks that a cook can do to finish the dish. Furthermore, by representing a task with a vertex on a graph, a recipe can be described as a graph [2] that we will further examine in later chapter.

A fine cuisine with complicated recipe needs a lot of effort to be put to the table. To serve complex dish with shortest time possible, a team of cook are required to apply teamwork and communication to execute each instruction in the recipe flawlessly. Hence, a hierarchy of chef inside the kitchen is applied to divide the task and each role have each task to focus on. According to [3] the role in a professional kitchen is as follows.

1. Executive Chef, rarely cooks but oversee the kitchen and its personnel.
2. Head Chef, similar to Executive Chef with more hands-on approach to the cooking process.
3. Deputy Chef (Sous Chef), second in command to Head Chef with direct cooking.
4. Station Chef, cook dish according to each food category
5. Junior Chef, works alongside station chef to deepen their understanding of kitchen environment.
6. Kitchen Porter, do basic preparation on ingredients used in main dish.
7. Purchasing Manager, managing food purchase and stock.

Each role can be assigned a task that contribute to completion of a dish or even a different dish overall. The main problem that this paper trying to analyze is the method to generate the most efficient task assignment possible with the theory of graph as the underlying theory. A task/shift assignment is one of the common problems to solve using discrete mathematics. But, to apply said theory in a kitchen context needs a few adjustments and adaptation to few aspects. One of which being the needs to categorized the difficulty of a task and assign it according to an experience level of a chef (with the most experienced chef being the highest ranking in the list role).

II. Theoretical Basis

A. Graph

Graph is a structure of discrete mathematics that consists of vertices and edges that connect the vertices. An edge always has either one or two vertices that connected by it. These vertices are called *endpoints [4]*. To provide an example, imagine there is a railway connecting major cities in North America. Each city is represented by a vertex in the graph while the railway connecting them is represented by the edges. When illustrated, the graph should have a structure as such.

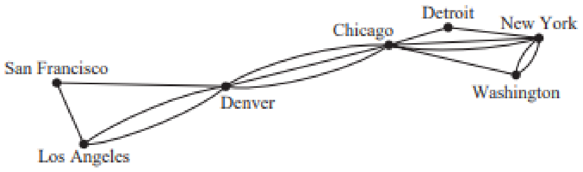

Figure I. Graph of North America Railway, source: [4]

Graph can be represented in form of tuple of vertices set and edges set:

$$G = (V, E)$$

G = Graph
V = Set of Vertices
E = Set of Edges

Classification of graph are plenty and is different between the factor that classify said graph. The most general one being the classification of graph with loop and double edges. A graph with an edge that connect two vertices and without another edge connecting the same vertices is called **simple graph**. Outside of simple graph is called **unsimple graph.** Unsimple graph has a classification of its own:

a. Multi Graph
Graph that contains two or more edges that connect the same vertices (double edge)

b. Pseudo Graph
Graph that contains edges that connect to one vertex or vertex that connected to itself (loop)

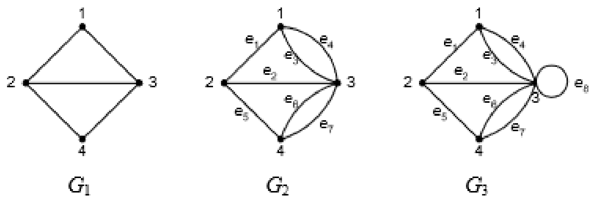

Figure II. G1: Simple Graph, G2: Multi Graph. G3: Pseudo Graph. Source: [10]

The graph depicted so far has been an **undirected graph** that is because the edges have no direction that point to either vertex that it connects to. A **directed graph** is a graph with edges that point to a vertex. Assume there is an edge with vertices A and B and edge pointing to A. The edge is said to be start at B and end in A. The illustration of said graph is to make the edge connecting them in the form of an arrow pointing to vertex A. Look at the example in Figure I, a railway is supposed to have a direction of the train shown in the graph. With directed graph, depiction in Figure I can be further completed as such.

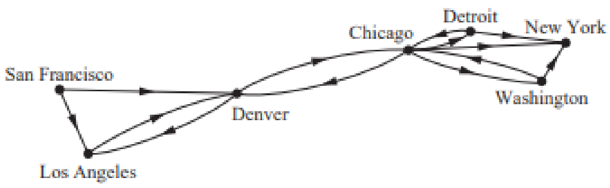

Figure III. Directed Graph of the example. Source: [4]

B. Topological Sort

Topological sorting is a linear order of vertices in a graph such that every directed edge with vertices A and B, in a topological list, A will come before B. Element of graph that Topologically sort will be linear ordered in a list. To further illustrate the use and meaning of topological sort, assume a cooking project to bake a cake. A cake has many ingredients and ingredients require action to be processed into a cake. Image the action required is a vertex scattered throughout a graph. Topological sort determined the action that a person must take first to bake a cake and the action after that. For example, a person must mix flour with water before mixing it with an egg or a person must knead the dough before putting it in the oven.

Is apparent how this algorithm can be applied to the task assignment in the kitchen, but to fully understand topological sort and to utilize it, one must examine the algorithm itself. The algorithm consists of multiple steps such as presented below [5].

1. Choose a vertex that have no predecessor in the graph (Assume X). The graph must have at least one vertex that has no predecessor.
2. Erase X from the graph and the edges along with it. Put X as the next element of the result list (if list empty put X as the first element).
3. Repeat process 1 and 2 until the graph if empty.

The steps of algorithm above are applied to the graph below and will return result as depicted.

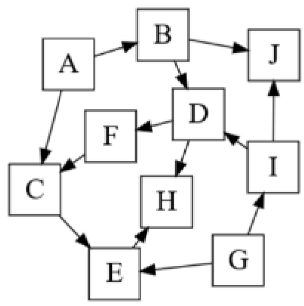

Figure IV. An example Graph. Source: [5]

Result: A, B, G, I, J, D, F, C, E, H. Note that the result of topological sort can differ according to the vertex choice. Therefore, the result can be more than one but is still the correct answer.

III. Method

A. Vertices and Edge Element in Graph of Recipe

A recipe act as a guide to direct the workflow in the kitchen. Therefore, a task assignment refers to a recipe for the task it assigns and the workflow it produces. For the topological sort and the assignment algorithm to work, recipe must be in the form of a graph first. As described previously, a recipe typically comes in a form of ingredients and instructions. The list of ingredients is often used for purchasing and initially checking availability of food as ingredients. For the purposes on focusing the problem to task assignment, from here on out, the ingredients necessary for the recipe is assumed to be available and in stock in our kitchen. Hence, the list of ingredients section of the recipe can be ignored and excluded from the graph of recipe. The instruction in a recipe often be numerically ordered and presented as a set of steps. This representation of the recipe can be translated to a series of vertices and edges to then converted to a graph. First, the definition of vertex and edge in the context of recipe should be determined. From analyzing each step of a recipe, it become apparent that there is two most important element in instruction steps. The first being the task itself and the second is the duration of the process that being performed. Not every instruction includes the duration of the process, but for the crucial aspect of cooking, such as fry, boil, and oven, the time in which the process is carried out is essential to the workflow and will affect the final dish tremendously. For instruction that does not include time duration, the time will be assumed based on the knowledge of the graph constructor and extensive research. To keep it uniform, the time format for duration is set to minute. In conclusion, the vertices in the context of a recipe are being defined as.

$$V = (T, D)$$

V = vertex label
T = task description
D = duration of the process in minute

As an example, the citation of an instruction steps in a recipe below is being converted to a vertex.

"Cook butterflied quail skin-side down for 2-3 minutes until golden" [6]

The instruction above is converted into a vertex and will render result as such.

A = ("*Cook butterflied quail skin-side down", 3*)

One more important aspect of a graph is edges. Edges in the context of recipe can be defined as pointer of a vertex to another vertex that represent an instruction. This simulates the ordered pattern of a recipe instruction where there are steps that should be done before proceeding to the next step. To further clarify the sequence, edge on the graph will be in the form of directed edge where edge will start on certain vertex and end on another vertex which are the next steps in the recipe. Mathematically, directed edge will be defined as.

$$E = (V1, V2)$$

$V1$ = starting vertex
$V2$ = ending vertex (next step)

Both vertices and edges defined above will be applied directly to the construction of the graph of recipe.

B. *Construct a Graph of Recipe*

Definition of vertices and edged that defined above is used to construct a full graph representing a recipe called Graph of Recipe. To simplify the conversion, construction is divided into several steps below.

1. Determined each step in the recipe as discrete as possible. Often times, recipe contains multiple steps in one instruction and that can lead to confusion in the conversion later on.
2. Define which steps that can be done simultaneously. Both graph and recipe are non-linear and can contain elements that do not depend on one another. Spot this element to maximize efficiency.
3. Convert all the steps into vertices as shown in sub-chapter previously.
4. Start from the first vertices, if the next processes can be done simultaneously, make edges connecting the vertices to the current one. If the next vertex depends on the completion of the current one only connects the next vertices and the current one.
5. Move to the next steps in the recipe with a vertex representation. If the current vertex depends on the completion of one or multiple vertices prior, connect those vertices and current one with edges.
6. Repeat steps 4 and 5 until there is no other instruction that has not yet represented in the graph of recipe.

Now, the recipe is now in the form of graph and applying discrete mathematics theory is now possible.

C. *Assignment Algorithm*

In reference to [7], the algorithm that underlying the base theory of this assignment algorithm is typically used in context of creating a schedule. But with little modification, this algorithm is the perfect solution to creating an efficient kitchen workflow. The algorithm is as such.

1. Start on the first vertices and apply topological sort to the graph. But, instead of putting it in a list of result, put the erased vertex in the table of assignment for the cook.
2. If there is still cook that have zero task, assign erased vertices until all cooks have a task.
3. If all cook already have a task, choose a cook that have the least amount of task, timewise, and put it there.
4. If there are task that require certain completion of another task, place current task after the predecessor task has done in the list of assignment.
5. Repeat steps 1 to 4 until there no vertices left in the graph.

Assume there is two cooks in a kitchen. Recipe for the cooks is already in the form of graph as depicted below. To further clarify the process of assigning, this example will be followed by a table that represent the cooks and task they do with each cell being one unit of time.

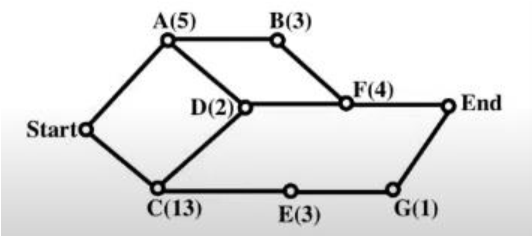

Figure V. Graph of Recipe. Source: [7]

Now start from the start vertex and erase that vertex from the graph and assign it to one of the cooks.

<table>
<tr><td>P1</td><td colspan="5">A</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>P2</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
</table>

Table I. Assign first task

Examine the next vertex, that being C, and assign it to cook that not yet have a task.

<table>
<tr><td>P1</td><td colspan="5">A</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>P2</td><td colspan="13">C</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
</table>

Table II. Assign C to the second cook

Vertex B is assigned to the cook with the least amount of task according to the total duration of their task.

<table>
<tr><td>P1</td><td colspan="5">A</td><td colspan="3">B</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>P2</td><td colspan="13">C</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
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Table III. Assign B vertex

The next vertex, D, require the task A and C to be both finished. Therefore, task C is assigned after both tasks has finished and still on the cook that has the least amount of work.

<table>
<tr><td>P1</td><td colspan="5">A</td><td colspan="3">B</td><td></td><td></td><td></td><td></td><td></td><td colspan="2">D</td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>P2</td><td colspan="13">C</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
</table>

Table IV. Assign D vertex after A and C is done

Move on to the next one, the vertex E is assigned to the second cook because due to last step, second cook has the least amount of work currently.

<table>
<tr><td>P1</td><td colspan="5">A</td><td colspan="3">B</td><td></td><td></td><td></td><td></td><td></td><td colspan="2">D</td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>P2</td><td colspan="13">C</td><td colspan="3">E</td><td></td><td></td><td></td><td></td></tr>
</table>

Table V. Assign E

The F vertex is assigned to the first cook according to step 3 in the algorithm.

<table>
<tr><td>P1</td><td colspan="5">A</td><td colspan="3">B</td><td></td><td></td><td></td><td></td><td></td><td colspan="2">D</td><td colspan="4">F</td><td></td></tr>
<tr><td>P2</td><td colspan="13">C</td><td colspan="3">E</td><td></td><td></td><td></td><td></td></tr>
</table>

Table VI. Assign F

Finally, the vertex G is assigned to second cook.

<table>
<tr><td>P1</td><td colspan="5">A</td><td colspan="3">B</td><td></td><td></td><td></td><td></td><td></td><td colspan="2">D</td><td colspan="4">F</td><td></td></tr>
<tr><td>P2</td><td colspan="13">C</td><td colspan="3">E</td><td>G</td><td></td><td></td><td></td></tr>
</table>

Table VII. Assign G

Instead of doing the recipe step by step, utilize human resource to the fullest is necessary for the kitchen workflow. Assignment algorithm can help improve kitchen efficiency by a wide margin. Example above is one of the simpler one of the bunches. To see how it perform in the real world, we must seek real world cases and analyze it using algorithm that established above.

IV. Case Study

A. Scenario

A kitchen environment as chaotic as it is, can be source of people entertainment. One of which that make whole reality show based on the chaos and tension of a professional kitchen, is "Gordon Ramsay's Hell's Kitchen" [8]. In the show, cooks divide into two team and compete to serve the best dish to their respective costumer. Each episode the contestants one by one is being eliminated based on their performance and teamwork in the kitchen.

The recipe from the show includes complex technique and multiple steps to get done. Hence, its crucial for the chefs to work as a team and assign task in the most efficient way possible. The use of discrete mathematics can be a help for assigning task faster and better than plain discussion alone.

To analyze our case, note there are 3 chefs in the kitchen. 2 station chefs and 1 head chef. Each chef is equally skilled and very experienced. Their task is to make "Sesame Crusted Seared Tuna" [9] with the recipe that, the shows host, Gordon Ramsay makes. Production of the dish cannot be over a time constraint of 10 minutes. Apply the assignment algorithm to find the most efficient way to complete the dish based on the recipe given.

B. Recipe to Graph

First the recipe provided must be converted to a graph form. Vertices on the graph must be determined first with the definition of depicted before. Result of the vertices is as follows.

a) A = ("Coat tuna with egg wash and sesame seeds", 1)
b) B = ("Heat oil in a pan", 1)
c) C = ("Sear the tuna", 4)
d) D = ("Slice the tuna", 1)
e) E = ("Cook soba noodles", 3)
f) F = ("Drain and rinse the noodles", 1)
g) G = ("Put noodles in a bowl and add sesame dressing", 1)
h) H = ("Make the sesame dressing", 2)
i) I = ("Plate the noodle and tuna", 1)

All vertices that represent steps in the recipe has been defined. To construct the graph, use the algorithm in "Construct a Graph of Recipe" subchapter. The result will render as follows.

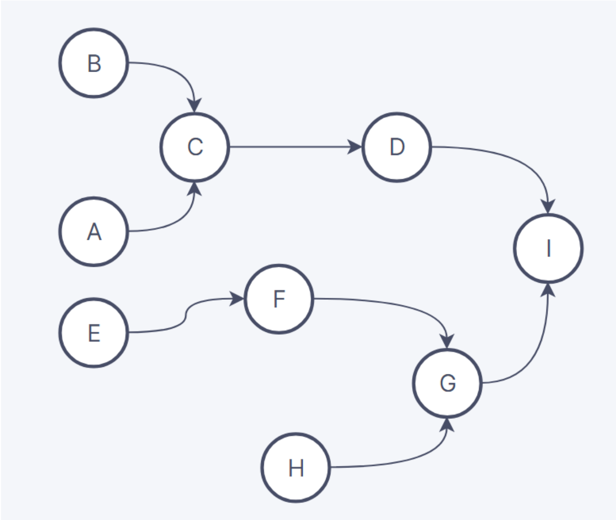

Figure VI. "Sesame Crusted Seared Tuna" recipe graph

Graph recipe above can be processed with the task assignment algorithm that already defined before.

C. Task Assignment

First, all resources in the case are modelled in a form of a table. The table below consist of the chef and 10 minutes time constraint with each cell is equivalent to 1 minute.

Chef 1										
Chef 2										
Chef 3										

Table VIII. Resources table model

Algorithm use in this analysis is based on "Assignment Algorithm" sub-chapter. Begin by examining the A vertex and assign it to the first chef.

Chef 1	A									
Chef 2										
Chef 3										

Table IX. Vertex A Assignment

Continue to the next vertex with no predecessor, B, and assign it to the second chef.

Chef 1	A									
Chef 2	B									
Chef 3										

Table X. Vertex B Assignment

Vertex E also have no predecessor and chef 3 does not have any task yet. Therefore, assign E to the third chef.

Chef 1	A									
Chef 2	B									
Chef 3	E									

Table XI. Vertex E Assignment

Move on to vertex C and assign it to chef 1 because they have equally least amount of work and sorted before chef 2.

Chef 1	A	C								
Chef 2	B									
Chef 3	E									

Table XII. Vertex C Assignment

Vertex H can be directly assigned to the least working chef in the table.

Chef 1	A	C								
Chef 2	B	H								
Chef 3	E									

Table XIII. Vertex H Assignment

F vertex has to be placed after the E vertex has done, Therefore F is assigned to chef 2.

Chef 1	A	C								
Chef 2	B	H		F						
Chef 3	E									

Table XIV. Vertex F Assignment

The vertex G must be worked on after the F and H completed. So G is assigned to chef 3.

Chef 1	A	C								
Chef 2	B	H		F						
Chef 3	E				G					

Table XV. Vertex G Assignment

D vertex must be placed after the C vertex has done, in regard to that, D vertex is assigned to chef 2 after C has done.

Chef 1	A	C								
Chef 2	B	H		F		D				
Chef 3	E				G					

Table XVI. Vertex D Assignment

Finally, the I vertex to complete the dish is assign to chef 3 after D has done according to "least amount of work time rule".

Chef 1	A	C								
Chef 2	B	H		F		D				
Chef 3	E				G		I			

Table XVI. Vertex I Assignment

The time to create the dish in a recipe is roughly 7 minutes according to resource table result. Meanwhile, if it done sequentially, the dish will take 14 minutes to serve. 7 minutes different and 50% increase in efficiency. Saving a priceless resource, time, while guaranteed increase in costumer satisfaction. Perhaps in the Hell's Kitchen competition it can be a different between elimination and make it through this week.

V. Conclusion

Discrete mathematic is a broad knowledge field consist of many theories. All of which have been applied in the real world and in everyday lives. Theory of graph and topological sort have nothing in common with the elegant world of culinary arts. But through this paper and study of the real-world cases, can be concluded that discrete mathematics is applicable in even the most different field of work.

The effectiveness of a commercial kitchen can be increase up to 50%. A more efficient restaurant can produce more high-quality foods in least amount of time. That can lead to more satisfied costumer, increasing the restaurant reputation, and eventually lead to more profit. In the context of competition, using assignment algorithm depicted in this paper, can lead to less chaotic environment in the kitchen and hopefully help a team to win the prize. Although not every recipe and every kitchen workflow are suitable to apply this algorithm, theory in which the assignment is done can be referenced to add or even perfect the existing workflow. In the research and experiment on this paper, the most suitable recipe is turn out to be more complex dishes that require multiple process of the food and supporting ingredient. On a recipe that only require a few amounts of ingredient and straight forward cooking this algorithm typically fall short in terms of effectiveness and even can a hindrance to creating the perfect dish.

In conclusion, the theory of graph can be applied to task assignment in the kitchen and can be concluded that it increases its efficiency to a certain degree. The theory forms this paper is the best version that the writer can do at the time. Even so, further research and, especially, experiment in a real-world kitchen is desperately needed to find out how the algorithm perform when presented a lot of random variables in a commercial kitchen. Future research on the topic can include a computer program that use the assignment theory or Internet of Things installment in the kitchen.

Finally, the topic and result discussed within this paper is with flaw. Considering the writer insufficient experience in both discrete mathematic and culinary field, a correction in the logic and fact is to be expected. Hopefully, this paper can raise the understanding of both field for the reader because it is certainly the case for the writer.

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STATEMENT

I hereby declare that the paper I am writing is my own writing, not an adaptation or translation of someone else's paper, and not plagiarism.

Bandung, December 12th, 2022

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