Solving ‘24’ Card Game Using Combinatorics

Alexander Jason - 13521100[1]
Program Studi Teknik Informatika
Sekolah Teknik Elektro dan Informatika
Institut Teknologi Bandung, Jl. Ganesha 10 Bandung 40132, Indonesia
[1]*13521100@std.stei.itb.ac.id*

***Abstract*—‘24’ card game is a mathematical and logical games that are really exciting. People can play it for fun or even for competition. This game has simple rules, it’s famous to be played in school students, or even as a learning module in schools. This paper will focus on the making decision and strategy based on the combination solutions. Critical and fast thinking are the main factor of this game. Using combinatorics, people can find ‘24’ solution way much faster and win the game. 1 deck of Poker card or playing card always have a fixed total of cards and fix value of each card. This paper will show how combinatorics can help winning in every single ‘24’ games.**

***Keywords*—‘24’-card-game, combinatorics, solutions**

I. Introduction

Card games are one of the most favorite games in the world. There are a lot of variation of card games, such as Chinese Poker, Bridge, BlackJack, Solitaire, ‘24’ game, and many more. In this paper, writer will discuss more about ‘24’ game. ‘24’ card game is one of arithmetical games which its objective is to find a way of to set up and operate 4 numbers that shows up in all four face-up cards that resulting 24. For example, if the four cards were 8, 2, 4, 6 then the possible solutions would be (8 x 6) ÷ (4 ÷ 2) or (8÷4) x (6 x 2) and many more.

People usually use Poker card to play ‘24’ game. Poker card itself consists of 52 cards with 4 symbols (diamond, club, heart, and spade). Each symbol consists of 13 cards from number 2 to 10, Jack, Queen, King, and Ace. In ‘24’ game, we only care about the value of the card, not the symbol. Ace has 1 value, Jack has 11 values, Queen has 12 values, King has 13 values, and numbers have value based on the number.

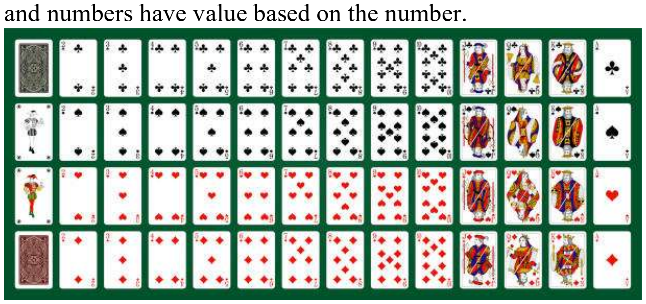

Fig. 1 One Pack of Poker Cards (source: https://www.istockphoto.com/id/foto-foto/kartu-remi)

The rule of this game is simple. Each player will draw a card from the stack and place them face up in front of all other player to form a square four face up cards. After all four cards have been faced up, each player tries to think of a way to combine the four numbers only using four basic operators, which is addition, subtraction, multiplication, and division, to get a result of 24. Each number can be only use once. The first player who come up with a solution raise his hand and tell the other players his/her solution. If the solution is correct, the four cards will be given equally to other players. Otherwise, the player who give the wrong solution will take the four cards as penalty. This action will be repeated until the stack has no cards. In the end of game, player with the least cards will be the winner. If neither of the player can find the solution, they may agree to redraw card(s) and replace it.

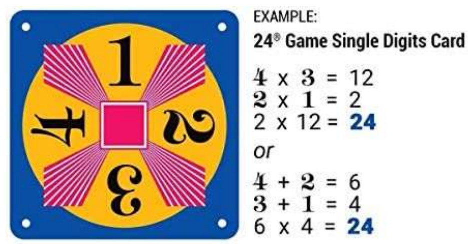

Fig. 2 ‘24’ Gameplay example (source: https://www.amazon.com/24-Game-Factors-Multiples-Ages/dp/B0009GU294)

II. Fundamental Theorem

A. Combinatorics

Combinatorics is a branch of mathematics that focuses on the research of countable discrete problems. Combinatorics has implementation in computer science, physics, and efficiency. Combinatorics can help to count number of ways an event could occur by every outcome. There are two basic counting principles in combinatorics, which are sum rule and product rule.

1) Sum Rule
   If a task can be done in n ways **or** m ways which both are different ways, then there is ($n + m$) ways to solve the tasks
2) Product Rule
   If a task can be done in n ways **and** m ways which both are different ways, then there is (n x m) ways to solve the tasks

For example, how many ways can a person pick a card from 2 decks of Poker card? The answer is simple, sum up the total of 2 decks: 52 + 52 = 104 ways.

Beside from the basic counting principles, combinatorics also has another principles:

1) Combination
   Combination is a counting principal to arrange a set of objects regardless its order. A combination of n things

taken r at a time, written C(n,r) is any subset of r things from n things regardless of the order. The formula of combination is:

$$C(n,r) = \frac{n!}{r!\,(n-r)!}$$

2) Permutation
Permutation is the total arrangement of certain ways or objects that depending on the order of the arrangement. A permutation of n things taken r at a time, written P(n, r) is an arrangement in a row of r things, which taken from a set of n things. The formula of permutation is

$$P(n,r) = \frac{n!}{(n-r)!}$$

3) Inclusion-Exclusion
From the sum-rule principle mentioned that if there are multiple sets of ways of doing a task, there wouldn't be any similarity between the sets, because if there are, it will be counted twice, and the result would be wrong. With Inclusion-Exclusion principle, we can count only unique ways of doing a task. For two sets of ways $A1$ and $A2$, the formula would be:

$$|A1 \cup A2| = |A1| + |A2| - |A1 \cap A2|$$

For example, how many binary strings of length 8 either start with '1' bit or end with two bits '00'? First, we can count all the string starts with '1' which is $1 \times 2^7 = 128\ ways$. If the string ends with '00' then the other 6 characters can be filled in $2^6 \times 1 \times 1 = 64\ ways$. Now if we just add both sets, we will get wrong answers, because there are strings that start with '1' and end with '00'. So, we need to subtract the previous total sets with the total of the strings that both satisfy the criteria, which is $1 \times 2 \times 2 \times 2 \times 2 \times 2 \times 1 \times 1 = 2^5 = 32\ ways$. With inclusion-exclusion principal we will get 128+64-32 = 160 ways.

4) Star and bars
It is used to find how many ways can one distribute k identical objects into n identical bins. We can imagine this as finding the number of ways to fit k balls into n boxes. For example, we have 6 objects to fit in 3 boxes

$$*** \mid * \mid **$$
$$** \mid ** \mid **$$
$$* \mid ** \mid ***$$

The number of ways to do that are,

$$C(6+3-1, 3-1) = C(8,2) = 28\ ways$$

Generally, the formula is:

$$C(n+k-1, k) = C(n+k-1, n-1)$$

B. *'24' Game*

'24' card game is one of exciting and challenging mathematical games. With this game, people can increase our intelligence, basic math skills, and many more. This game can be play for fun or even for competition.

'24' game have objective to arrange 4 random number cards to get 24 as the final result with using some of basic math operators. People usually play it using Poker Card that consists of 52 cards. The deck of card split into 4 symbols, each symbol consists of 13 cards (Ace, 2, 3, 4, 5, 6, 7 ,8, 9 ,10, Jack, Queen, King). In '24' game, we only care about the value of the card, not the symbol. Ace has 1 value, Jack has 11 values, Queen has 12 values, King has 13 values, and numbers have their respective face value.

Here are the rules of '24' game:

1) In the beginning of the game, dealer or one of the players will take 4 card from the shuffled deck. The game will end if the deck has no card anymore.
2) Using basic math operators; such as parentheses (()), subtractions (-), addition (+), multiplication (x), and division (÷); players have to arrange the numbers with the operators to get 24 as the end of the result
3) Every card can only be used once and must be used.
4) The fastest player who can come up with the solution won't take the card, but the other player does.
5) If none of the player can't find the solution, the cards will be put back to the deck, reshuffled it, and take another 4 cards.
6) The game is over if the deck has no more card and player with the least of card win.
7) If there is a draw, there will be 1 match point round to find the winner.

The strategy of this game is to find cards that can make 24 factors first, such as 1 and 24, 2 and 12, 3 and 8, 4 and 6.

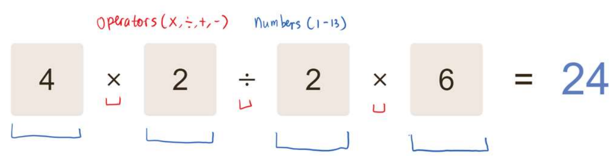

Fig.3 Example of '24' Game Arrangement (source: writer's archive)

Why the game use 24 as the result? It is because 24 is the smallest natural number with at least 8 divisors. This means, that if we draw 4 cards randomly from a deck, there is a big chance that it's solvable. The following chart shows the number of solvable random quadruple and its probability for the game using n as its result.

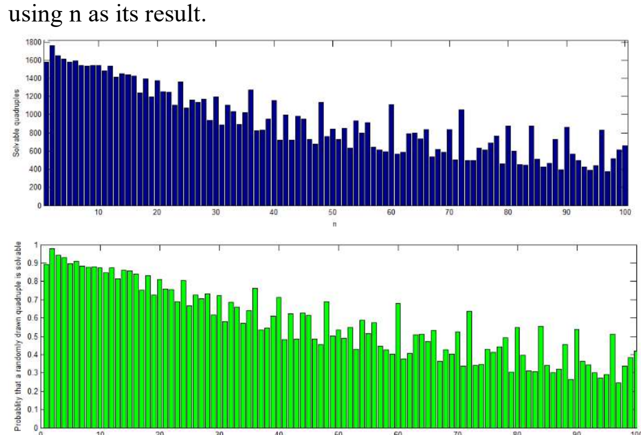

Fig.4 Chart Solvable Quadruple for n as its Result (source: https://www.4nums.com/24_game/facts/)

III. Implementation

A. Combination Sets of '24'

'24' game requires 4 cards to play. The player's goal is to arrange the 4 cards to get 24 as the result. Poker cards have value between 1 to 13. In between every number will be a math operator, so there will be total 3 operators that will be used.

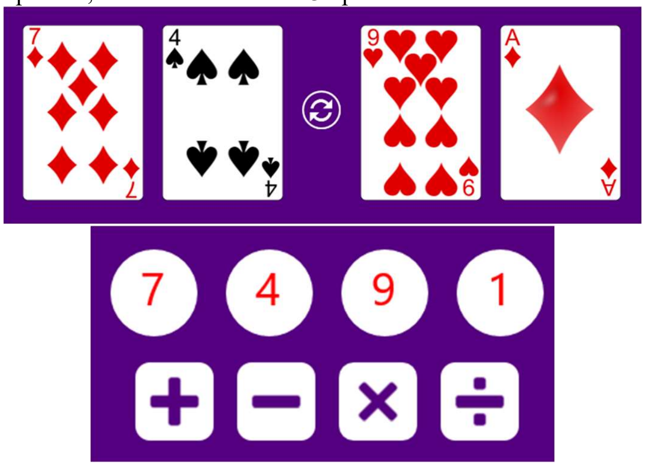

Fig.5 '24' Card Game Setup (source: writer's archive)

Assume n_i will be the number of cards with *i* as it value. While '24' game only needs 4 cards to be showed up, meanwhile there are 13 numbers, where each number can be showed up to 4 times, in the deck, we can make the equation:

$$n1 + n2 + n3 + n4 + n5 + n6 + n7 + n8 + n9 + n10 + n11 + n12 + n13 = 4\ cards$$

Using star-and-bars equation, we will get:

$$C(13 + 4 - 1, 4)$$

$$C(16,4) = \frac{16!}{(16-4)!\,4!}$$

$$C(16,4) = 1820\ combinations$$

Around 1362 out of 1820 combinations are solvable, that means around 75% of cards combination. This doesn't mean that randomly draw 4 cards from deck, 75% chance of it solvable. We can observe the possibility of each type of combination. Assume *a, b, c, d* are any 4 different values between 1 to 13, and let P be the probability drawn from a deck of a card:

$$P(\{combinations\}) = \frac{face\ up\ cards}{1\ deck\ of\ cards} \times combinations$$

$$P(\{a,a,a,a\}) = \frac{4}{52} \times \frac{3}{51} \times \frac{2}{50} \times \frac{1}{49} \times 1 = 3.69 \times 10^{-6}$$

$$P(\{a,a,a,b\}) = \frac{4}{52} \times \frac{3}{51} \times \frac{2}{50} \times \frac{4}{49} \times 4 = 5.91 \times 10^{-5}$$

$$P(\{a,a,b,b\}) = \frac{4}{52} \times \frac{3}{51} \times \frac{4}{50} \times \frac{3}{49} \times 6 = 1.33 \times 10^{-4}$$

$$P(\{a,a,b,c\}) = \frac{4}{52} \times \frac{3}{51} \times \frac{4}{50} \times \frac{4}{49} \times 12 = 3.54 \times 10^{-4}$$

$$P(\{a,b,c,d\}) = \frac{4}{52} \times \frac{4}{51} \times \frac{4}{50} \times \frac{4}{49} \times 24 = 9.45 \times 10^{-4}$$

After going through every solvable and adding the probability, the chance that a combination is solvable is 0.8046 if player randomly draw 4 cards from a deck.

Here is the list of some distinct solutions:

	Solvables	Solution 1	Solution 2	Solution 3	Solution 4
1	1 1 1 8	(1+1+1)×8			
2	1 1 1 11	(11+1)×(1+1)			
3	1 1 1 12	(1+1)×12×1			
4	1 1 1 13	(13-1)×(1+1)			
5	1 1 2 6	(1+1)×6×2	(2+1+1)×6		
6	1 1 2 7	(7+1)×(2+1)			
7	1 1 2 8	(2+1)×8×1			
8	1 1 2 9	(9-1)×(2+1)			
9	1 1 2 10	(10+2)×(1+1)	(10+1+1)×2		
10	1 1 2 11	11×2+1+1	(1+1)×11+2	(11+1)×2×1	
11	1 1 2 12	12×2+1-1	12/(1-1/2)		
12	1 1 2 13	13×2-1-1	(1+1)×13-2	(13-1)×2×1	
13	1 1 3 4	(1+1)×4×3			
14	1 1 3 5	(5+1)×(3+1)			
15	1 1 3 6	(6+1+1)×3	(3+1)×6×1		
16	1 1 3 7	(7-1)×(3+1)	(7+1)×3×1		
17	1 1 3 8	8×3+1-1			
18	1 1 3 9	(9+3)×(1+1)	(9-1)×3×1		
19	1 1 3 10	(10-1-1)×3			
20	1 1 3 11	(11+1)×(3-1)			
21	1 1 3 12	(3-1)×12×1			
22	1 1 3 13	(13-1)×(3-1)			
23	1 1 4 4	(4+1+1)×4			
24	1 1 4 5	(4+1)×5-1	(5+1)×4×1		
25	1 1 4 6	6×4+1-1			
26	1 1 4 7	(7-1)×4×1	(7+1)×(4-1)		
27	1 1 4 8	(8+4)×(1+1)	(8-1-1)×4	(4-1)×8×1	
28	1 1 4 9	(9-1)×(4-1)			
29	1 1 4 10	(1+1)×10+4			
30	1 1 4 12	12×4/(1+1)	(4-1-1)×12		
31	1 1 5 5	(5+1)×(5-1)	(5×5-1)×1		
32	1 1 5 6	(5-1)×6×1	(6-1)×5-1		
33	1 1 5 7	(7+5)×(1+1)	(7-1)×(5-1)		
34	1 1 5 8	(5-1-1)×8			
35	1 1 6 6	(6+6)×(1+1)	(6-1-1)×6		
36	1 1 6 8	8×6/(1+1)			
37	1 1 6 9	(1+1)×9+6			
38	1 1 6 12	(1+1)×6+12			
39	1 1 7 10	(1+1)×7+10			

Fig.6 Some Distinct Solution from Solvable Combinations (source: https://www.4nums.com/solutions/allsolutions/)

From the data, there are:

- 515 solvable combinations have 1 solution
- 427 solvable combinations have 2 solutions
- 216 solvable combinations have 3 solutions
- 125 solvable combinations have 4 solutions
- 31 solvable combinations have 5 solutions
- 17 solvable combinations have 6 solutions
- 17 solvable combinations have 7 solutions
- 8 solvable combinations have 8 solutions
- 2 solvable combinations have 9 solutions
- 3 solvable combinations have 10 solutions
- 1 solvable (2, 4, 8, 10) has 11 solutions

In '24' game, every solution of card combinations can be solved using combinatorics and brute force algorithm. Here are the steps of the code implementation:

- Assume *a, b, c, d* as the numbers and *opr* as the math operator
- We can split the operation into some sections

$$a\ opr\ b\ opr\ c\ opr\ d = 24$$

$$Group\ of\ 2$$

$$(a\ opr\ b)\ opr\ c\ opr\ d = 24$$

$$a\ opr\ (b\ opr\ c)\ opr\ d = 24$$

$$a\ opr\ b\ opr\ (c\ opr\ d) = 24$$
$$(a\ opr\ b)\ opr\ (c\ opr\ d) = 24$$

$$Group\ of\ 3$$
$$(a\ opr\ b\ opr\ c)\ opr\ d = 24$$
$$a\ opr\ (b\ opr\ c\ opr\ d) = 24$$

- From the number itself, we will have:
  All cards are different: (*a, b, c, d*)
  $$4\ \times 3\ \times 2 \times 1 = 24\ combinations$$
  1 pair are the same (*a, b, c, c*):
  $$\frac{4!}{2!} = 12\ combinations$$
  1 triple are the same (*a, c, c, c*):
  $$\frac{4!}{3!} = 4\ combinations$$
  All are the same (*a, a, a, a*):
  $$\frac{4!}{4!} = 1\ combinations$$
  There are 2 pair that have the same number (*a, a, b, b*):
  $$\frac{4!}{2!\,2!} = 6\ combinations$$

- From the operators, we have 4 kinds of it (subtractions (-), addition (+), multiplication (x), and division (÷)). So, the combination we will have:
  $$4 \times 4 \times 4 = 64\ combinations$$

- We will iterate in every combination to find every solution that possible. Here is the implementation of the code using python language

```
def solve(digits):
     solutions = []
     digit_length = len(digits)
     expr_length = 2 * digit_length - 1
     #permutation combination
     digit_perm        =        sorted        (set
(permutations(digits)))
     #operator combination
     op_comb       =       list(product('+-*/',
repeat=digit_length-1))
     # parentheses combination
     brackets = ([()] + [(x,y) for x in range(0,
expr_length, 2) for y in range(x+4, expr_length+2,
2) if (x,y) != (0,expr_length+1)] + [(0, 3+1, 4+2,
7+3)])
     run= True
     for d in digit_perm:
       if (not run):
          break
       for ops in op_comb:
          if (not run):
             break
          if '/' in ops:
             d2 = [('F(%s)' % i) for i in d]
          else:
             d2 = d
          ex    =    list    (chain.from_iterable
(zip_longest(d2, ops, fillvalue='') ))
          for b in brackets:
              exp = ex[::]
              for insert_point, bracket in zip(b,
'()'*(len(b)//2)):
                  exp.insert(insert_point,
bracket)
              txt = ''.join(exp)
              try:
                 num = eval(txt)
              except ZeroDivisionError:
# preventing zero division
                 continue
              if num == 24:
# if the result is 24, then show the solution and
stop the loop
                 if '/' in ops:
                   exp    =    [(term    if    not
term.startswith('F(') else term[2]) for term in
exp]
                 ans = ' '.join(exp).rstrip()
                 print("Solution found:", ans)
                 solutions.extend(ans)
                 run=False
                 break
  if (len(solutions)==0):
    print("No    solution    found    for:",    '
'.join(digits))
```

Fig.7 Implementation of '24' Solver Code (source: writer's archive, reference: https://stackoverflow.com/questions/61114206/how-to-list-all-solutions-in-24-game-using-python)

IV. Analysis

A. Decision Making

To show how combinatorics of the cards could lead to winning '24' game, this paper will analyze multiple cases of '24' game. The card that the player decides to arrange first will be

Here are the strategies:

1) Finding arrangement that makes 24 factors such as 1 and 24, 2 and 12, 3 and 8, 4 and 6.
2) If there are no such arrangement that satisfy the first steps, sum up all the numbers
3) If there are still no arrangement that get 24 as the result, find the multiple of 24 and use division to find the result

Here are some '24' game examples:

a) Game Example A

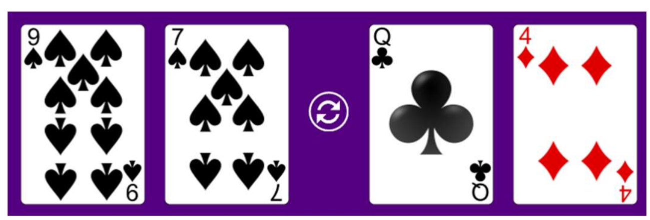

Fig 8 Game Example A (source: writer's archive)

In this sets, there are number 9, 7, 12 (Queen), and 4. Based on the strategy, player will find the arrangement that create '24' factors.

We could list down all the factors first:

$$24\ and\ 1{:}\ not\ possible$$
$$12\ and\ 2{:}\ possible$$
$$8\ and\ 3{:}\ not\ possible$$
$$6\ and\ 4{:}\ not\ possible$$

Solution: $12 \times (7 + 4 - 9) = 24$
There is also another solution, which using only addition and division: $12 - 4 + 7 + 9$

b) Game Example B

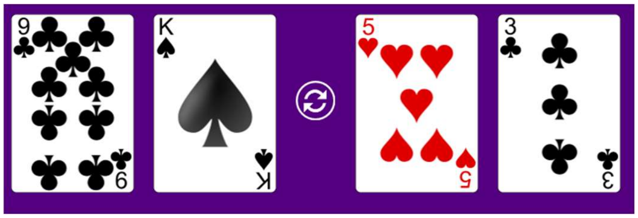

Fig 9 Game Example B (source: writer's archive)

In this sets, there are number 9, 13 (King), 5, and 3. Based on the strategy, player will find the arrangement that create '24' factors. We could list down all the factors first:

$$24 \text{ and } 1: \textit{not possible}$$
$$12 \text{ and } 2: \textit{not possible}$$
$$8 \text{ and } 3: \textit{possible}$$
$$6 \text{ and } 4: \textit{not possible}$$

Solution: $(13 - 5) \times (9 \div 3) = 24$
There is also another solution, which using only addition and division: $9 - 3 + 13 + 5$

c) Game Example C

Fig 10 Game Example C (source: writer's archive)

In this sets, there are number 10, 1 (Ace), 12 (Queen), and 4. Based on the strategy, player will find the arrangement that create '24' factors. Unfortunately, there are no such arrangements that satisfy '24' factors.

$$24 \text{ and } 1: \textit{not possible}$$
$$12 \text{ and } 2: \textit{not possible}$$
$$8 \text{ and } 3: \textit{not possible}$$
$$6 \text{ and } 4: \textit{not possible}$$

Even using only addition and subtraction we can't get 24. Here where the third strategy come in. We can find the multiple of 24, which is: $(12 \times 10) = 120$
Then, we know that 120 is 5 times 24, so we can get this equation.$(12 \times 10) \div (4 + 1) = 24$

B. Code Analysis

As mentioned before, the algorithm used to solve '24' game is adapted from brute force search (BFS). The BFS algorithm first accepts 4 numbers from input.

```
def input4Number():
   num1 = input("Enter First Number: ")
   num2 = input("Enter Second Number: ")
   num3 = input("Enter Third Number: ")
   num4 = input("Enter Fourth Number: ")
   digits = [num1, num2, num3, num4]
   return list(digits)
```

Fig 11 Code Initialization (source: writer's archive)

After accepting the numbers, the algorithm will initialize list of digits, digits' permutation, operators, and possible brackets position.

```
def solve(digits):
    solutions = []
    digit_length = len(digits)
    expr_length = 2 * digit_length - 1
   #permutation combination
    digit_perm  =  sorted  (set  (permutations
(digits)))
   #operators combination
    op_comb = list (product ('+-*/', repeat=
digit_length-1))
   # parentheses combination
    brackets = ([()] + [(x,y) for x in range(0,
expr_length, 2)
       for y in range (x+4, expr_length+2, 2)
       if  (x,y)  !=  (0,expr_length+1)] + [(0,
3+1, 4+2, 7+3)])
```

Fig 12 Code Initialization (source: writer's archive)

Then using the brute force search algorithm, the algorithm will iterate all the possible combination to find 24 as the result. The worst case, the algorithm will search around:

All numbers are different: (*a, b, c, d*)
$$4! \times 4^3 = 1536 \textit{ iterations}$$
1 pair are the same (*a, b, c, c*):
$$\frac{4!}{2!} \times 4^3 = 768 \textit{ iterations}$$
1 triple are the same (*a, c, c, c*):
$$\frac{4!}{3!} \times 4^3 = 192 \textit{ iterations}$$
All are the same (*a, a, a, a*):
$$\frac{4!}{4!} \times 4^3 = 64 \textit{ iterations}$$
There are 2 pair that have the same number (*a, a, b, b*):
$$\frac{4!}{2!\,2!} \times 4^3 = 384 \textit{ iterations}$$

```
run= True
for d in digit_perm:
    if (not run):
      break
    for ops in op_comb:
       if (not run):
          break
       if '/' in ops:
          d2 = [('F(%s)' % i) for i in d]
       else:
          d2 = d
       ex    =    list    (chain.from_iterable
(zip_longest(d2, ops, fillvalue='')))
       for b in brackets:
```

```
        exp = ex[::]
        for insert_point, bracket in zip(b, '()'*(len(b)//2)):
            exp.insert(insert_point, bracket)
        txt = ''.join(exp)
        try:
          num = eval(txt)
        except ZeroDivisionError:
        # preventing zero division
          continue
        if num == 24:
        # if the result is 24, then show the solution and stop the loop
          if '/' in ops:
            exp = [(term if not term.startswith ('F(') else term[2])for term in exp]
          ans = ' '.join(exp).rstrip()
          print("Solution found:", ans)
          solutions.extend(ans)
          run=False
          break
if (len(solutions)==0):
   print("No solution found for:", ' '.join(digits))
```

Fig 13 BFS Algorithm (source: writer's archive)

The algorithm will start to iterate in every combination of the sets. If the algorithm finds a solution arrangement, the solution will be printed and merge into list of solutions.

Here are some test cases using the code implementation:

a) Game Example A

Fig 14 Solution Game A using Python (source: writer's archive)

b) Game Example B

Fig 15 Solution Game B using Python (source: writer's archive)

c) Game Example C

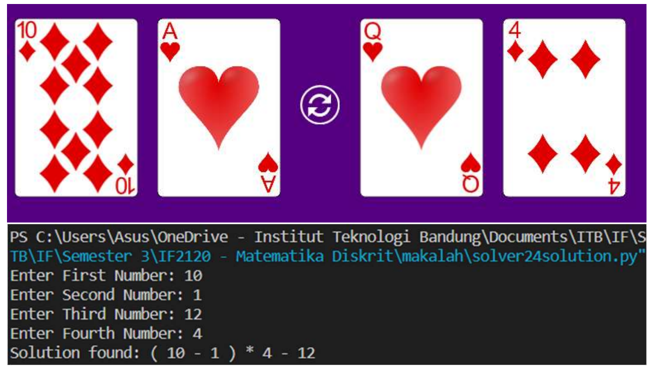

Fig 16 Solution Game C using Python (source: writer's archive)

V. Conclusion

'24' is not just a mathematic game, by using combinatorics can help people to win this game by using some strategies, such as finding '24' factors arrangement and so on. From this paper analysis, it shows there are 1820 sets of quadruple card combinations and among of them there are 1362 (around 75%) sets that are solvable. Knowing that '24' is the smallest natural number with at least 8 divisors, it makes the games more possible to be solve. This paper shows that combinatorics can help people to make fast decision based on list of combinations. Combinatorics has proven that this game can be win even we're not that fast at counting numbers. With practicing, people can increase the probability of winning '24' game and their skills.

VI. Acknowledgment

The writer would like to thank all IF2120 lecturers especially Dr. Nur Ulfa Maulidevi, S.T., M.Sc as lecturer in first class IF2120 for Discrete Mathematics, for teaching and supporting students to write these paper. I have gained much better understanding in combinatorics and its application. I also would like to thank Dr. Ir. Rinaldi, M.T, who provided students with plenty of resources on Discrete Mathematics at the website.

References

[1] Admin, 24T. (2012, September 12). Facts about 24 the math game - 4nums.com, solves 24 the math game. Retrieved December 12, 2022, from https://www.4nums.com/24_game/facts/

[2] Foundation, C. K.-12. (2018, August 28). 12 foundation. Counting with Permutations and Combinations. Retrieved December 12, 2022, from https://flexbooks.ck12.org/cbook/ck-12-college-precalculus/section/14.2/primary/lesson/counting-with-permutations-and-combinations-c-precalc/

[3] Geeks, G. F. (2021, July 16). Mathematics: Combinatorics basics. Mathematics | Combinatorics Basics. Retrieved December 9, 2022, from https://www.geeksforgeeks.org/mathematics-combinatorics-basics/

[4] Halpern, J. (2004). Discrete mathematics in computer science. CS 280. Retrieved December 12, 2022, from https://www.cs.cornell.edu/courses/cs280/2004FA/

[5] Online, A. P. S. (2020). Page. Ball-and-urn. Retrieved December 9, 2022, from https://artofproblemsolving.com/wiki/index.php/Ball-and-urn

[6] R. Munir, "Kombinatorial Bagian 1," IF2120 Matematika Diskrit. Retrieved: December 10, 2022, from https://informatika.stei.itb.ac.id/~rinaldi.munir/Matdis/2020-2021/Kombinatorial-2020-Bagian1.pdf

[7] R. Munir, "Kombinatorial Bagian 2," IF2120 Matematika Diskrit. Retrieved: December 10, 2022, from

https://informatika.stei.itb.ac.id/~rinaldi.munir/Matdis/2020-2021/Kombinatorial-2020-Bagian2.pdf
[8] Yuan, N. (2020, April 1). How to list all solutions in 24 game using Python. Retrieved December 10, 2022, from https://stackoverflow.com/questions/61114206/how-to-list-all-solutions-in-24-game-using-python

PERNYATAAN

Dengan ini saya menyatakan bahwa makalah yang saya tulis ini adalah tulisan saya sendiri, bukan saduran, atau terjemahan dari makalah orang lain, dan bukan plagiasi.

Bandung, 12 Desember 2022

Alexander Jason 13521100