Applications of Decision Tree in Hare and Hounds Game

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***Abstract*—Hares and Hounds is a variant of Hare games, which is one of the traditional European hunt games, with not so clear origins. At a glance, it seems like a simple strategy game, similar to Tic Tac Toe, with an asymmetric gameplay. But those with keen senses will realize that there are situations which has a definitive conclusion. Using trees we learn from Discrete Mathematics, more specifically the decision tree, we can learn what is the best play in specific situations, which will cause the hare or hound to win.**

***Keywords*—situation, turn to move, tree, decision.**

I. Introduction

Hares and Hounds is a variant of Hare games, which is one of the traditional European hunt games. Nowadays, people play it as a simple strategic board game, such as tic tac toe or a simpler version of chess or checkers. Other variant of Hare games include "The Soldier's Game" or small Hare Game, large Hare game, "Game of Dwarves" or "Catch the Giant". These games were common in northern Europe, but nowadays almost forgotten. These games exist since the medieval times[1].

Hare and hounds is a somewhat improved variant of the Hare games, where the rules have been revoked to make the game more balanced. However, the design of the game has caused it to have a somewhat deterministic outcome, where there is a situation where the hounds will always win regardless what the hare do, and where hare will win by stalling no matter what the hounds do.

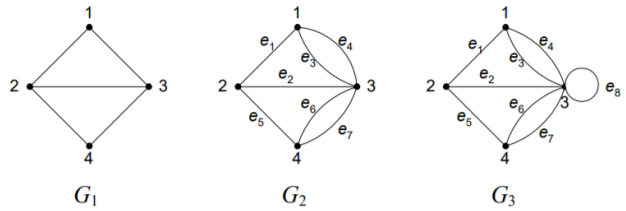
In Discrete Mathematics, we learn about graphs, and it's various types, simple graph, unsimple graph (which includes multi graph and pseudo graph), and directed and undirected graph. There are also other terms for graphs, including connected, unconnected, planar, circuits, path and more. A particular type of graph is called a tree, where the graph is connected, undirected and doesn't contain any circuit.

In this paper, we use a tree to decide the best course of action for both the hounds and the hare in an ongoing game. In Discrete Mathematics, this is called a decision tree, one of the many uses of trees. However, we will only be looking on situations that will often occur in this game, the rest of the situations are able to be resolved with common sense, which will allow the hare or hound to be the victor. [2]

II. Theoretical Basis

2.1 Graph

Graphs are used to represent discrete objects and the relations between them. Graphs consists of a non-empty set of vertices, and a set of edges, which will define the relation between the vertices[2]. Here, we will learn several type of graphs and several terms necessary to understand a tree.



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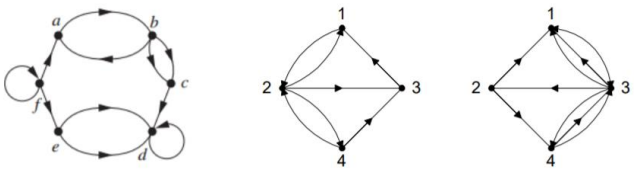
The picture above shows several type of graphs. The first one (G1) is a simple connected, undirected graph. The second one (G2) is a connected, undirected multi graph. The third (G3) one is a connected, undirected pseudo graph (because it has a loop). Directed graphs are graphs with arrows to show it's a one way relationship. Examples of directed graphs would be such as the ones below.



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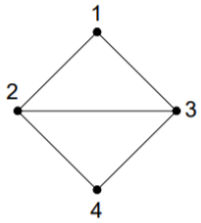
A path is a series of edges that connects one vertex to another[2]. For a more clear illustration, look at graph G1 below. Path 1,2,4,3 means it's a path from vertex 1 to 3 with (1,2), (2,4) (4,3) as the edges.

A circuit is a path which starts and ends in the same vertex[2]. So path 1,2,4,3,1 is a circuit.



G_1

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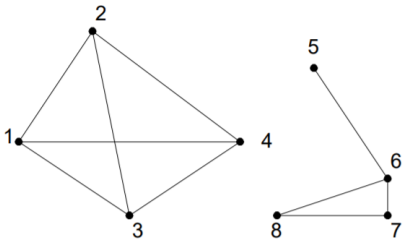
A graph is connected if and only if for every vertex vi and vj, there is a path from vi to vj[2]. An example of an unconnected graph would be such as the one below.



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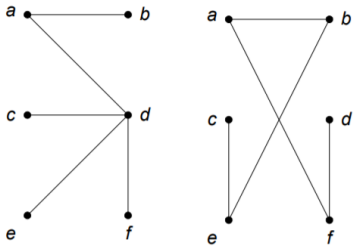
2.2 Tree

A tree is defined as an undirected connected graph with no circuits. Examples of trees are such as ones below.



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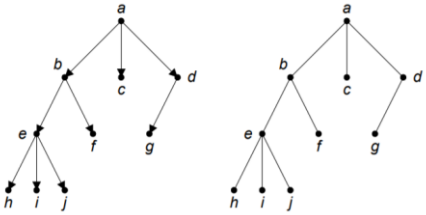
Normally however, trees are represented with one vertex as the root and is given direction. This is called a rooted tree[2]. As a an agreement the arrows in a tree can be removed.



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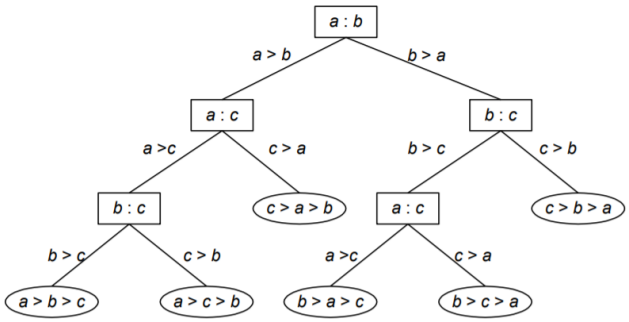
There are lots of uses for trees. It can be used as a tree for expression (to check syntax for example), to make a prefix code (such as Huffman code), it can also be used to store information in a certain way, and then be read quickly when finding information, using the binary search tree. What we're interested in this paper is a decision tree.

Decision tree is a tree that is used to an action or draw a conclusion based on the current situation. An example of this is like the decision tree to sort the biggest number between 3 numbers. This tree is show below.



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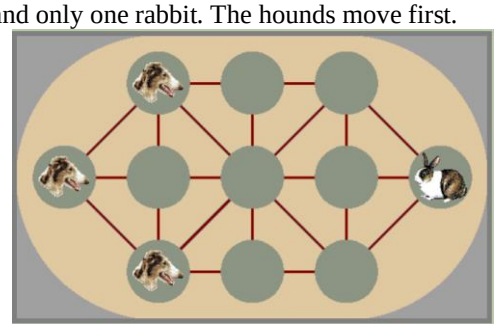
In this paper, the format of tree will be a little bit different, the arrow(directions) will not be removed, and for the decision tree part, the if will be written in one of the node.

For example, in the decision tree above, it will look more like a:b →a > b →
→ b>a →

2.3 Hare and Hounds

Hare and Hounds is a asymmetrical strategy game, where one player control the hounds, and the other control the hare. The hounds is able to move forward, or up and down, but not backwards, according to the board. The hare is allowed to move anywhere according to the lines on the board. There are 3 hounds, and only one rabbit. The hounds move first.



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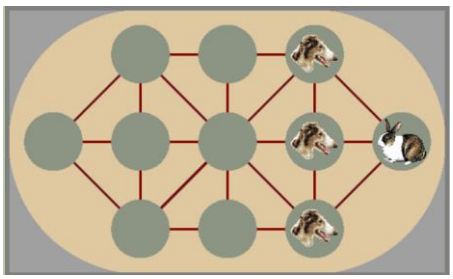
http://www.lutanho.net/play/harehound.html

The picture above is the start positions of the hounds and the hares. The objective is for the hounds to entrap the hare so that the hare have no tiles to move from. The objective for the hare is to "escape" the hounds, or more specifically to be in the same column or to the left of the most left hound.

Here is an example if the hounds win.



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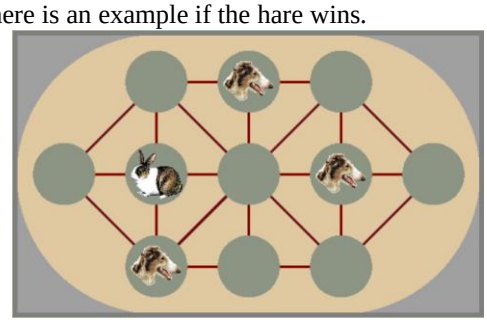
And here is an example if the hare wins.



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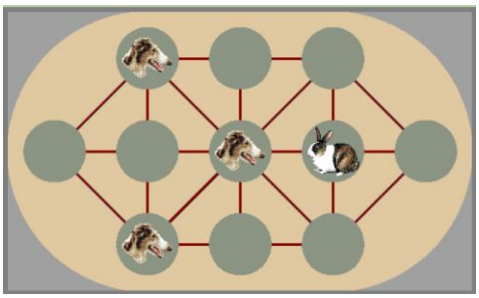
There is also one subtle but really important rule, called stalling. Stalling is when the hound doesn't advance, and instead move up and down several times in a row. If the hounds move up and down 10 times without moving forward(right), then the hare wins. Also, the hare and the hounds must move each turn, there can be no turn where they just stay still.

As you can guess, there are lots and lots of ways the game can be played out. However, most of the plays will often result in one of 3 patterns. It includes…

First one:



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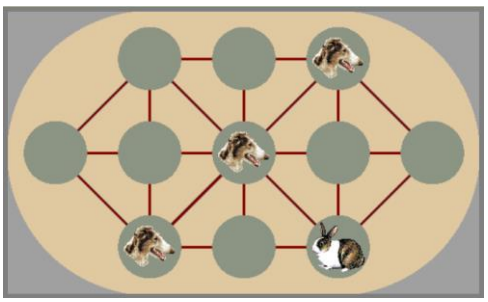
Second one:



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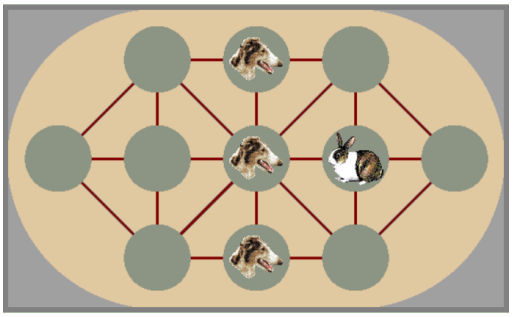
Third one:



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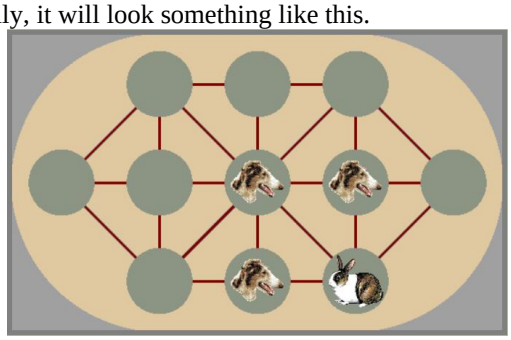
In this paper, we will only be looking in the last 2 situations, because depending on whose turn it is, this situation is solved either for the hare or for the hound to win.

III. Applications of Decision Tree in Choosing the Right Move.

3.1 Stalemate

The first thing we should note is the more subtle ways for the hare to win. Apart from being the left side of the hounds, the hare can also win if the chessboard come to a stalemate. Generally, it will look something like this.



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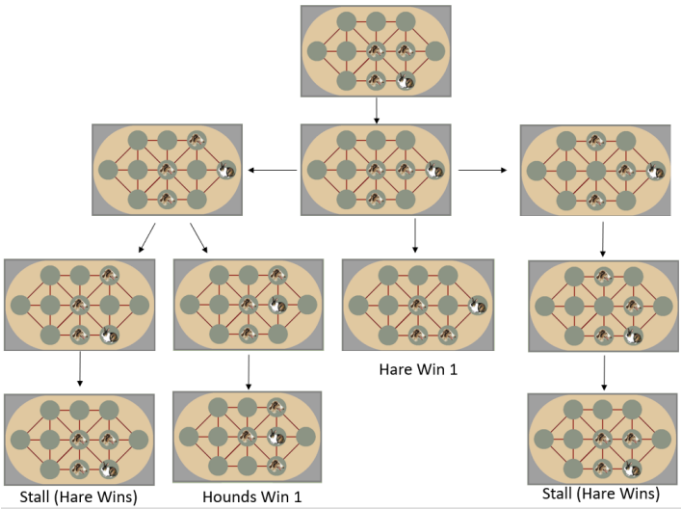
In the above situation, with the *hare's turn to move*, all the hare need to do is to not go to the middle circle. Here is the tree to show what's possible.

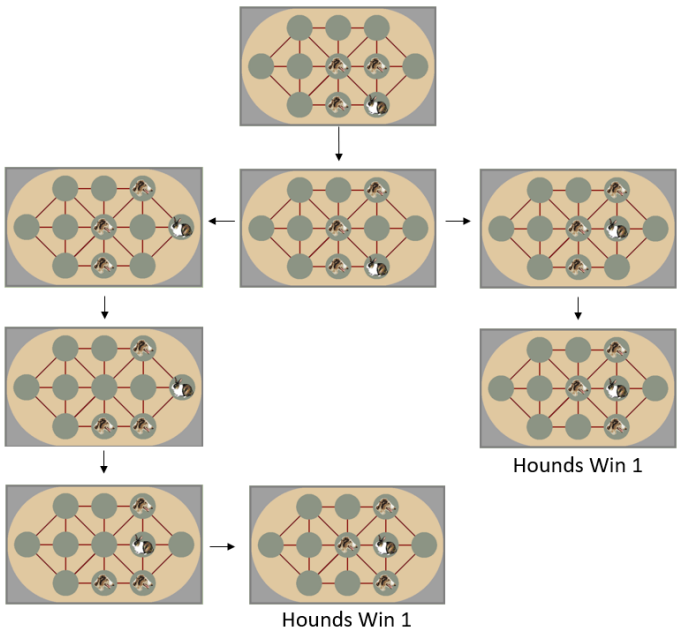


3.1.1 Stalemate Situation

The tree here shows all the steps that the hare can take, and all the logical steps the hounds would take. If the hounds choose to not be into one of these positions, the hare could almost guarantee a win, assuming that the hare do the correct move set. The stall may win because it is just wasting time for the hounds. On Hounds Win 1, the hounds will win because the hare don't have any other way to go. On Hare Win 1, the hare will win because if the hare goes left, the hound will create an opening that is easy for the hare to pass by. If the hare goes up, then the hounds can't block the hare from going to the left.

As you can see, the hare will always win this type of situation. The situation described above will from now on be referred to as stalemate.

However, the same situation, if it was the *hound's turn to move*, would be a conclusive win for the hounds. The tree below will describe the process.

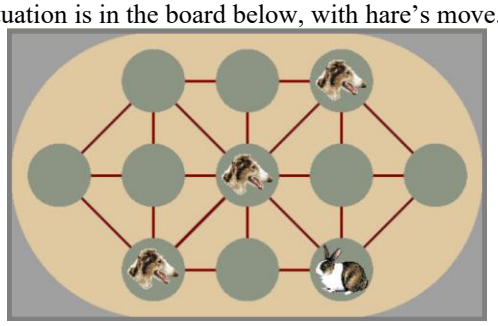


3.1.2 Hounds Win Situation

As you can see, if it's the hounds turn to move at that current situation, then regardless of what the hare do, the hounds will definitely win. Hounds Win 1 is a win because the hare will be forced to move back, then the hounds can entrap the hare.

3.2 Situation One

As discussed in chapter 2.3, we will only discuss these 2 situations, as the first one don't have a definitive conclusion. The situation is in the board below, with hare's move.



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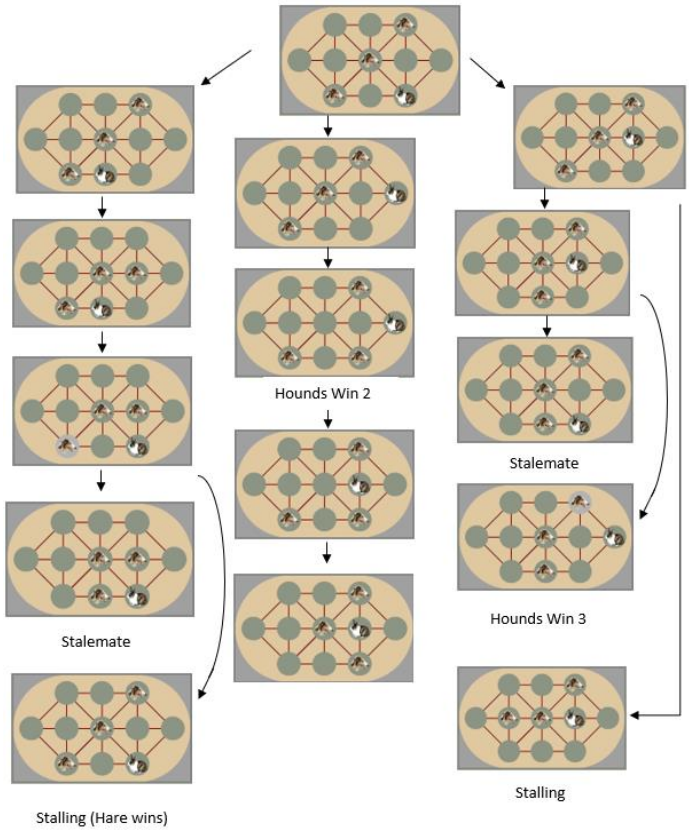
In the above situation, with the *hare's turn to move*, there are several steps that the hare can take. There are more steps to this tree, but it will be listed here as stalling.



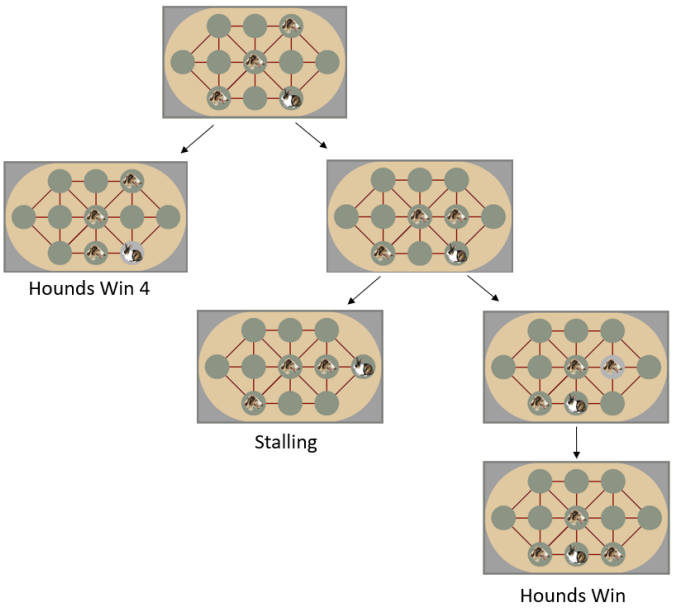
3.2.1 Situation One Resolution (Hare).

The tree above shows all the possible first steps the hare can take, and all the logical steps the hounds would take. There are more steps that hounds can take, but it would just lead the rabbit to victory. There are some steps that should be explained. Stalemate is a situation we have already discussed beforehand, on picture 3.1.1. Hounds Win 2 is a state where it's the hare's turn to move, but as shown in the tree, is a definite win for the hounds. So this situation later on will be called as Hounds Win 2. Hounds Win 3 is a state where it's the hounds turn to move.

One of the branches of a Hounds Win Situation, described in picture 3.1.2. The situation on the bottom right part of the tree, I call as stalling, and indeed it is if the hare plays it right. There are more ways for that situation to resolve, which can't be fitted into this tree.

As you can see, the hare can win by stalling if they take the path of the left branch of that tree. If the hounds ever decide to not do what the tree does, then the hare can really easily win.

In the same situation, if it's the *hounds turn to move*, this tree below describes the moves that the hounds can take.



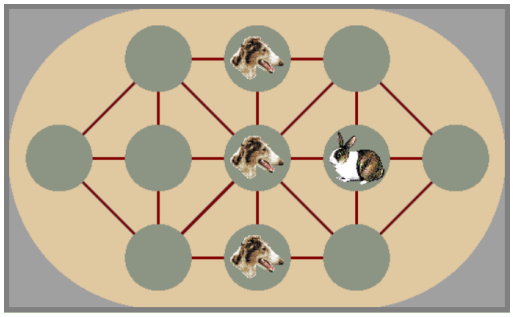
3.2.2 Situation One Resolution (Hounds)

The tree in picture 3.2.2 lists all the moves the hounds would take, as other moves just won't make sense. There are some explanations that needs to be made. Hounds Win 4 is a branch of 3.1.2 Hounds Win Situation, whilst the stalling part here, there is too much move set, but the hare can stall the game one way or another if this does happen.

As you can see, if it is the hounds turn to move, as long as the hounds move to the hounds win 4 situation, then the hounds would win. The other choice is listed out just to show, that it's not a risk going after, although some might go choose this way.

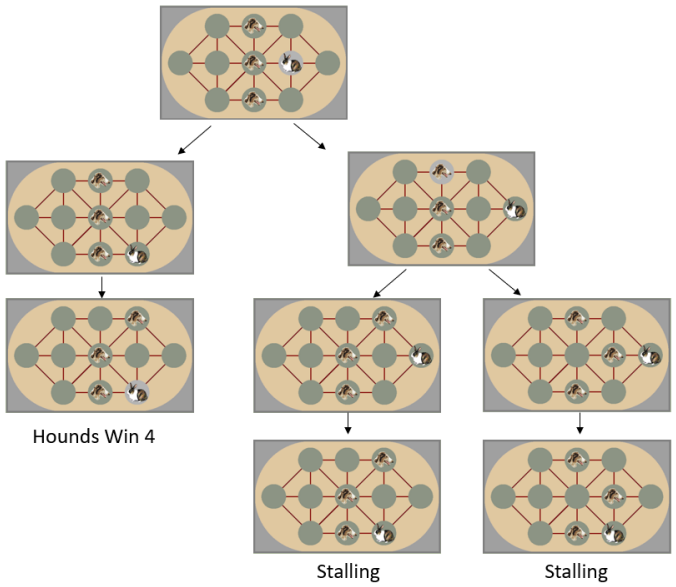
3.3 Situation Two

We will now discuss the second situation that appears a lot in the game. The situation can be seen below.



Source:
http://www.lutanho.net/play/harehound.html

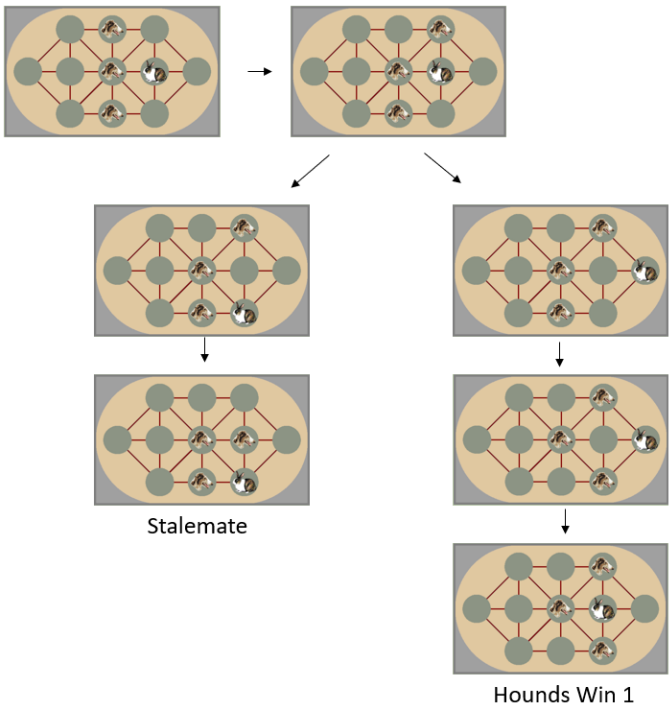
For the steps that is coming on this situation, the hare going up and down will be regarded as one of the same, because it is symmetrical, all the moves the hounds make then can be mirrored.



3.3.1 Situation Two Resolution (Hare)

The tree above shows all the possible hare moves and all the moves hounds would do if it is Situation Two, *on hare's turn to move*. There is not much explain needed, both stalling situation are a part of the stalemate tree. The Hounds Win 4 situation is the exact same situation in picture 3.2.2, where the hounds would decisively win.

Since the hare moves first, the hare can choose which path to follow. Because of it, the hare can always win as long as it choose the right path.



3.3.2 Situation Two Resolution (Hounds)

The tree in 3.3.2 shows all the possibility that can happen in Situation Two, if it's *hound's turn to move*. There is no other explanation needed.

This situation is interesting, as the hounds are forced to only move forward, and the one deciding which path to go is the hare. So either way, in Situation Two, the hare will win by stalling.

IV. Conclusion

Some interesting points that we can take is, while both of the situations have their corresponding trees, the turn (hare's turn or hound' turn to move) really matters. With the same situation, the difference between the turns can mean a conclusive victory or defeat. So the strategy shifts from just choosing a favorable position to a favorable position and turn. Some of the turn can be shifted by the hare by going to by going to middle or the top, and then to the middle. The hounds can shift the turns by having the top hounds go to the middle, then the last hound go forward, then go to the top, or just go to the middle immediately.

Acknowledgement

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References

[1] http://mlwi.magix.net/bg/haregames.htm accesed in 6th of Decmber 2020

[2] http://informatika.stei.itb.ac.id/~rinaldi.munir/Matdis/2020-2021/matdis20-21.htm accesed in 7th of December 2020

[3] http://www.lutanho.net/play/harehound.html accesed in 10th of December 2020.

Pernyataan

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Jakarta, 11 Desember 2020

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