Implementation of Modular Arithmetic for The "Last Two Cards Match" Card Trick

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***Abstract*—Number theory could be said as the most prominent subject in mathematics, one of the studies in this subject is modular arithmetic. There are many implementations of modular arithmetic in daily life of people. One of them is being the secret to the "Last Two Cards Match" card trick. To put it simply, the card trick is about matching a pair of cards while shuffling the piles every time one spelling out the keyword. The original version only needed modular arithmetic to determine the length of the keywords that are used in the trick. On the other hand, the modified version utilizes also modular arithmetic and Chinese Remainder Theorem (CRT) with the use of Gauss' algorithm on CRT to solve the problem and determine the length of the one keyword used in the trick.**

***Keywords*—Card Trick, Chinese Remainder Theorem (CRT), Gauss' Algorithm on CRT, "Last Two Cards Match", Modular Arithmetic, Number Theory**

I. Introduction

Card trick is one of the branches of magical illusion by creating effects utilizing cards. Generally speaking, card trick technique could be divided into two parts: sleight of hand and mathematical theorems (such as the number theory) for the performance trick. For this particular paper, the card trick that is going to be researched is The "Last Two Cards Match". The card trick was first invented by Howard Adams in 1984, in his book "Oh, I See You Have ESP". The original card trick for this paper is different from Howard's, as the author preferred the modified version to maximize the prominence of Chinese Remainder Theorem. The modified card trick is performed first by setting up two piles of cards of N number. The spectator would randomly select one of the piles to be 'shuffled'. However, after all the shuffling of those two piles, when the magician opened up the faces of the cards pair by pair, all of them would match exactly as the face of the card from the opposite pile. This card trick is of course not using 'real magic', instead it uses modular arithmetic and the Chinese Remainder Theorem to make the set up ready and thus createx the effects desired at the end. This paper will thoroughly explain the secret of this particular magic trick which is the implementation of modular arithmetic particularly the Chinese Remainder Theorem.

II. Basic Theory

For better understanding of this paper, it is a must to review the basic theory of both card trick and modular arithmetic.

2.1. Number Theory

In this chapter, there will be explanation regarding the definition of number theory, the division properties of integers, modular arithmetic, congruence, Chinese Remainder theorem

2.1.1. Definition of Number Theory

Number theory is the branch of pure mathematics focusing on the properties of the integers and any function that returns integers. For the nature of mathematics is manipulating numbers and all the numbers are actually based by integers, there is one quote that could describe this characteristic of number theory. "Mathematics is the queen of the sciences - and the number theory is the queen of the mathematics." Carl Friedrich Gauss.

2.1.2. Division Properties of Integers

In this subchapter, there will be an explanation regarding the properties of dividing integers followed by the examples.

Let a and b are any two integers and $a \neq 0$. Then a divides b if there there is an integer c in such a way $b = ac$.

Notation:

$$a \mid b \text{ (if } b = ac;\ a,b,c \in \mathbf{Z};\ a \neq 0)$$

Example:

$$4 \mid 12, \text{ because } 12 = 4 * 3$$

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2.1.3. Modular Arithmetic

Modular Arithmetic is an arithmetic system of integers that, where the result value of the operation using modulo "wraps around" a certain number which is the divider. To get a better understanding, see the following explanation.

Let a and m are any two integers and m ≠ 0, the operation

$$a \bmod m$$

would return the remainder of a divided by m.

Notation:

$$a \bmod m = r$$

such a way $a = mq + \mathrm{r}$ (with $0 \leq r < \mathrm{m}$)

m is called a *modulus* value, it is included in the set {0, 1, 2, 3, 4, 5, 6, …}. q is an integer value, it could be a negative or positive number.
Example:

23 mod 5 = 3	(23 = (5)*4 + 3)
17 mod 2 = 1	(17 = (2)*8 + 1)
8 mod 9 = 8	(8 = (9)*0 + 8)
-43 mod 7 = 6	(-43 = (7)*(-7) + 6)

Notice that -43 mod 7 = -1 (-43 = (7)*(-6) + (-1)), because the value of r should be positive.

The most known modular arithmetic usage is in the 12-hour clock system. For instance, if right now it is 09.00 AM then 4 hours later it would be 1.00 PM. Simple addition would result in 9 + 4 = 13, but a clock only has 12 digits on its face, that is why it shows 1 since 13 mod 12 = 1.

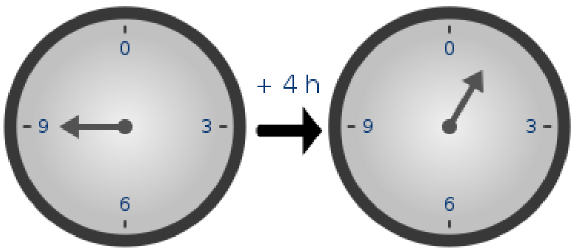

Figure 1. Usage of Modular Arithmetic in Time Keeping
Source:https://upload.wikimedia.org/wikipedia/commons/thumb/a

2.1.4. Congruence

Another expression for modulo operation is by using the congruence notation. Congruence is similar to the normal modulo operation, however it determines whether two integers have the same remainder if divided by a modulus. To get a better understanding, see the following example:

Let a and b are any two integers, m is the *modulus*. Then, a is congruent to b of a modulus m.
Notation:

$$a \equiv b \pmod{m}$$
if a mod m = b mod m.
if and only if $m \mid (a - b)$

Example:

$$38 \equiv 13 \pmod{5}$$
$$14 \equiv 2 \pmod{12}$$
$$10 \equiv 6 \pmod{4}$$

Congruence will be the foundation for this particular card trick that the author is going to explain. However, for the setup of this card trick, it is compulsory to understand the linear congruence system.

Linear congruence system is a set of congruence expressions, with the value of the first integer is the same throughout all the expressions. For better understanding, see the following example.

Let x and a_1, a_2, a_3,…, a_n are any integers, m_1, m_2, m_3,…, m_n are the modulus of all the expressions. Then the below expressions is a linear congruence system.

Notation:

$$x \equiv a_1 \pmod{m_1}$$
$$x \equiv a_2 \pmod{m_2}$$
$$\ldots$$
$$x \equiv a_n \pmod{m_n}$$

Example:

$$8 \equiv 2 \pmod{3}$$
$$8 \equiv 3 \pmod{5}$$
$$8 \equiv 0 \pmod{4}$$

2.1.5. Chinese Remainder Theorem

In the 3rd century AD, a chinese mathematician named Sun Tzu stated a problem as the following:

"Determine an integer that if divided by 5 remains 3, if divided by 7 remains 5, and if divided by 11 remains 7."

Remembering the previous subchapter, this problem could be interpreted as a linear congruence system:

$$x \equiv 3 \pmod{5}$$
$$x \equiv 5 \pmod{7}$$
$$x \equiv 7 \pmod{11}$$

Sun Tzu stated the theorem as such

$$x \equiv a_1 \pmod{m_1}$$
$$x \equiv a_2 \pmod{m_2}$$
$$\ldots$$
$$x \equiv a_n \pmod{m_n}$$
where m_1,...,m_n relatively coprime for each pair

Then, the linear congruence system has a solution for x from $0 \le x \le m$, where $m = m_1 * m_2 * \ldots * m_n$ and all the other solutions that are congruent of a modulus m.

As stated from Sun Tzu, such a system above is solvable. The steps to calculate x should be explained below. The steps below were originally formulated by the mathematician Gauss.

1. Find $m = m_1 * m_2 * \ldots * m_n$
2. Find $n_i = m/m_i$ for each expression
3. Find u_i from the equation $n_i u_i \equiv 1 \pmod{m_i}$
4. Then x should be sum of $a_i * n_i * u_i$

For better understanding, see the following example:

$$x \equiv 3 \pmod{5}$$
$$x \equiv 5 \pmod{7}$$
$$x \equiv 7 \pmod{11}$$

We could follow the steps above by constructing a table such as below:

m_i	a_i	n_i	u_i
5	3	385/5 = 77	$77u_1 \equiv 1 \pmod{5}$, u_1=3
7	5	385/7 = 55	$55u_2 \equiv 1 \pmod{7}$, $u_2 = 6$
11	7	385/11 = 35	$35u_2 \equiv 1 \pmod{11}$, $u_3 = 6$

$$x = \Sigma\, a_i * n_i * u_i$$

Thus, x = 3*77*3 + 5*55*6 + 7*35*6 = 3813 ≡ 348 (mod 385)

Although the Gauss Steps alone could solve the Chinese Remainder Problem, for this particular card trick it is mandatory to understand another method to calculate x where the system does not have coprime pairwise for each m_i. Another method includes this property:

$$\text{if } x \equiv a \pmod{mn}, \text{ where } m \text{ and } n \text{ are coprime,}$$
$$\text{then } x \equiv a \pmod{m} \text{ and } x \equiv a \pmod{n}$$

By having the above property, elimination of modulus values that are not coprime is possible. For instance,

$$\begin{array}{lcl} x \equiv 1 \pmod 2 & & x \equiv 1 \pmod 2 \\ x \equiv 1 \pmod 3 & \Rightarrow & x \equiv 1 \pmod 3 \\ x \equiv 1 \pmod 6 & & \end{array}$$

The above removal is legal since 6 is 2*3 then $x \equiv 1 \pmod 6$ is $x \equiv 1 \pmod 2$ and $x \equiv 1 \pmod 3$. Due to the existence of the previous expressions, then $x \equiv 1 \pmod 6$ can be eliminated from the system.

2.2. Card Tricks

In this chapter, there will be an explanation about the definition of card tricks, play styles in card tricks, the "Last Two Cards Match" card trick for both the original version and modified version.

2.2.1. Definition of Card Tricks

Card tricks or card manipulation is the branch of magical illusion that is often used in close up magical performance that deals with creating desired effects by playing cards.

2.2.2 Play Styles in Card Tricks

Card tricks could be divided generally into two styles: Sleight of Hand and Self-working Card Trick. The latter is the one used for the card trick that is going to be explained.

a. Sleight of Hand

Like its name, Sleight of Hand is a style where the performer manipulates the cards by using sets of skills of fast hand movements and misdirections. There are so many techniques involved in this style such as: Lifts, False deals, Side slips, Passes, Palming, False shuffles, False cuts, Color change, Crimps, Jogs, Reverses, Forces.

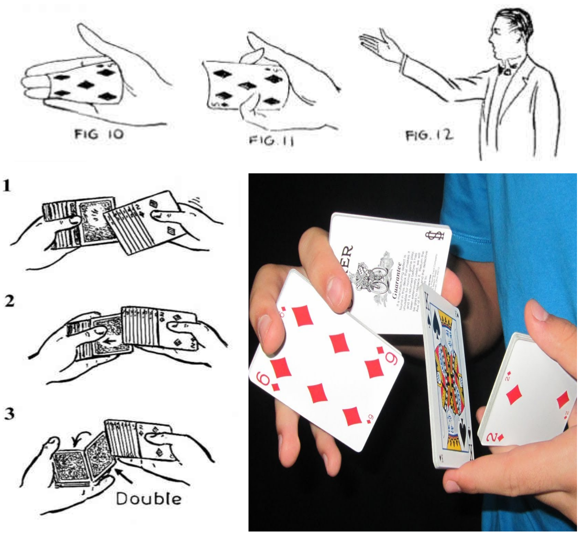

Figure 2. Backpalm Technique (First image), Double Lift (Left Bottom Image), Sybil False Cut (Right Bottom Image)
Source:https://www.magicalapparatus.com/silk-handkerchief-3/

b. Self-working Card Trick

On the other hand, Self-working Card Trick is not something that is really hard to perform. However, it is so hard to create a new original self-working card trick. This kind of style is hard to be invented because it needs the creativity and of course mathematics formula that could be implemented while playing the card trick for the performance. For this play style, magicians don't need to do many things when performing the trick. However, for some particular tricks, they need to set up the deck of cards so it could work as desired.

Figure 3. A Marked Ace of Heart
Source:https://spielkartenshop.com/media/image/product/1169/

Mainly in this style, it could be divided into two parts: using gimmick and pure mathematics. Magicians usually use gimmicks in their deck of cards such as marked decks (marked with symbols to recognize the face without actually seeing it), stripper decks (decks with different size of cards), and so on. It is true that pure mathematics for set up is rarely seen because it is not as amusing as gimmick with really amazing effects or sleight of hands that is just pure commitment for those skills or sometimes a new original trick is just really hard to reproduce. However, the author found a good card trick that its secret actually utilizes Chinese Remainder Theorem.

2.2.3. The "Last Two Cards Match" Card Trick

The "Last Two Cards Match" card trick was first invented by Howard Adams in 1984, in his book "Oh, I See You Have ESP". The card trick is actually really simple, the big idea is to match pairs of cards on 2 piles while shuffling the piles as the spectator likes. This card trick usually is led by sleight of hand performance before to set the deck of cards as the magician desires. In this subchapter, there will be an explanation regarding the original version and the modified version.

a. Original Version

In this sub-subchapter, it is going to be explained a set of instructions on how to perform the original card trick.

For the original version, the magician needs to set up 5 cards from the red suit (could be hearts or diamonds) and the other 5 matched cards (e.g. Ace of Heart and Ace of Spade) from the black suit.

After that, split those 10 cards into 2 piles with 5 cards each facing down and set the sequence of the cards as the following (For example, the author use the cards Ace, 2, 3, 4, 5)

The Spade Suite Pile: Ace, 2, 3, 4, 5
The Diamond Suite Pile: 5, 4, 3, 2, Ace

Figure 4. Example Set Up for The Original Card Trick

Moving on, the magician will say to the spectator that he will spell “Last Two Cards Match” one letter by one. While spelling the words, the magician will shuffle any of the pile (By shuffling means, putting the top card of the pile to the bottom of the pile).

Before shuffling the piles, the magician asked the spectator to say what pile should be shuffled each time an alphabet is spoken out from the magician’s mouth. If the spectator says nothing, then the pile before would be shuffled again.

For instance, a dialogue of the magician and the spectator would be written below (M as the magician, S as the spectator):

M: “L!”
S: “Shuffle the left pile!” (M shuffle the left pile)
M: “A!”
S: “...” (M still shuffle the left pile)
M: “S!”
S: “Right” (M shuffle the right pile)
M: “T!”
S: “Switch” (M shuffle the left pile)

After a word is spelled out, the magician takes the top cards of each pile, takes it as a pair and sets it on the side. Following this, the magician will spell the next word which is “Two”.

The magician will continue doing the previous instructions until all words from the sentence “Last Two Words Match” is spelled out.

After finishing the spelling, the magician will reveal each of the pair and “Voila!”, all the cards match, even with all that shuffling.

b. Modified Version

In this sub-subchapter, it is going to be explained a set of instructions on how to perform the modified version of the card trick.

For the modified version, the magician needs to set up 4 cards from the red suit (could be hearts or diamonds) and the other 4 matched cards (e.g. Ace of Heart and Ace of Spade) from the black suit.

After that, split those 8 cards into 2 piles with 4 cards each facing down and set the sequence of the cards as the following (For example, the author use the cards Ace, 2, 3, 4)

The Spade Suite Pile: Ace, 2, 3, 4
The Diamond Suite Pile: 4, 3, 2, Ace

Figure 5. Example Set Up for The Modified Card Trick

Moving on, the magician will say to the spectator that he will spell “ABRACADABRA” one letter by one. While spelling the words, the magician will shuffle any of the pile (By shuffling means, putting the top card of the pile to the bottom of the pile).

Before shuffling the piles, the magician asked the spectator to say what pile should be shuffled each time an alphabet is spoken out from the magician’s mouth. If the spectator says nothing, then the pile before would be shuffled again.

For instance, a dialogue of the magician and the spectator would be written below (M as the magician, S as the spectator):

M: “A!”
S: “Shuffle the left pile!” (M shuffle the left pile)
M: “B!”
S: “...” (M still shuffle the left pile)
M: “R!”
S: “Right” (M shuffle the right pile)
M: “A!”
S: “Switch” (M shuffle the left pile)
M: “C”!”
S: “...: (M still shuffle the left pile)
so on until “ABRACADABRA” *is spelled out*

After the word “ABRACADABRA” is spelled out, the magician takes the top cards of each pile, takes it as a pair and sets it on the side. Then, the magician will continue spelling “ABRACADABRA” while shuffling.

The magician will continue doing the previous instructions until all the cards is paired up

After finishing the spelling, the magician will reveal each of the pair and “Voila!”, all the cards match, even with all that shuffling.

By seeing the instructions before, we could see that only one word is used as the key for the magic trick. This actually could be a better trade-off from the original version because the set up actually is not limited to only 4 cards rather it could be any number as long as the magician found what is the keyword for that number of cards. To determine what is the keyword for the set up of the “Last Two Cards Match” card trick, the magician needs to know the Chinese Remainder Theorem. This is where modular arithmetic takes its important part of the trick.

III. Analysis on The "Last Two Cards Match" Card Trick

3.1. Original Version

The initial situation for the trick is 2 piles of 5 cards with the first pile sequence as Ace, 2, 3, 4, 5 and the other one is 5, 4, 3 ,2, Ace. In the figure 4, it could be seen that the cards are in a pile or rather a stack. However, to understand the working behind this trick, the cards should be seen as more of a clock.

Figure 6. Clock Perspective for a Pile of Cards

After seeing the cards as more of a clock, the numbers of the card should be seen as an id of itself (Ace as one). By doing that, we actually see the pile of cards as a clock but only with 5 face digits starting from 1 to 5. And a pointer choosing the top card of the pile as the Ace of Spade and 5 of diamonds in the clock.

The first word that the magician would spell is "Last". This word has 4 alphabets, it means the total of shuffling (shuffle can be seen as moving the pointer clockwise) for those 2 piles is 4. By this event, it could be concluded such statement:

Let l be the number of shuffles in the left pile (or Spades) and r be the number of shuffles in the right pile (or Diamonds). Then, the expression (1) below is formed:

$$l + r = 4$$

Take l as 3 for example, then r should be 1. If the Spade pile is shuffled 3 times then the pointer would be pointing 4 right now. On the Diamonds pile, the pointer would be pointing 4 right now if the pile is shuffled once. As the previous event, then the statement below could be formed:

Let *ValueL* be the value pointed by the left pile pointer and *ValueR*, vice versa. And m as the number of cards in each pile. Then, the expression (2) below is formed:

$$ValueL = l + 1$$
$$ValueR = m - r$$

Also denote that, this expression (3) could be produced:

$$4 \equiv -1 \pmod 5$$
$$l + r \equiv -1 \pmod m$$

4 is the number of the alphabets in the word "Last" and 5 is the number of the cards in each pile, while -1 denotes that after shuffling it is going to be moved to the bottom of the pile.

After getting the previous expression (3), it can be seen that by doing modular arithmetic calculations, the expression (4) below could be produced:

$$l + r \equiv -1 \pmod m$$
$$l \equiv -r - 1 \pmod m$$
$$l + 1 \equiv -r - 1 + 1 \pmod m$$
$$l + 1 \equiv -r \pmod m$$
$$l + 1 \equiv m - r \pmod m$$

By seeing the expression (4), it could be seen that $l + 1$ is congruent to $m - r$ in a modulus of m. Notice that from expression (2) *ValueL* is $l + 1$ and *ValueR* is $m - r$. It could be concluded that *ValueL* is actually congruent with *ValueR* when having $l + r \equiv -1 \pmod m$. Because $4 \equiv -1 \pmod 5$, then the first pair is of course a match even after shuffling 4 times total for both piles.

After taking a pair from the piles, each pile now has only 4 cards. This means that m is 4. However, $l + r$ is also changed because the word "Two" has only 3 alphabet. Because $3 \equiv -1 \pmod 4$, then the second pair is also a match after all the shuffling. Likewise for the third, fourth and the last pair is a match. This means that the alphabet for the each word in the sentences should follow the rule below:

Let x_1, x_2, x_3, x_4 be the length of the keywords for the card trick.

$$x_1 \equiv -1 \pmod 5$$
$$x_2 \equiv -1 \pmod 4$$
$$x_3 \equiv -1 \pmod 3$$
$$x_4 \equiv -1 \pmod 2$$

This means that,

$$x_1 \bmod 5 = 4$$
$$x_2 \bmod 4 = 3$$
$$x_3 \bmod 3 = 2$$
$$x_4 \bmod 2 = 1$$

Thus,

$$x_1 = \{4, 9, 14, 19, 24, \ldots\}$$
$$x_2 = \{3, 7, 11, 15, 19, \ldots\}$$
$$x_3 = \{2, 5, 8, 11, 14, \ldots\}$$
$$x_4 = \{1, 3, 5, 7, 9, \ldots\}$$

By the explanation above, it could be concluded that the keywords in the magic sentence could be changed if the length of the word follows the rule above. However, if one wants to increase the number of cards that are played for this trick, it is going to be hard to determine the length of the keywords every single time because one needs to calculate $m - 1$ value. This is where the modified version shines.

3.2. Modified Version

The initial situation for the trick is 2 piles of 4 cards with the first pile sequence as Ace, 2, 3, 4 and the other one is 4, 3 ,2, Ace. In the figure 5, it could be seen that the cards are in a pile or rather a stack. However, to understand the working behind this trick, the cards should be seen as more of a clock.

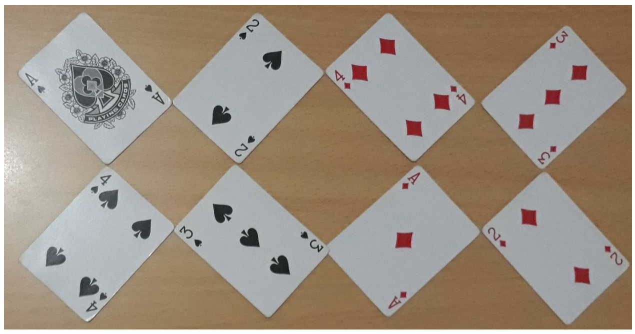

Figure 7. Clock Perspective for a Pile of Cards

After seeing the cards as more of a clock, the numbers of the card should be seen as an id of itself (Ace as one). By doing that, we actually see the pile of cards as a clock but only with 4 face digits starting from 1 to 4. And a pointer choosing the top card of the pile as the Ace of Spade and 4 of diamonds in the clock.

The word that the magician would spell is “ABRACADABRA”. This word has 11 alphabets, it means the total of shuffling (shuffle can be seen as moving the pointer clockwise) for those 2 piles is 11. By this event, it could be concluded such statement:

Let l be the number of shuffles in the left pile (or Spades) and r be the number of shuffles in the right pile (or Diamonds). Then, the expression (1) below is formed:

$$l + r = 11$$

Take l as 5 for example, then r should be 1. If the Spade pile is shuffled 5 times then the pointer would be pointing 2 right now. On the Diamonds pile, the pointer would also be pointing 2 right now if the pile is shuffled 6 times. As the previous event, then the statement below could be formed:

Let *ValueL* be the value pointed by the left pile pointer and *ValueR*, vice versa. And m as the number of cards in each pile. Then, the expression (2) below is formed:

$$ValueL \equiv l + 1 \pmod{m}$$
$$ValueR \equiv m - r \pmod{m}$$

Also denote that, this expression (3) could be produced:

$$11 \equiv -1 \pmod{4}$$
$$l + r \equiv -1 \pmod{m}$$

11 is the number of the alphabets in the word “ABRACADABRA” and 4 is the number of the cards in each pile, while -1 denotes that after shuffling it is going to be moved to the bottom of the pile.

After getting the previous expression (3), it can be seen that by doing modular arithmetic calculations, the expression (4) below could be produced:

$$l + r \equiv -1 \pmod{m}$$
$$l \equiv -r - 1 \pmod{m}$$
$$l + 1 \equiv -r - 1 + 1 \pmod{m}$$
$$l + 1 \equiv -r \pmod{m}$$
$$l + 1 \equiv m - r \pmod{m}$$

By seeing the expression (4), it could be seen that $l + 1$ is congruent to $m - r$ in a modulus of m. Notice that from expression (2) *ValueL* is $l + 1$ and *ValueR* is $m - r$. It could be concluded that *ValueL* is actually congruent with *ValueR* when having $l + r \equiv -1 \pmod{m}$. Because $11 \equiv -1 \pmod{4}$, then the first pair is of course a match even after shuffling 11 times total for both piles.

After taking a pair from the piles, each pile now has only 4 cards. This means that m is 4. Notice that $l + r$ is not changed because the keyword is still “ABRACADABRA”, different from the original version. Because $11 \equiv -1 \pmod{3}$, then the second pair is also a match after all the shuffling. Likewise for the third, fourth and the last pair is a match. This means that the alphabet for the each word in the sentences should follow the rule below:

Let x be the length of the keyword for the card trick, then the linear congruence system below could be formed:

$$x \equiv -1 \pmod{4}$$
$$x \equiv -1 \pmod{3}$$
$$x \equiv -1 \pmod{2}$$

Notice that the system before is actually a Chinese Remainder Problem and x is the length of the keyword for the magic trick. By solving the problem one could find the value of x and perform the card trick. The solving process is shown below:

Notice that the modulus are not all coprime pairwise, 4 is not a coprime of 2. Therefore one needs to simplify the equation. Remember the property below:

if $x \equiv a \pmod{mn}$, where m and n are coprime,
then $x \equiv a$ (mod m) and $x \equiv a \pmod{n}$

This means that the system could be changed into:

$$x \equiv -1 \pmod{4}$$
$$x \equiv -1 \pmod{3}$$

The removal of $x \equiv -1 \pmod{2}$ is legal, since the mod 4 also represents this equation. To solve the system above, we could use the Gauss steps and create the table below:

m_i	a_i	n_i	u_i
4	-1	12/4 = 3	$3u_1 \equiv 1 \pmod{4}$, $u_1 = 3$
3	-1	12/3 = 4	$4u_2 \equiv 1 \pmod{3}$, $u_2 = 1$

Thus, $x = (-1)*3*3 + (-1)*4*1 \equiv -13 \pmod{12} \equiv 11 \pmod{12}$.
Or simplified by showing the possible sets:

$$x = \{11, 23. 35, 47, 59, \dots\}$$

Notice the word “ABRACADABRA” has 11 alphabets. This means the rule above is correct.

By the explanation above, it could be concluded that the keywords in the magic sentence could be changed if the length of the word follows the rule above. Furthermore, if one wants to increase the number of cards that are played for this trick, one just needs to solve 1 Chinese Remainder Problem to determine the shortest possible length for the keyword instead of solving $m - 1$ modular arithmetic problem.

To conclude if one wants to determine the length x of the keyword for N number of cards. The following system should be solved:

$$x \equiv -1 \pmod{N}$$
$$x \equiv -1 \pmod{N - 1}$$
$$x \equiv -1 \pmod{N - 2}$$
$$\dots$$
$$x \equiv -1 \pmod{2}$$

IV. Conclusion

Modular arithmetic has many uses in mankind activities such as time keeping. A rare usage of this knowledge is to set up a particular card trick. By implementing the modular arithmetic in the “Last Two Cards Match” card trick, the effect of pairing two cards even when shuffling the piles is created.

Both the original and modified versions of “Last Two Cards Match” utilizes modular arithmetic. Nevertheless, the modified version also uses Chinese Remainder Theorem to determine the length of the keyword.

VI. Acknowledgment

Firstly, the author would like to thank God for His grace and guidance throughout the process of creating this paper. The author would also like to thank Dra. Harlili, M. Sc as the lecturer for IF2120 and Dr. Ir. Rinaldi Munir, MT. for creating the website http://informatika.stei.itb.ac.id/~rinaldi.munir/ for students so they can research and gather inspirations for the paper. The author would also like to thank all the colleagues and friends who supported the author. Lastly, the author would like to apologize if there are mistakes in this paper.

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PERNYATAAN

Dengan ini saya menyatakan bahwa makalah yang saya tulis ini adalah tulisan saya sendiri, bukan saduran, atau terjemahan dari makalah orang lain, dan bukan plagiasi.

Bandung, 3 Desember 2020

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