Rooted Tree Modelling for Price Equilibrium

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***Abstract*—The study of human behavior, in this case in the lens of the study of microeconomics, In this paper, the author presents an overview of the application of rooted tree as modelling basis for the field of microeconomics. In particular, the author looks at how rooted tree modelling could be used for effect of supply-and-demand on pricing, as well as simple pricing strategies utilized by firms.**

***Keywords*—microeconomics, rooted tree, price strategy, competition, supply and demand.**

I. Introduction

The study of human behavior is crucial for proper understanding of our society. It encompasses a wide variety from the study of incentives or motives underlying these behaviors to interactions between agents exhibiting them. Among other things, it allows us to model emergent behaviors in society and, hopefully, predict possible incentives or policies to improve them in accordance to our desired goals.

Human behaviors can be modeled in various ways. At individual or organizational basis, they might be represented through a specific branch of economics, the field of microeconomics. While economics is frequently derided as being solely concerned with money, it is in fact not necessarily restricted to the study of monetary incentives. Money, as far as economics is concerned, is merely a unit of value, not value in itself. More to the point, economics is about weighing different choices or alternatives, some of which might involve money. As thus, it provides an interesting insight to the system of human behavior.

Another important aspect in studying human behavior is regarding its modelling or representation. It's important to understand the relationship between different behaviors, as well as the effects they produce and likewise their relationship. While there are numerous ways to represent or model a behavioral theorem, in this paper the author looks at one way to represent such: through graph representation, specifically rooted tree representation.

Below the author will highlight how rooted tree graph might be used to model microeconomic behaviors between different individuals or firms in many ways, especially through extension-form game descriptions. This is useful especially because it allows us to see different outcomes produced by different actors committing different behaviors. The same outcomes, assuming sufficiently simple models, presumably could also be predicted by these actors prior, therefore influencing their decision-making.

One thing should be clarified beforehand: microeconomics rely on simple models—hence the usage of term *Ceteris paribus*, meaning all else equal—that might not accurately reflect reality but nonetheless reflects a tendency of outcome in reality. While real-life behaviors tend to manifest with more complex incentives, it should be stated beforehand that in no way this discounts the predictive validity of microeconomic theorems and models, which are grounded on perfectly logical basic assumption: that an individual is, generally, most of the time, a rational actor, concerned primarily with maximizing their self-interest.

II. ROOTED TREE MODELLING

A. Graph

Conceptually speaking, graphs are data structures comprised of two primary components: vertices and edges. They are also sometimes referred to as networks. Graphs express relationship between pairs of items. The items are represented through vertices, while the relationships are represented through edges.

Formally, a graph is a pair of sets (V, E), where V is the set of vertices and E is the set of edges, formed by pairs of vertices.

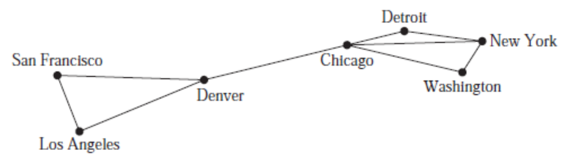

Figure 1. A graph that represents computer network. Source: Discrete Mathematics and Its Applications.

There are different kinds of graphs, depending on whether edges have directions, whether multiple edges can connect the same pair of vertices, and whether loops are allowed.

1. A graph with directed edges is called a directed graph.
2. A graph without directed edges is called an undirected graph.
3. A graph in which each edge connects two different vertices and where no two edges connect the same pair of vertices is called a simple graph.
4. A graph that may have multiple edges connecting the same vertices is called a multigraph.
5. Finally, a graph that may include loops, and possibly multiple edges connecting the same pair of vertices or a vertex to itself, is called a pseudograph.

Graphs could have paths or circuits. Paths are sequences of edges that begins at a vertex of a graph and travels from vertex to vertex along edges of the graph. Meanwhile, circuits are paths which end at the vertex it begins.

Graph is considered as among the most useful mathematical models, because it's widely applicable in almost any problems, especially those related to connectivity or relationship between objects. It is used in many fields, not strictly limited to mathematics, but also includes computer science, biology, and economics.

B. *Tree*

A tree is defined as a connected undirected graph that contains no simple circuit. Because a tree cannot have a simple circuit, a tree cannot contain multiple edges or loops. Therefore, by definition, any tree must be a simple graph.

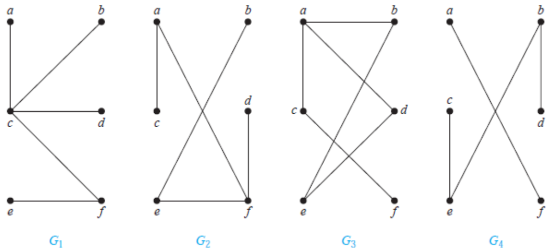

Figure 2. Example of trees and non-trees. Source: Discrete Mathematics and Its Applications.

Fig. 2 shows the example of trees and non-trees. G1 and G2 are trees, as both of them are connected graphs without simple circuit. G3 isn't a tree because it contains a simple circuit. G4 isn't a tree because it's not connected. Unconnected graphs without circuits are called forests instead.

For some trees, a particular vertex is designated as a root. Every edge in the aforementioned tree is then directed away from the root. This kind of tree is called a rooted tree.

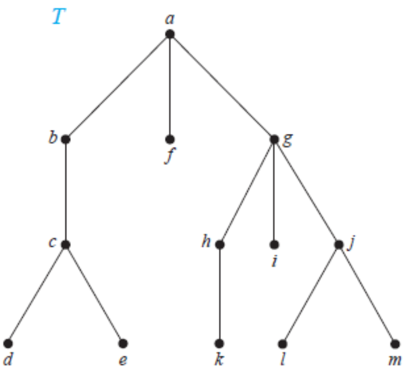

Figure 3. A rooted tree T. Source: Discrete Mathematics and Its Applications.

Suppose that T, like in Fig. 3, is a rooted tree. If there's a vertex called *v* that isn't the root, the parent of *v* is the unique vertex *u* such that there is a directed edge from *u* to *v*. Conversely, for the same relationship *v* is the child of *u*. If *u* has a child other than *v*, then the child is a sibling of v. The ancestors of a vertex other than the root are the vertices in the path from the root to this vertex, excluding the vertex itself and including the root. The descendants of a vertex *v* are those vertices that have *v* as an ancestor. A childless vertex of a rooted tree is called a leaf. The subtree with *v* as its root is the subgraph of the tree consisting of *v* and its descendants and all edges incident to these descendants.

C. *Decision Tree*

A decision tree is a rooted tree in which each internal vertex corresponds to a decision, with a subtree at these vertices for each possible outcome of the decision. In a decision tree, the possible solutions of the problem correspond to the paths of the tree's leaves.

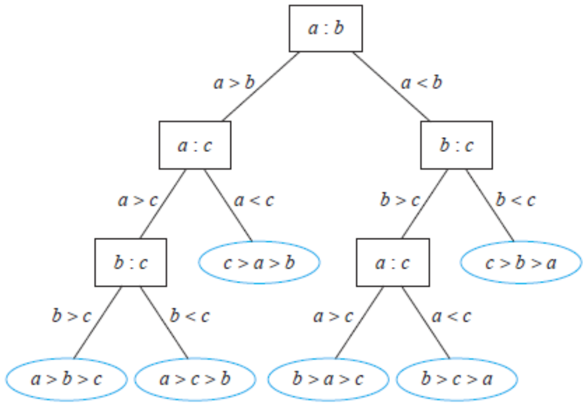

Figure 4. A decision tree that sorts three distinct elements a, b, and c. Source: Discrete Mathematics and Its Applications.

III. MICROECONOMICS

A. *Economics*

There are numerous contested definitions on economics. Narrowly speaking, it is the scientific study of aspects of society in some sense closely connected to money. But broadly speaking, it is best defined as a science which studies human behavior as a relationship between given ends and scarce means which have alternative uses. An economic analysis starts by distinguishing a set of preferences as well as a set of constraints.

By default, within economics the preferences of a certain individual are assumed to be almost entirely selfish—that is, individuals place vastly more value on their own interest than the interest of others. Nevertheless, selfish values don't necessarily preclude cooperative results, which are frequently observed and yet doesn't contradict the assumed principle of self-interest.

Economic conception of constraint originates from scarcity. Scarcity is the gap between theoretically limitless individual want and the limited—that is, scarce—resources. The flip side of scarcity is opportunity costs, whereas an individual necessarily loses potential gain from an act when pursuing another act. And by necessity, the concept of constraints and opportunity costs creates incentives, that is, motivation to behave in a certain kind of way instead of others.

Together, the concepts of preferences, constraints, and incentives form the fundamental basis of the study of economics.

B. *Microeconomics*

As a branch of economics, microeconomics is generally distinguished from macroeconomics from the scope of their study. Microeconomics orient more towards decision-making formulated by individuals or firms, while macroeconomics study the behavior of larger systems: entire countries or entire financial or commercial networks.

In a nutshell, microeconomics generally involves the concepts of supply, demand, and equilibrium. The discipline then branches into various sub-fields: labor economics, agricultural economics, international economics, public finance, public choice, etc.

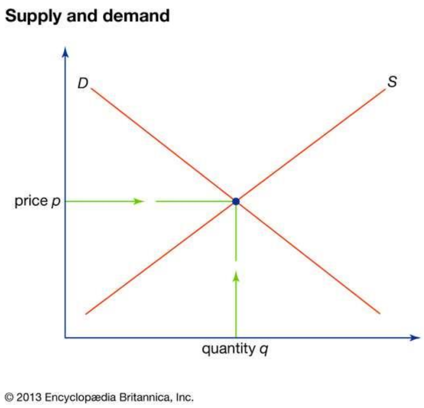

Figure 5. Supply and demand curve. Source: Encyclopedia Britannica.

Supply and demand are main determinants of the price equilibrium, represented in Fig. 5 as the intersection between supply and demand curve. As supply increases, price falls. The opposite happened when supply decreases. Likewise, when demand increases, price also rises, and the opposite occurs when demand declines.

Microeconomic analysis frequently utilize different models on market structure, depending on the context. Market structure refers to features of a market, including the number of firms in the market, the distribution of market shares between them, product uniformity across firms, how easy it is for firms to enter and exit the market, and forms of competition in the market.

Competition generally exists as regulatory mechanism within certain market structure, with varying degree of control. Competition is the act of endeavoring to gain what another endeavors to gain at the same time.

There are also several other terms to be familiarized with before the models are generated. A cartel is a group of producers that colludes, that is, they attempt to restrict output in order to keep prices higher than the competitive level. A monopoly is an enterprise that is the only seller of a good or service.

C. *Extensive-Form*

In the field of game theory, the extensive form is a way of describe a game using a game tree, unlike the strategic form where the game is described using a matrix. Similar to decision tree, a game tree has a vertex that functions as a root, from which other vertices branch in outwards direction.

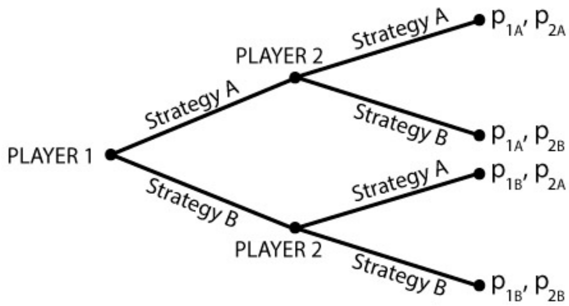

Figure 6. Example of an extensive-form game. Source: https://policonomics.com/lp-game-theory1-extensive-form/

While extensive-form could be used to describe either sequential or simultaneous games, it's more commonly used for the latter instead, as it could be more easily applied to represent decisions made at different times. Most microeconomic models, as they represent behaviors by different actors similar to a game, are best represented through extensive-forms.

IV. ROOTED TREE MODELLING

One of the simplest application of decision tree or game tree in microeconomics is a model generation for supply-and-demand and their effect on pricing. Fig. 7 shows the rough modelling below. It should be noted beforehand that the vertex placement of supply and demand, either as root or not, are interchangeable, or at least produce similar outcomes regardless of whether supply or demand is placed as the root.

Figure 7. Tree modelling of supply, demand, and pricing.

Fig. 7 is approximately based on the supply and demand curve as seen in Fig. 5. In this representation, the author observes different dynamics between supply and demand change, and what occurs thenceforth.

Overall, there are 13 possible paths from root to the terminal, with three likely outcomes: price decrease, price increase, or price stagnation. The end result depends on the mechanism of supply and demand as specified.

- If supply increases, *ceteris paribus*, then price will fall. This mechanism is maintained as demand falls or stays.
- If supply decreases, *ceteris paribus*, then price will rise. This mechanism is maintained as demand rises or stays.
- If demand increases, *ceteris paribus*, then price will rise. This mechanism is maintained as supply falls or stays.

- If demand decreases, *ceteris paribus*, then price will fall. This mechanism is maintained as supply rises or stays.
- Other scenarios, i.e. demand and supply simultaneously rising or falling, must be addressed through comparing the degree of change for which demand and supply rise or fall. Larger change procured by either one will determine the direction of the price's change.

It should be noted that this is an approximate modelling and does not account for the equilibrium equation. Further refining of this model is necessary to make it reflective of reality.

Another possible application for rooted tree modelling, also oriented on pricing, is by looking at how competition drives down price. This model assumes once again human rational self-interest, broken down into the following axioms:

- *Ceteris paribus*, consumers generally favor cheaper prices.
- *Ceteris paribus*, firms generally favor higher profits and avoid losses.

This model will also assume that the goods produced by the firms are equal in quality. In other words, this model represents Bertrand/Cournot competition, where firms produce homogeneous goods.

Our first model, Fig. 8, shows the simplest model—there's only one firm, or in other words, a monopoly market. In the absence of government intervention, a monopoly is free to set any price it chooses and will usually set the price that yields the largest possible profit. The reason for that is immediately evident upon observing this simple tree structure.

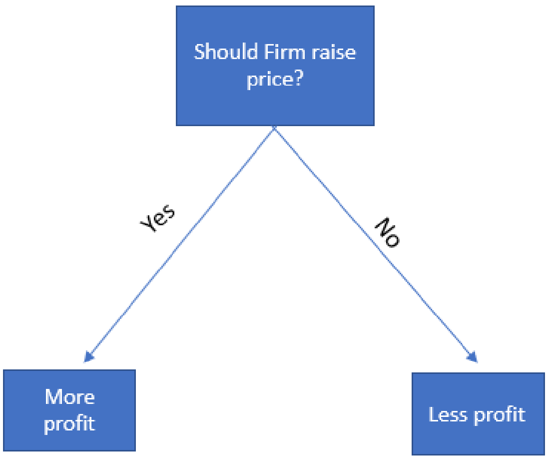

Figure 8. Decision tree for a single monopoly firm to raise price.

Fig. 8 is simple because the firm has no competitor (it is also assumed that it will not have competitors in the foreseeable future). As the result, the firm will raise larger profit when it raises the price of its products and gain less profit when it doesn't do so. Obviously, this isn't yet taking account that if the cost is raised too high, consumers might just decline to purchase the product altogether, implying a ceiling for which the product's price might be raised.

The next model is complicated by the presence of another firm in the market. As another firm exists as a potential competitor, in order for the firm to keep the price high first it must collude with another firm, forming a cartel that coordinates to maintain the market price high so that neither will be harmed by competition.

Fig. 9 shows two firms competing in a mechanism not dissimilar to the famous Prisoner's Dilemma, where they decided whether to "collude" (both raise prices), or wage "price war" (lowers the price to make own firm more competitive) instead. The considerations for both of the firms are presented below.

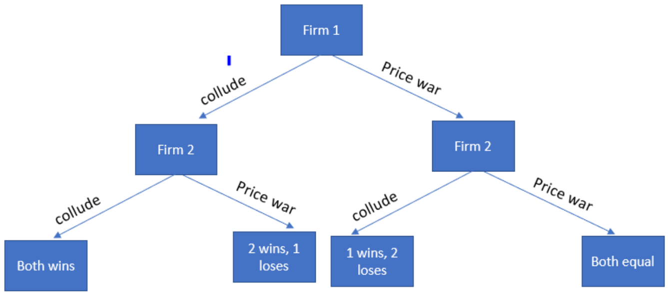

Figure 9: Price competition dynamic between 2 firms.

Here, the following observations can be made.

- The pareto-optimal solution, that is, solution that benefits all firms involved, occur when both firms collude.
- However, if only one firm colludes while another wages price war, then the firm that colludes (i.e. increases price) loses, while the firm that wages price war wins—in other words, the firm that wages price war gains larger market share for selling cheaper products with similar/homogeneous quality with another firm.
- If both firms wage price war, then neither wins or loses, both remains in their position or determined by the extent of their price war, which will bid the price down to market equilibrium.

It can be concluded from this observation that all else equal, without two firms being capable to commit themselves to collude, there's only a quarter chance for both firms to succeed in colluding and keeping the prices high. This is still a relatively high chance compared to scenarios where there are increasing number of firms. However, it can be concluded that without communication or reliability/trust between the firms, the optimal strategy is to break off the collusion and wage price war instead, exploiting the market share of another firm.

In a market with just two firms competing, it's still possible to expect horizontal collusion, as the firms could communicate with each other and it's not improbable to coordinate to the extent that both firms collude to profit from raising prices. However, as more firms enter the market, it is increasingly become more difficult to coordinate and expect optimal outcome. Fig. 10 shows what happens when a third firm enters the market.

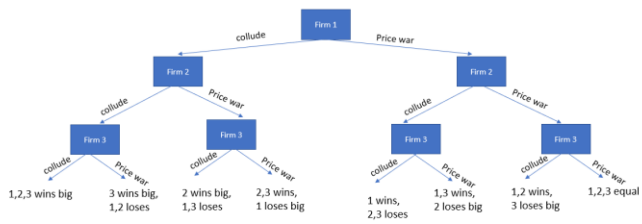

Figure 10. Price competition dynamic between 3 firms.

As we can see, collusion becomes increasingly difficult as the number of participants in the market increases, as the market dynamic becomes increasingly complicated.

- Again, the pareto-optimal outcome occurs when all firms agree to collude and maintain their collusion.
- However, if one firm betrays the collusion and wages price war instead, the firm will win out as it captures larger share of consumers who shift to cheaper goods produced by this firm.
- If two firms betray the collusion and wage price war, then there's only one firm that remains colluding. It'll lose massively to two more competitive firms that capture its market share.
- If all firms betray the collusion and wage price war, then it returns to the previous *status quo* determined by the extent of their price war, driving the market price down to equilibrium.

While in the scenario of two firms competing there's a ¼ chance of success in price collusion, competition of three firms further reduces the chance of price collusion success into just 1/8 success chance. This makes the chance of collusion success, all else equal, to be $\frac{1}{2^n}$, with n being the number of firms present in the market.

With an increasing number of firms, even with ability between firms to communicate intent to collude with each other, it's becoming increasingly difficult to maintain the collusion because the incentive for every firm to break off is becoming increasingly larger and more apparent.

In summary, a decision tree for a firm to decide whether to collude or not might look like Fig. 11 below.

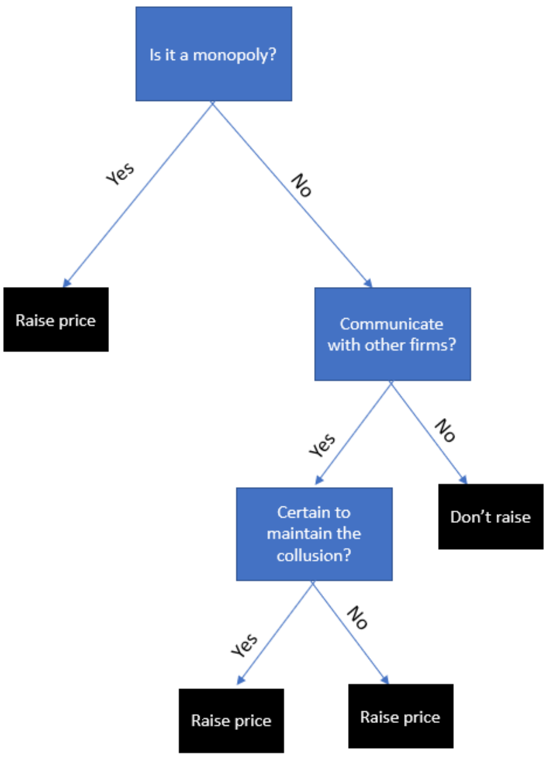

Fig. 11. Decision tree for firm collusion.

The details are as follows.

- If the firm is a monopoly, all else equal, then there's no rational reason for it not to raise price. By all means it should do so.
- If the firm isn't a monopoly, then if it couldn't communicate its intent to collude with other firms, or if they don't agree to collude, then the optimal solution is not raising the price.
- If there's no certainty on whether the firms that agree to collude beforehand will continue to agree to collude afterwards, then don't raise the price. In other words, in absence of means to coerce firms previously agree to collude in the cartel to commit themselves by consistently remaining in the cartel, then don't raise the price.

This is the result of a simple price mechanism model.

V. Conclusion

Henceforth, the author concludes that rooted tree models in the form of game trees and decision trees could be applied to represent simple models in the field of microeconomics, especially in representing how economic behaviors affect prices. In this case, the author has modeled how price reacts from supply and demand, as well as from competition dynamics between varying number of firms. This allows simpler observation of outcomes from behaviors produced by different actors, be they firms or individuals conducting commercial transactions. In turn, this also allows the generation of decision tree for a firm to determines its course of action under the scenario presented.

Nevertheless, the models presented in this paper are still relatively simple, necessitating many assumptions for them to function. Improvements to these models could be made by accounting more variables in designing the trees, with less simplistic assumptions made.

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