# The Usage of Karnaugh Map as an Alternative to Determine Tyre Strategy in Formula 1 Australian Grand Prix

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***Abstract*— This paper will discuss about boolean algebra, Karnaugh map, Formula 1 tyre strategy, and Formula 1 Australian Grand Prix. Formula 1 is a type of racing motorsport where the drivers drive their car around the track for a couple of laps. During the race, tyre strategy is needed to determine which lap the driver needs to pit and change tyre in order to be able to finish the race faster. Karnaugh map can be used as a method to determine the optimum strategy in the ideal condition of the track.**

***Keywords*— Formula 1, pitstop, strategy, tyres**

## I. Introduction

Motorsport is any of several sports involving the racing or competitive driving of motor vehicles. It includes many different activities with different vehicles. The vehicles included in motorsport ranges from cars and motorcycles to monster trucks and lawnmowers, while the activities included varies between drifting, rallying, demolition derby, gymkhana, tractor pulling, and racing.

Racing is one of the sports where the athletes compete to determine the fastest. It comes from word race comes from Norse word '*raz*' which means swift water which then interpreted as the contest of speed. This includes running, swimming, skating, motorised vehicles, etc. However, the type of racing included in motorsport is just racing using motorised vehicles, which is called motor racing.

There's a variety of vehicles used in motor racing, which includes motorcycles, planes, karts, boats, drones, and cars. Racing using cars or automobiles is called auto racing, which has many variations of vehicles, including touring cars, open-wheel cars, sports cars, production cars, stock cars, rally cars, and off-road cars. Formula 1 cars fall into the category of open-wheel cars.

In a race, drivers are placed in a track. The drivers then drive their vehicle around the track for a certain number of laps and the one who finishes that first is the winner. There are a couple of factors that decide the athletes' or drivers' quickness in the race. Some of which are the drivers' ability to maximize the potential of the vehicle, the quality of the vehicle, luck, and strategy.

In Formula 1, as well as some other auto racing, pit stop is a part of the strategy. A pit stop is a place where the racing car can stop and make changes. It is located one in every track so that drivers have a chance to visit it every lap. Inside the pit stop, pit crews which on the same team as the driver make changes into the driver's car that can be changed during the race to increase the condition of the car. The activities differ in which auto sport is discussed, but in Formula 1, most of the activities in the pit stop are just changing tyres.


*Fig 1. Formula 1 Tyres (https://www.skysports.com/f1/news/12515/11166962/f1-2018-pirelli-select-tyres-for-australian-bahrain-and-chinese-gps)*

Tyres are a key component which affects the speed of the car. The tyres are made of rubber which degrades as they make contact to the track due to the heat and pressure it gets to grip the asphalt and create better traction for the car. That means, the longer the time a driver uses the tyres, the less the grip the tyre has and will create longer time to finish a lap. Tyres also have the limit called "pit window", which indicates the number of laps that could be done in a race in a certain set of tyres. When it is passed, the tyres could blow up due to intense heat and pressure. Even if the tyres don't blow up, they can't get a proper grip on the road, resulting an even worse lap time. That's where the pit stop comes in. The old set of tyres are replaced with a newer set of tyres during the pitstop to create a better time to finish a lap.

But that doesn't mean that every driver can just pit every lap because the pit stop also costs time. To reach the optimum time, drivers usually use their tyres until the tyres reach the limit, then they go into the pit to change into new tyres and use it until the limit of the tyres. The process keeps repeating until the end of the race. But it's not just pitting and changing the tyres because there are different compounds of the tyres used in the race. That way, before the race, the team creates a strategy which compound of tyres to use and how many of the tyres should be used during the race.

Boolean algebra is a rule that creates basic logic rules. Using boolean algebra, we can logically determine if type of tyres and the number of tyres that is prepared could make it until the end of the race or not. After that, using the data about time to complete a lap and pit stop time, we can determine the optimum combination of tyres to minimize the time to finish the race. To simplify the process, Karnaugh Map will be used as a method to determine if the tyres that is planned to be used can reach the end of the race or not.

## II. BOOLEAN ALGEBRA THEORIES

### A. Boolean Algebra Definition

Boolean Algebra is an idea found by George Boole in 1954 after realizing that proposition logic algebra and set algebra have similar characteristics. He then introduced the basic laws of logic that has the characteristics represented in both proposition logic algebra and set algebra, which then constructs mathematical structure called Boolean Algebra.

Boolean Algebra is a division of mathematics which deals with operations on logical values and incorporates binary variables. Binary variables are variables that have only two types of value, true or false. These values usually represented by 1 and 0 respectively.

Boolean algebra is constructed with two types of values, two binary operators, a unary operator, and the properties that define the rules that apply in this type of algebra. The two types of values that is represented in boolean algebra are 0 and 1 which can be interpreted as false and true in propositional logic algebra or empty set and universal set in set algebra. While the properties and operators of boolean algebra will be explained in the next two sections.

### B. Properties of Boolean Algebra

Assume that x, y, and z are the variables of boolean algebra, then these properties apply.

1. Identity
   $x + 0 = x$
   $x \cdot 1 = x$
2. Commutative
   $x + y = y + x$
   $x \cdot y = y \cdot x$
3. Distributive
   $x \cdot (y + z) = (x \cdot y) + (x \cdot z)$
   $x + (y \cdot z) = (x + y) \cdot (x + z)$
4. Complement
   For every x ϵ B, there exists x' ϵ B so that
   $x + x' = 1$
   $x \cdot x' = 0$

### C. Operators of Boolean Algebra

As mentioned before, boolean algebra has two kinds of binary operators and a unary operator. Binary operator is the operator that operates on two operands, manipulates them, and gives results while the unary operator only operates on one operands to give a result.

The two binary operators are + and ·, which represents 'or' and 'and' in propositional logic algebra or 'union' and 'intersection' in set algebra. Since the way the operators work are the same in propositional logic and set logic, there isn't any difference in boolean algebra.

+ operator has disjunctive property. The operator gives the value of 1 if at least of the operands have 1 as the value. Other than that, the value will be 0. On the other hand, · operator's result will be 1 if both of the operands have 1 value and 0 if the condition isn't met.

The unary operator in the boolean algebra is ', which represents 'negation' in propositional logic algebra or 'complement' in set algebra. The operator operate the same as its counterpart in propositional logic or set.

The unary operator gives the opposite value of the operand. Which means 0' will give the result of 1 and vice versa.

To make things easier, if x and y are considered as the variables in the boolean algebra, then the operators work as follows.

*Table I. Boolean Algebra Operator*

| a | b | a + b | a · b | a' |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 1 |
| 0 | 1 | 1 | 0 | 1 |
| 1 | 0 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 | 0 |

With two unique values, those four properties, and three types of operators which follows the table, the condition of boolean algebra could be met.

### D. Boolean Function and Canonical Normal Form

Boolean function is used to evaluate boolean type input and manipulate it to create one boolean output using logical type calculations. The function is composed of a combination of boolean algebra's operators and operands. The example of boolean function is as follows:

$$f(x, y) = x + xy'$$

The x and y are called the variables or literals in the equation. The x and y acts as an input of the function and the function will create an output value based on the value of x and y. For instance, if the value of x is 1 and the value of y is 0, then the value of the function will be 1.

The function of boolean algebra can be expressed in another way called canonical normal form. There are two types of canonical normal form, which are canonical disjunctive normal form (CDNF) and the dual, canonical conjunctive normal form (CCNF).

CDNF is also called minterm canonical form, sum of minterms, or Sum of Products (SoP). Minterm defines the result of · operator applied on the varibles / literals in which each variable only appears once. For example, if you have three variables a, b, and c, the minterm would be a·b·c. Each minterms that is created in the solution is then summed using + operator to create CDNF or SoP. The example of SoP is $xy + yz'$.

CCNF is the dual, or counterpart of the CDNF. It is also called maxterm canonical form, product of maxterms, or Product of Sums (PoS). Maxterm is the opposite of minterm. Maxterm is the result of applying the operator + in the literals. Each maxterms that is created, then timed by using · operator, which acts as 'AND' to create CCNF or PoS. The example of PoS is $(x + y)' \cdot (y + z)$.

Using these two canonical forms, the boolean function can be represented more easily and easier to understand as well. In order to make it easier to create this canonical normal form, Karnaugh map is used as the method, which will be explained in the next section.

E. Truth Table and Karnaugh Map

Truth table is a way to list every input and output created from an unidentified function. After listing the inputs and outputs, the function could be identified. But to create a canonical form, truth table isn't the most effective method. That's where Karnaugh map comes in.

Karnaugh map is a method to simplify expression of boolean algebra. The Karnaugh map reduces the need of extensive counting using the rules of boolean algebra by using pattern recognition.

In the Karnaugh map, n variables ranging from 1 to 4 could be put into the map and be checked. With 2 types of values of each variable, there will be $2^n$ slots in Karnaugh map. The slots in the map will then be filled with value of 0, 1, or X. The three values represent as follows. 0 represents that the combination of the variables' values is not minterm. 1 represents the opposite, while X represents a 'don't care' state. A 'don't care' state means that the condition is impossible to achieve or wouldn't be achieved so it is not included in the calculation.

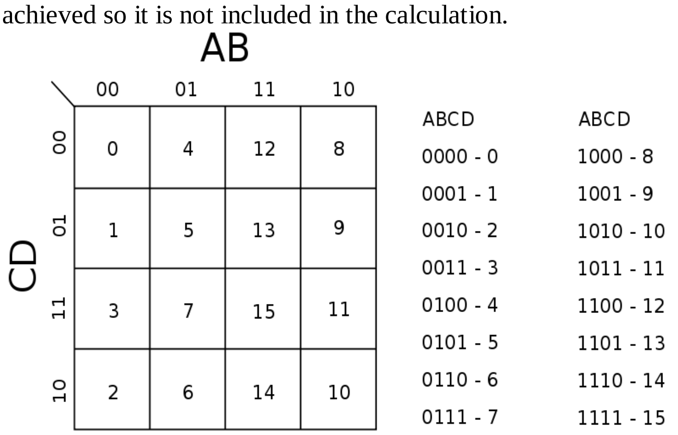


*Fig.2. Karnaugh Map (https://en.wikipedia.org/wiki/File:K-map_minterms_A.svg)*

The figure above explains the Karnaugh map. The map will then be filled with 0, 1, and X on the box / slot that represent the value of the variables.

After the map is filled, we can now see the pattern and group those with value of 0 and those with the value of 1. The slot that has value X doesn't have much meaning, otherwise known as 'don't care'. The slot can be treated as 1 or as 0.

The group should have the size of $2^n$ and should be made as big as possible while still only contains the same type of value. The other rule is when grouping a certain value, every same type in the Karnaugh Map must be used too. The figure below will further explain that.

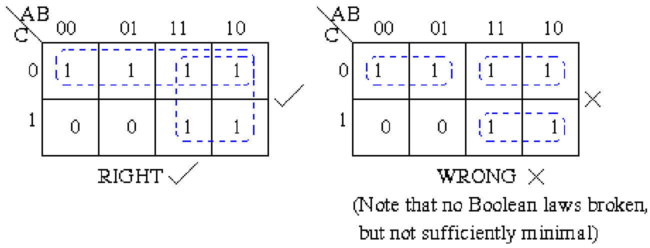


*Fig.3. Grouping Karnaugh Map (http://www.ee.surrey.ac.uk/Projects/Labview/minimisation/karrules.html)*

After grouping, the result can be converted into SoP or PoS. If the value 1 is grouped in Karnaugh Map, the SoP could be made. On the other hand, if the grouped value is 0, PoS could be created.

On the figure above, after grouping value 1 in the Karnuagh Map, it could be seen that the value when variable C has 0 value always considered as minterm. This also applies to the value inside Karnaugh Map when variable A's value is 1. That means the minterms are A and C'. Thus, the SoP of the Karnuagh Map above is A + C'.

On the other hand, if the zeros are grouped, the value of the slot in Karnuagh Map will be 0 when the value of A is 0 and the value of C is 1. That means A'C is not the minterm and its negation, (A+C') is a maxterm of the Karnuagh Map. Thus, the PoS of the Karnuagh Map above is A + C'.

## III. PROBLEM IDENTIFICATION

A. Scope of Problem

As mentioned in the introduction, the problem that will be identified is tyre strategy. Tyre strategy is the preparation of choosing the right compound of tyre and how many of them needed for the race.

There are many types of vehicles to race, so the type of vehicle is also narrowed down to Formula 1 vehicles, which is a type of open-wheel vehicles.

In Formula 1, there are also a lot of races with different layout, height, and tyre compound available for the race. That's why the results will differ for each track and the track chosen should be narrowed down to one. The chosen track will be the first track in F1 calendar in 2018, which is Albert Park in Melbourne, Australia.

The solution created only contribute towards tyre strategy, which is the ideal target that can change throughout the race. In addition, this solution doesn't take qualification into account. Plus, this solution is on assumption that the car will start with fresh tyres.

To summarize, the problem is to determine the tyre strategy in Formula 1 race in Albert Park, Melbourne, Australia.

B. Tyres and Pit Stops

The race in Formula 1 is basically drivers driving their cars around the track until the finish line to find out who is the fastest, which then will be decided as the winner. The drivers start from their starting position and make their way through the circuit or track for a prescribed number of laps.

The starting position of the drivers is decided by qualification. But this paper doesn't discuss that part as mentioned in the scope of problem.

During the race, there are some parts of the car that will deteriorate due to intense heat and pressure during the race, mainly tyres.

As mentioned in the introduction, tyres are part of the car that is connected towards the asphalt of the track to create traction. The grippier the tyre is, the better the traction is. Better traction means the time needed to do a lap is faster which contributes to the time needed to finish the race.

But things are not that simple. The tyres that make contact to the racetrack will deteriorate as mentioned before. As tyres deteriorate, the power of the grip will be less and less, creating less traction, thus worsening the lap time. Tyres also have the limit of their lifetime. Once they reached the limit of their lifetime, the grip will lessen even quicker as the rubber reaches the yield, which means it becomes less and less elastic. At the same time, the tyres' bad elasticity will make the tyres become vulnerable to blow up due to its inability to contain the heat and pressure generated by the speed of the car and the contact it makes with the racetrack. Because of that, it is necessary to change the tyres during the race.

In order to change the tyres during the race, a pitstop is needed. As mentioned before, pitstop is a place where the race car can stop and the changes, that is allowed to be made on the car, is made. This includes one or a combination of the following: tyres, fuel, car adjustments, and parts changing. In Formula 1, refuelling is banned, but cars still can change the part of the car, which is front wing, do some adjustments to repair the car if it's not functioning well, and change tyres. Assuming that every part of the car is functioning well, a pit stop is used just to change tyres. But the pitstop costs time, which is called pitstop delta. Pitstop delta will be counted as the time deficit because driver loses time because of pitstop. So strategically, drivers should drive their car until the end of the tyre life, then do a pitstop.

To make things more complex, there are a couple of compounds of tyres available marked by the colour of the tyres. The compounds differ in the hardness of the tyres. The compounds range from softer tyres to harder tyres. Softer tyres have high grip on the track on the expense of lower durability. In contrast, harder tyres have higher durability than the softer tyres, but lower grip. High grip creates faster time to complete a lap but high durability means more laps could be done in that tyre compound.

C. Tyre strategy

The types of tyres needed to use and the number of pitstops needed to be done during the race creates a tyre strategy that contributes towards the speed that the drivers and their vehicles speed during the race, which then contributes to determine the fastest.

The tyre strategy that will be discussed is about what compound should be used and how many of them needed for the car to finish the race.

As mentioned before, every tyre has limit for the lifetime, in which if it is passed, the grip will be much lower and the tyres will become vulnerable. After that, a pit stop is needed. But the different tyre compound will create a different strategy which could result in different time needed to complete the race.

The strategy could be using the softer tyre compound to create better laps, using the harder tyre compound to minimize the amount of pitstop needed to finish the race, or the combination of both of them. In the end, the strategy should state which tyre compound to be used at which point of the race to create the fastest time to finish the race.

But put in mind that tyre strategy works ideal condition which is nearly impossible to find due to many changes happen in the racetrack. In practice, there would be a deviation of the strategy a bit if the condition doesn't change much. If the track condition changes a lot, then the tyre strategy will change a lot. For example, rain isn't something that is expected on the racetrack. Thus, if, for example, the rain occurs, the strategy will deviate a lot.

D. Track Data

Albert Park is located in the city of Melbourne, Australia. The track has been used as the venue of one of Formula 1 races ever since 1996. The track is designed to encircle the lake in the centre of the track, which is named Albert Park Lake and some part of the circuit is sometimes used for daily roads.

The track has the length of 5.303 km and the drivers need to drive around the circuit for 58 laps to finish the race. The track layout is as follows.

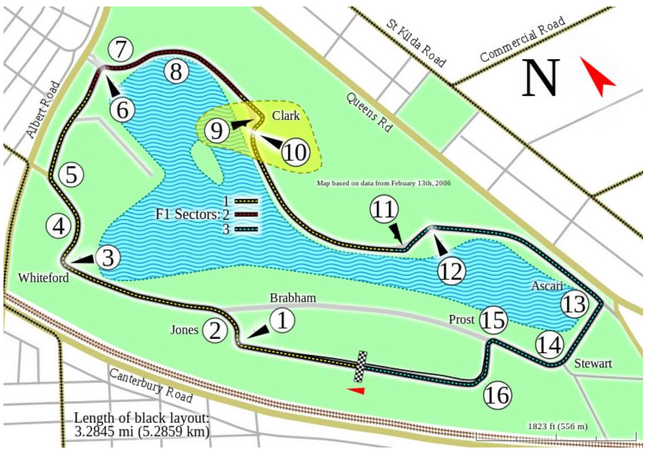


*Fig.4. Albert Park Circuit, Melbourne (https://en.wikipedia.org/wiki/File:Albert_Lake_Park_Street_Circuit_in_Melbourne,_Australia.svg)*

The tyre compounds that is used by Pirelli, Formula 1 tyre supplier, for Australian Grand Prix in 2017 and 2018 are Ultrasoft, Supersoft, and Soft. The difference between these three tyres are the same as the difference between tyre compounds, which is durability and the grip it creates between car and the asphalt.

Ultrasoft are the softest tyres, followed by Supersoft and Soft respectively. Ultrasoft has the highest grip between these three followed by Supersoft and Soft. On the other hand, Soft tyres have the highest durability followed by Supersoft and Ultrasoft.

That means, theoretically, Ultrasoft tires can create faster time to do a lap but it can only be used for fewer number of laps. In contrast, although Soft tyres can't create better traction, they are more durable, thus can be used for more laps before having to pitting due to tires having reached the limit.

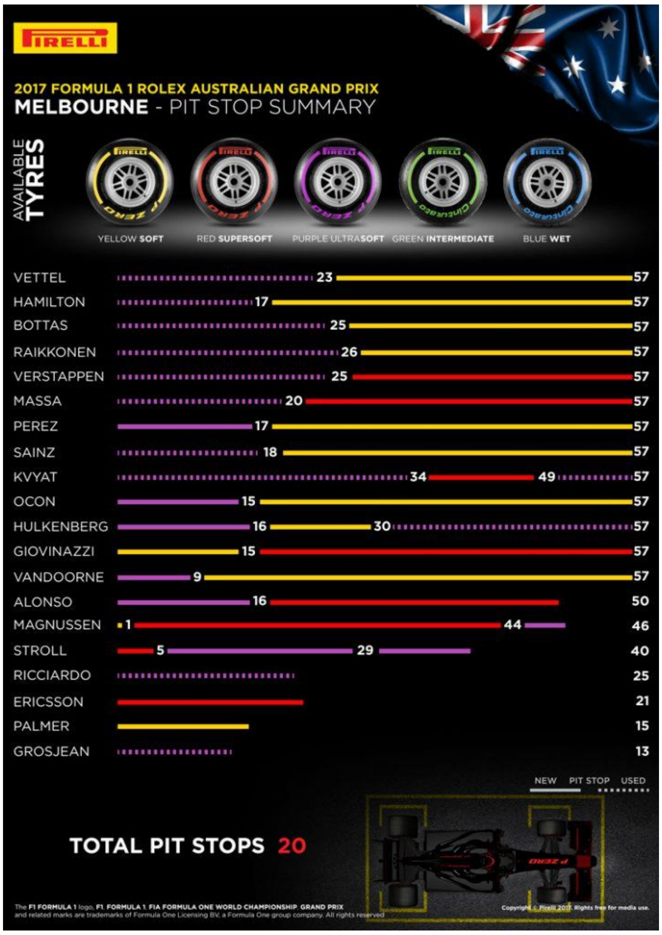


*Fig.5. F1 Australian GP 2017 Tyre Strategy (http://f1strategyreport.com/2017/03/29/f1-strategy-report-australian-grand-prix-2017/)*

The figure above explains the tyre strategies that the team used in Formula 1 Australian GP 2017. That data in 2018 isn't taken because there is an event that activates Virtual Safety Car (VSC) in Australian GP 2018. Virtual Safety Car acts similar as safety car which makes racing slower and disrupting race strategy from the ideal since pitting while VSC is active creates lesser pit delta.

To explain the graphics above, the colour represents the type of tire that each drivers' use during a certain part of the race. The straight line represents the tyre that is fresh and hasn't been worn before while the dotted line represents the tyre that is not fresh, which still contributes into the calculation since the tyres are used for qualification. The number represents the lap while the line which isn't number at both ends means that the driver has crashed during a point in the race. There are some data that won't be regarded aside from the tire that is used when the driver crashed, it is the number of laps that is too small to be ideal, which is the durability of Soft tyres being 15 laps or less. Most probably, on that occasion there's an accident so the tyres have to be changed immediately.

From the data above, it can be estimated that the average amount of laps can be done without pitting if the driver is using the Ultrasoft type of tyre is around 20 laps. For the Supersoft tires, the average durability is around 29 laps. The average durability of Soft tires is 38.25 laps, rounded down to 38 laps.

After knowing the difference in durability of the tyre, difference between the grip of the tires needs to be found out. The way to discover it is through the differences of lap time when using a certain type of tyre. According to Pirelli after preseason test in 2018, for 2018 tyres, every lap, the Supersoft tyres are faster than Soft tyres by 0.4s while the Soft tyres are faster than the Medium type of tyres by 0.8s.


*Fig.6. Pirelli Tyre name change (reddit.com/r/formula1)*

At the same time, Pirelli changed the tyre compound's name for 2018 so that the Medium compound for 2018 means Soft compound in 2017. The Soft compound in 2018 means Supersoft compound in 2017 and the Supersoft compound in 2018 means Ultrasoft compound in 2017. Thus, if the Ultrasoft is regarded as the fastest tyres, the time deficit per lap for Ultrasoft tyres will be 0 second. Supersoft does a lap around 0.4 seconds slower than Ultrasoft tyres and Soft tyres does a lap around 1.2 seconds slower than Ultrasoft tyres. The summarization is on the table below.

*Table II. Tyre Compounds Differences*

| Tyres | Durability (laps) | Time deficit (seconds/lap) |
|---|---|---|
| Ultrasoft | 20 | 0 |
| Supersoft | 29 | +0.4 |
| Soft | 38 | +1.2 |

## IV. SOLUTION OF THE PROBLEM

### A. Condition Limitation

There are some limitations and assumptions in this solution of the problem which are:

1. The time deficit between tyre compounds data from Pirelli is regarded as absolute.
2. All the tires are regarded as fresh tires.
3. Drivers' are expected not to push tires further than the average durability from the data.
4. The condition of the racetrack isn't included. The rain or being held up by the car in front isn't regarded in the solution.

B. Problem Solution

Using the data in Table II, it can be determined if the tyre compounds used could reach until the end of the race or not by addition and multiplication of tyre durability. As for the additional information, the laps needed to be done to finish Australian GP is 58 laps while the average pitstop time (or pitstop delta) in the race is about 22 seconds.

The total durability of the chosen tyres should be the same or more than 58 laps. For example, 2 Ultrasoft tyres can't make it until the end of the race because $2 * 20 = 40 < 58$ while 3 Ultrasoft tyres can make it until the end of the race because $3 * 20 = 60 \geq 58$.

From the example above, it can be seen that if the total number of tires of Ultrasoft compound exceeds 3, the tyres can reach the end of the race. Since Ultrasoft tyres are the least durable between the 3 tyres, that means if the total of any compounds of tyres is 3 or more, then it would be possible to finish the race. That means if the total amount of tires is 4, it would be irrelevant. Also, the truth table could be made to check every possible combination of tires which is as follows.

*Table III. Truth Table of Tyre Strategy*

| Tyre Compounds (number of tires) | | | Result |
|---|---|---|---|
| Ultrasoft | Supersoft | Soft | |
| 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 |
| 0 | 1 | 0 | 0 |
| 0 | 0 | 1 | 0 |
| 2 | 0 | 0 | 0 |
| 0 | 2 | 0 | 1 |
| 0 | 0 | 2 | 1 |
| 1 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 1 | 1 |
| 3 | 0 | 0 | 1 |
| 0 | 3 | 0 | X |
| 0 | 0 | 3 | X |
| 2 | 1 | 0 | 1 |
| 1 | 2 | 0 | X |
| 0 | 2 | 1 | X |
| 0 | 1 | 2 | X |
| 2 | 0 | 1 | X |
| 1 | 0 | 2 | X |
| 1 | 1 | 1 | X |

*Number of laps : 58*
*Ultrasoft durability (laps) : 20*
*Supersoft durability (laps) : 29*
*Soft durability (laps) : 38*

The result is 1 if the total durability of the tires is the same as or more than the number of laps. The result is 0 if the total durability of the tires is less than the number of laps. The result is X if the result is unnecessary because without one of the tires, it still can reach the end of the race.

Using the truth table above, we can create Karnaugh Map of four variables containing the number of tyres in binary value. Since the maximum value is 3, that means 2 digits to represent their amount as binary number is needed. For example, 2 is represented by 10. That means there's 2 variables used to represent the number of the tires. Since it is useless using every types of tyres at least once during the race, as shown in the truth table, it can be divided into 3 types of strategy. Strategy using Ultrasoft and Supersoft tyres, strategy using Supersoft and Soft tyres, as well as strategy using Ultrasoft and Soft compound tyres. To make it simple, the number of Ultrasoft tyres is represented by variable a and b, the number of Supersoft tyres is represented by variable c and d, and the number of Soft tyres is represented by variable e and f. The three Karnaugh maps will be as follows.

*Table IV. Karnaugh Map of Tyre Strategy using Ultrasoft and Supersoft Tyres*

| Ultrasoft and Supersoft | | ab | | | |
|---|---|---|---|---|---|
| | | 00 | 01 | 11 | 10 |
| cd | 00 | 0 | 0 | 1 | 0 |
| | 01 | 0 | 0 | X | 1 |
| | 11 | X | X | X | X |
| | 10 | 1 | X | X | X |

*Table IV. Karnaugh Map of Tyre Strategy using Supersoft and Soft Tyres*

| Supersoft and Soft | | cd | | | |
|---|---|---|---|---|---|
| | | 00 | 01 | 11 | 10 |
| ef | 00 | 0 | 0 | X | 1 |
| | 01 | 0 | 1 | X | X |
| | 11 | X | X | X | X |
| | 10 | 1 | X | X | X |

*Table VI. Karnaugh Map of Tyre Strategy using Ultrasoft and Soft Tyres*

| Ultrasoft and Soft | | ab | | | |
|---|---|---|---|---|---|
| | | 00 | 01 | 11 | 10 |
| ef | 00 | 0 | 0 | 1 | 0 |
| | 01 | 0 | 1 | X | X |
| | 11 | X | X | X | X |
| | 10 | 1 | X | X | X |

After finishing the Karnaugh Map, The SoP and PoS which represents the boolean function of each Karnaugh map could be made. In this paper, the SoP of the Karnaugh Map will be made. After that every SoP is to be summed to create the overall solution of the problem.

So the SoP of each Karnaugh map would be:

For Table IV : c + ab + ad
For Table V : c + e + df
For Table VI : e + ab + bf

If all of the SoP summed, the overall solution of the problem would be c + ab + ad + c + e + df + e + ab + bf. The solution can be simplified into c + e + ab + ad + df + bf. This function is the solution of the problem.

To translate the solution into normal language again, we could just see what each variable represents above the table. After translating back, the solution is two Supersoft tires, two Soft tires, three Ultrasoft tires, two Ultrasoft plus a Supersoft tires, a

Supersoft plus Soft tires, or an Ultrasoft plus a Soft tires.

After looking at the six solutions, we can now determine the fastest combination of tyre compounds for the race through counting at the least time deficit. The time deficit would be counted as follows:

$$T = a * m + b * n$$

T = total time deficit
a = average pitstop delta
m = number of pitstops
b = time deficit per lap of a certain tyre compound
n = number of laps using a certain tyre compound

For example, if the strategy is using 2 Ultrasoft and 1 Supersoft tyres, then there will be 2 pitstop, 40 laps using Ultrasoft tyres, and 18 laps using Supersoft tyres. Thus, the deficit time would be (2 * 22 seconds) + (0 * 4 seconds) + (0.4 * 18 seconds) = 51.2 seconds.

The strategy that has the least time deficit would be the best strategy ideally. Using that equation, we can total the number of time deficit of each possible strategy.

*Table VII. Time Deficit of Tyre Strategies*

| Tyre Strategy (number of tyre compounds) | | | Time Deficit (seconds) |
|---|---|---|---|
| Ultrasoft | Supersoft | Soft | |
| 3 | 0 | 0 | 44 |
| 0 | 2 | 0 | 45.2 |
| 0 | 0 | 2 | 91.6 |
| 2 | 1 | 0 | 51.2 |
| 0 | 1 | 1 | 68.4 |
| 1 | 0 | 1 | 67.6 |

## V. CONCLUSION

Boolean Algebra and Karnaugh Map can be used as an alternative to determine the ideal tyre strategy of a Formula 1 Race. After knowing the tyre strategies that could be used for the race, we can count the time deficit by summing all the time lost because of the tire compounds and the pitstop the strategy that has the least time deficit, which then concluded as the ideal strategy for the race. The ideal race strategy for the Formula 1 Australian Grand Prix is by using 3 Ultrasoft tyres.

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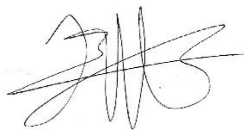
## PERNYATAAN

*Dengan ini saya menyatakan bahwa makalah yang saya tulis ini adalah tulisan saya sendiri, bukan saduran, atau terjemahan dari makalah orang lain, dan bukan plagiasi.*

Bandung, 9 Desember 2018



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