Application of Graph Coloring on Sudoku Solver

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***Abstract*— This cover with discuss mainly about how to solve Sudoku games using graph coloring theory. Sudoku games with size n^2 x n^2 can be solved by using graph coloring. The Sudoku was first transformed into a Sudoku graph which had vertices n^2 x n^2 and each vertex is labeled a number from 1, 2, 3, to n^2 x n^2. While the edge of the Sudoku graph is the edges that correspond to the rules that apply in the Sudoku graph. The Sudoku graph obtained is a regular graph which has a degree of $3n^2$ - 2n - 1. Applying graph coloration, especially point coloration, on Sudoku graph and then will be able to find a solution from the Sudoku game. The number of chromatics obtained is n^2.**

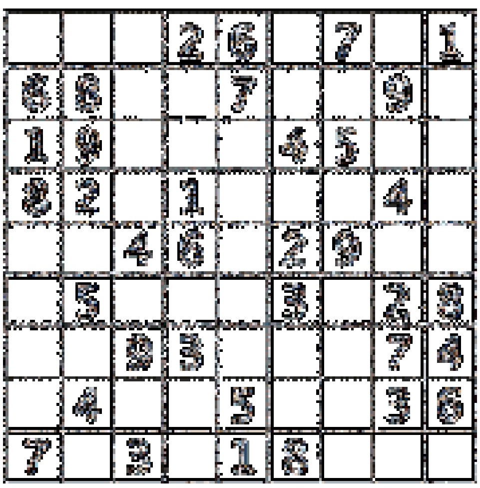
***Keywords*—Sudoku, Graph, Coloring, Chromatic Number.**

I. INTRODUCTION

Sudoku, also known as Number Place or Nanpure, is a kind of logic puzzle. The goal is to fill in the numbers from 1 to 9 into 9 × 9 nets consisting of 9 3 × 3 boxes without any repeated numbers in a row, column or box. First published in a French newspaper in 1895 and possibly influenced by Swiss mathematician Leonhard Euler, who made famous Latin square.

The modern version of the game began in Indianapolis in 1979. Then it became famous again in Japan in 1986, when the Nikoli publisher discovered this riddle created by Howard Garns.

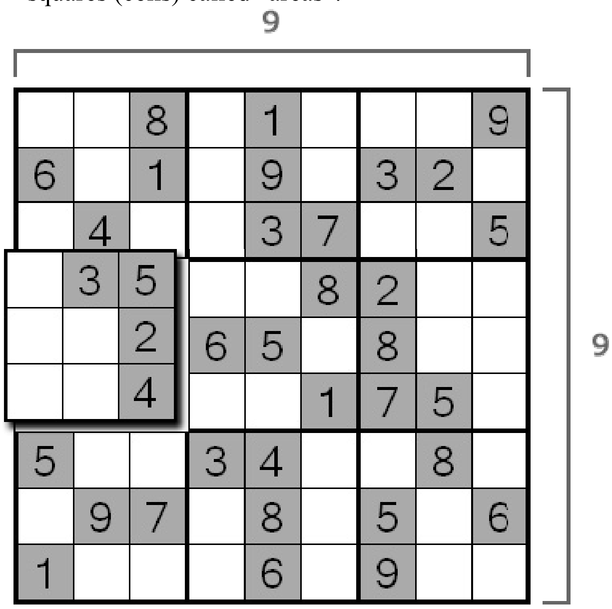
The name "Sudoku" is a Japanese abbreviation of "Suuji wa dokushin ni kagiru" (数字 は 独身 に 限 る), meaning "the numbers must remain single".



Picture 1 Example of Sudoku Game Board

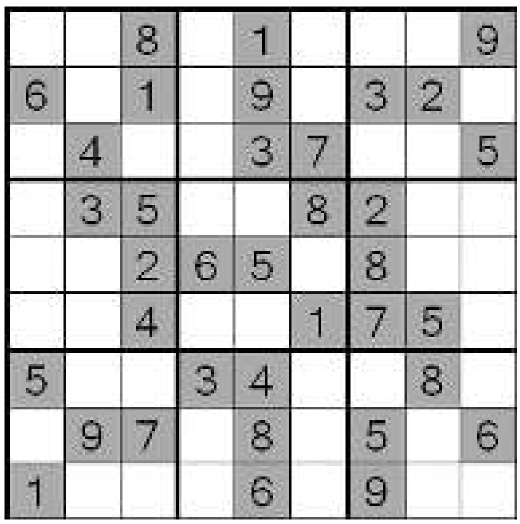
There are some rules that are applied in sudoku.

- Sudoku is played in 9x9 boxes divided into 3x3 small squares (cells) called "areas":



Picture 2 Sudoku Board (http://lifeslittleinspirations.com/sudoku-rules-for-the-game-of-life)

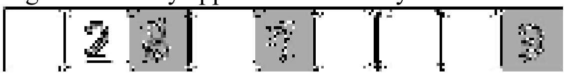
- Sudoku starts with several cells already filled with numbers:



Picture 3 Sudoku Board Column (http://lifeslittleinspirations.com/sudoku-rules-for-the-game-of-life)

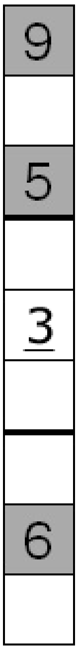
- The goal of the Sudoku game is to fill in empty cells with numbers between 1 and 9 (one-mark only 1 number) according to the following instructions:

a. Figures can only appear once in every row:



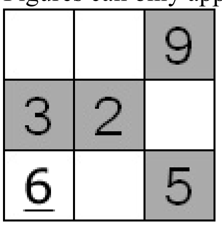
Picture 4 Sudoku Board Row (http://lifeslittleinspirations.com/sudoku-rules-for-the-game-of-life)

b. Numbers can only appear once in each column:



Picture 5 Sudoku Board Column (http://lifeslittleinspirations.com/sudoku-rules-for-the-game-of-life)

c. Figures can only appear once in each area:



Picture 6 Sudoku Board Column (http://lifeslittleinspirations.com/sudoku-rules-for-the-game-of-life)

- Summary of the rule is a number should appear once in each row, column, and area.

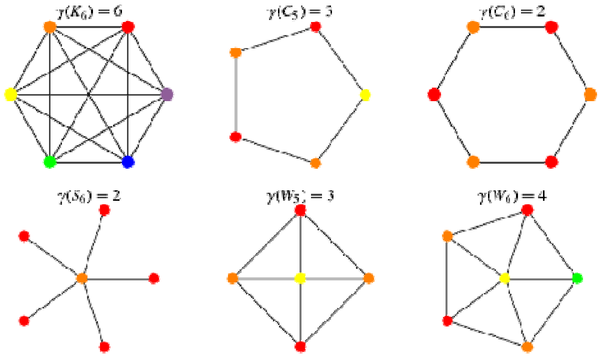
II. CHROMATIC NUMBER

A graph can be colored by giving different colors to each vertex. In fact, most graphs can be colored less than the number of vertices in the graph. This can lead to the question of what is the minimum color that can be used to color a graph. Then the term chromatic number appears.

Chromatic number G, denoted by () is the smallest number k so that G can be colored (Wilson & Watkins, 1989: 235).

The concept of chromatic number is a development of the concept of partition dimensions and coloring graph. The point coloring on the graph is = () → {1,2,3, ...,} with conditions for each neighboring point must have a different color. Minimum the number of colors used for point coloring on graph G is called chromatic number, denoted by ().

The following is given the definition of the chromatic number of the location of the graph taken from Chartrand et al. (2002). Let c be a point coloring on graph G with () ≠ () for u and v next to G. Suppose the set of points - the point that is colored i, hereinafter referred to as the color class, then Π = {,, ...,} is a set consisting of color classes from V (G). The color code () of v is k-ordered ((,), (,), ..., (,)) with (,) = min {(,) | ∈} for 1 ≤ ≤. If every G has a different color code, then c is called location coloring G. The minimum number of colors used for location coloring is called chromatic number location of G, and denoted by (). Because every location coloring is also a coloring, then () ≤ ().



Picture 7 The chromatic number of graphs (http://mathworld.wolfram.com/ChromaticNumber.html)

The algorithm that can be used to color a G graph is the Welch Powel algorithm. However, this algorithm only gives a base limit for the value of K (G), meaning that this algorithm does not always give the minimum number of colors needed, to refer to G. The Welch-Powel algorithm is as follows:

1) Sort the vertices on graph G from vertices of high degree to low degree.
2) Take the highest degree node that has never been visited, if the color is colored then the color i with a unique color.
3) Color it with the same color as a knot that is not adjacent to it and has never been colored.
4) Repeat steps number 2 and 3 until all nodes have been colorized.

Theorem 1:

A graph G does not have a side with an odd length, if and only if it can be colored with 2 colors.

Proof:

As explained above, if G has a side with an odd length l, then G stain requires at least 3 colors. Suppose G doesn't have an odd length side. Select a vertex V that is given a red color. Then at each vertex adjacent to V is colored blue. Now, at vertices adjacent to the blue dot, given the color red. Can one of these red points. say vertex W, close to point V which is also red?

Theorem 2:

If d is a maximum degree of point in graph G, then K (G): 5 d + I.

Proof:

To prove this, it is shown that we can do a coloring (d + I) on G. Suppose that we are given a number (d + 1) of a different color. The pill is any point v0 and the color i ti is with any color from (d + 1) color. Select a color that has not yet been colored, for example v1. Fill this v1 with a color that has not been given at the points next to it. because the degree of v 1 or dv1 <d, means that the number of d colors that have been used by the point k is set aside v0, so i v 1 can be colored with color to (d + I). Repeat this process by specifying a new point v 1 until all the points have been colored i. So in other words K (G): 5d + I.

Note:

The chromati number k for a complete graph and the longest particle of starch l is d + I or K (G) = d + I.

III. CHROMATIC POLYNOMIAL

For example, G is a simple graph, and PG (k) is a lot of ways to color vertex G with k colors so that no two adjacent vertices get the same color. The function of PG (k) is called polynomial chromatic G or many chromatic tribes G.

Example :

If G is the P3 path, then to label the graph point, the point in the middle can be named any of A. colors. The color cannot be used again to color the other point so that it meets the dye requirements. So that the points that are set aside by these points can be colored with one color from (A. - I) the remaining color, then P (G, A) = A. (A. - 1) 2

Example :

If G is a complete graph K3, then the top tick can be any arbitrary from A. color, the other points can be named with A - I color because the point is set aside with the top point. On the other k points (leftovers) can be given a color range from A. - 2 colors that have not been given kao to the other two points, then P (G, /,) = /, (I, - I) (A .- 2)

Example :

Take graph G as a blank graph with p points each of which can be colored by A. way, then the graph is valid P (G, A.) =) P.

Theorem 4:

G is a simple graph, eg A and B two non-fractional pins in the G. G '

is a graph obtained from graph G by connecting a line between RA and AB. While G "is a graph of the gain obtained by tapping A and B, which becomes a single point, then P (G, A.) = P (G ', A.) + P (G' ', A.)

Proof:

The number of ways of staining from G can be grouped n in two groups. Group I is coloration G where litik A and B have the same color and group 2 is G coloration where t and t A and B have different wats. A lot of coloration -A. d ima na titi k A and B have different colors. If a line connects points A and B, then a lot of coloration - /,. from G '. Number of colorants n- A. of G where the tick A and B are the same color is not pronounced, ji ti k-titi k A and B is united so it will be equal to the number of dye n - A. from G ". So that the value of a G is: P (G, A.) = P (G '. A.) + P (G ", /,). To illustrate the writing of P (G, /,) = P (G ', A.) + P (G ", / ..), I am pictured where graph G is taken simple graph By using the above theorem by re-signing i A and B as titi k which should be observed at every step.

From the results obtained, it appears that the polynomial of the chromite from a graph is simplified to be a biological factorial.

Theorem 4:

G is a simple graph, missal G 'and G "there is a graph obtained by G G by removing and tuning between points A and B into a thole mat l, then P (G, A.) = P (G', A.) - P (G ".A.)

Proof:

In the previous theorem of the process, the process of adding the line and ending with the chromatic polynomial expressed in the form of factoria on the complete graphs. But in this theoretical process that is done is the process of removal of the line. Apabi la A and B are the two points that are split in G ma, then G = G '- (A, B) and G "are obtained from G by synthesizing (A, B) in a single pole. will be r in the form of polynomial blank graphs From the description is obtained

P (G, A.) = A. 4 -4 / .. 3 + 6 / .. 2 -3 A.

III. GRAPH COLORING

The line or rib coloring on a graph is the determination of the color of the ribs of a graph so that each adjacent rib gets a different color. The size of the coloring of a graph is defined as the size of point coloring, which refers to the number of colors that are possible so that each rib with a tan gets a different color. The color minima l can be used to color the ribs in a graph G called chromatic chromatic G.

Definition 4:

The coloring A of graph G. From a graph G is a given A. color on each line of G is so that two or more lines that meet at a point are given a different color. If G has a coloring A. , then G is said to be A colorized.

Definition 5:

The chromatic index of G is notated as K (G). is the minimum number A needed when G has the A colorized.

Definition 6:

K(Km,n) =d = max (m , n)

IV. DISCUSSION

A. Sudoku Graf

Each square (small box) of Sudoku is represented by a dot. So that the graph of sudoku 9 x 9 has 81 points that correspond to the number of boxes. Each point of the sudoku adjasen graph with a point in line, a column, and a box (3x3).

2			3			7		9
	8			5				
5		9						
						4		
7				9		8	6	
		1						2
	1			4		9		6
				7			2	
9					8			

Figure 8 Sudoku 9 x 9 Puzzle
Information:
row = column = box (3x3)
(http://viemagazine.com/article/thinking-inside-the-box/)

Steps that can be used for Sudoku work using graphs are as follows:

1) Changing Sudoku elements into Sudoku graphs where each element is a vertex of the graph and denoted by vi, j with Vertex vi, j and vi, j are said to be neighbors if i i = i 'or j = j' which is connected by an edge. Edge-edge is seen as the relation of each number element in Sudoku
2) Giving color to each Sudoku graph vertex. The trick is to give a certain color to a vertex, then find another vertex that has not been given a color and do the same process. This search is continued on other vertices that have not been colored until all vertices have been properly colored. In graph theory, if the vertexes connected by an edge do not have the same color, they are called 'the right color'.
3) Re-change Sudoku graphs that have been colored in the early form of Sudoku games, namely Sudoku tables. The colors on Sudoku graphs are converted into numbers in Sudoku games.

B. Example case of Sudoku 9 x 9

Sudoku 9 x 9 is a regular graph with n = 9, so Sudoku 9 x 9 has 9 colors and the degree of chromatic number is 10 such that chromatic polynomial is obtained as follows:

$$P(G,\lambda) = \lambda(\lambda-1)^{18}(\lambda-2)^{18}(\lambda-3)^{10}(\lambda-4)^{10}(\lambda-5)^{10}(\lambda-6)^{10}(\lambda-7)^{10}(\lambda-8)^{10}$$
$$= 9(9-1)^{10}(9-2)^{10}(9-3)^{10}(9-4)^{10}(9-5)^{10}(9-6)^{10}(9-7)^{10}(9-8)^{18}$$

= 1.0219 x 1047

If given a Sudoku case like the picture below

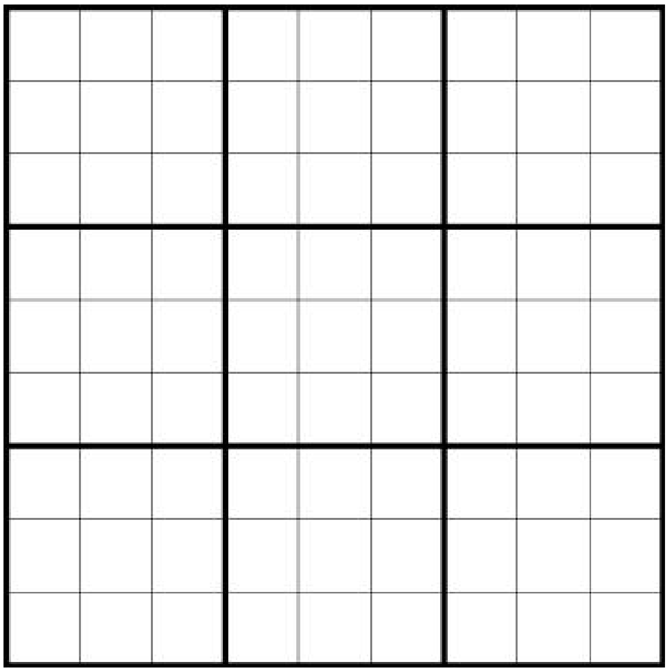


Figure 9 Initial Sudoku 9 x 9
(http://www.sudoku.4thewww.com/grids.php)

To find a solution, a graph coloring technique is based on the known chromatic number. The minimum color that must be owned by Sudoku 9 x 9 is 9 colors, so Sudoku has a solution:

Here's one way to solve the problem of Sudoku Figure 3.2.

Given 9 colors as follows

- 1 = blue
- 2 = red
- 3 = green
- 4 = yellow
- 5 = purple
- 6 = orange
- 7 = brown
- 8 = ash
- 9 = pink

Vertex v2,2, v3,5, v1,8, v5,3, v6,6, v4,9, v8,1, v9,4 and v7,7 are not neighboring so the initial color is blue, so the initial stage The following picture is obtained:

Figure 10 Blue coloring (number 1)

Then v1,9, v2,3, v3,6, v4,1, v5,4, v6,7, v7,8, v8,1 and v9,4 are adjacent to vertices that have been given a blue color and between the nine vertices are not neighboring, so that it is given a red color.

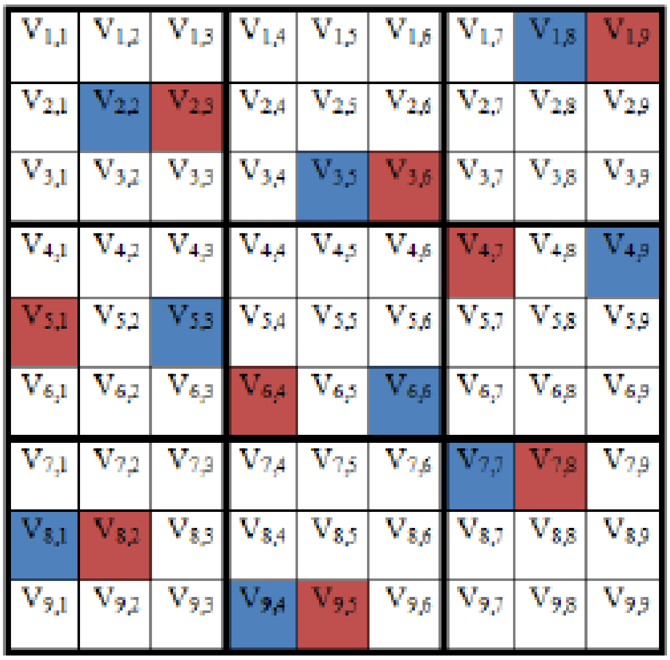


Figure 10 Red coloring (number 1)

Vertex v1,2, v2,5, v3,8, v4,1, v5,4, v6,7, v7,3, v8,6 and v9,9 are adjacent to vertices that have been given blue and red but between the nine vertices are not neighboring, so they are given a green color.

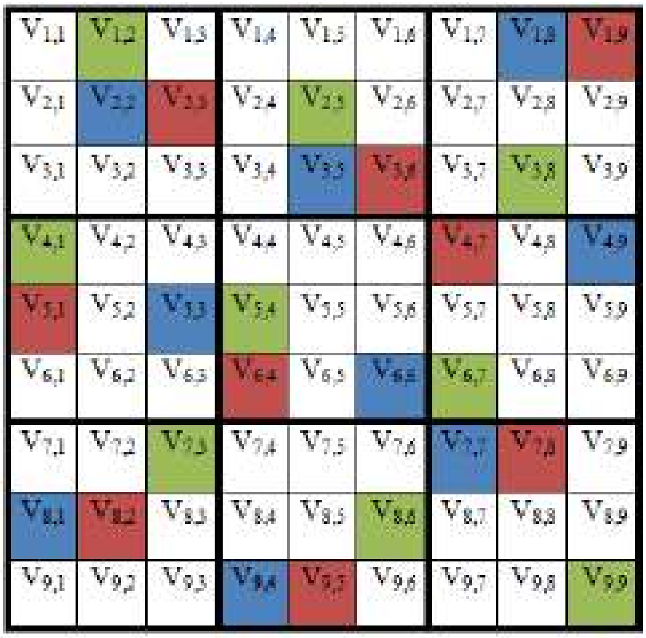


Figure 11 Green coloring (number 3)

Vertex v1,7, v2,1, v3,4, v4,8, v5,2, v6,5, v7,9, v8,3 and v9,6 are adjacent to vertices that have been given blue, red and green but between the nine vertices they are not neighboring, so they are given a yellow color.

Figure 12 Yellow coloring (number 4)

Vertex v1,3, v2,6, v3,9, v4,2, v5,5, v6,8, v7,1, v8,4 and v9,7 are adjacent to vertices that have been given blue, red, green and yellow but the nine vertices are not neighboring, so they are given a purple color.

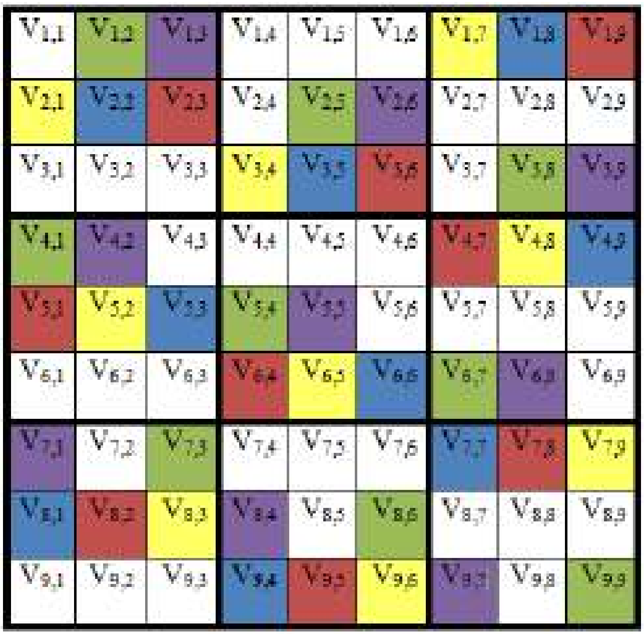


Figure 13 Purple coloring (number 5)

Vertex v3,3, v1,6, v2,9, v6,1, v4,4, v5,7, v9,2, v7,5 and v8,8 adjacent to vertices that have been given blue, red, green , yellow and purple but the nine vertices are not neighboring, so they are given an orange color.

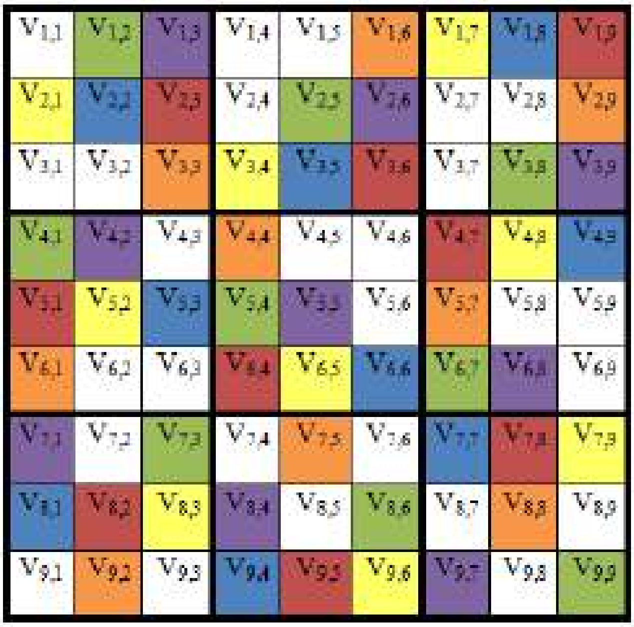


Figure 14 Orange coloring (number 6)

Vertex v1,3, v2,4, v3,7, v4,3, v5,6, v6,9, v7,2, v8,5 and v9,8 are adjacent to vertices that have been given blue, red, green , yellow, purple and orange but the nine vertices are not neighboring, so they are given a brown color

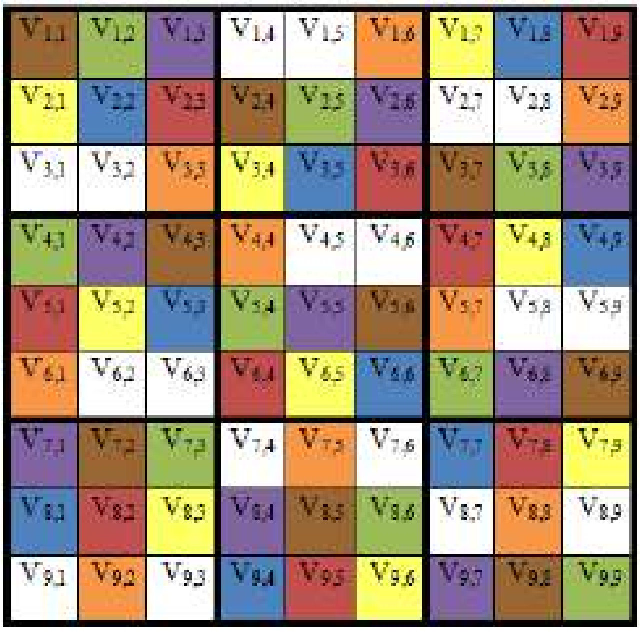


Figure 15 Brown coloring (number 7)

Vertex v3,1, v1,4, v2,7, v6,2, v4,5, v5,8, v9,3, v7,6 and v8,9 are adjacent to vertices that have been given blue, red, green , yellow, purple, orange and brown but the nine vertices are not neighboring, so they are given the color gray.

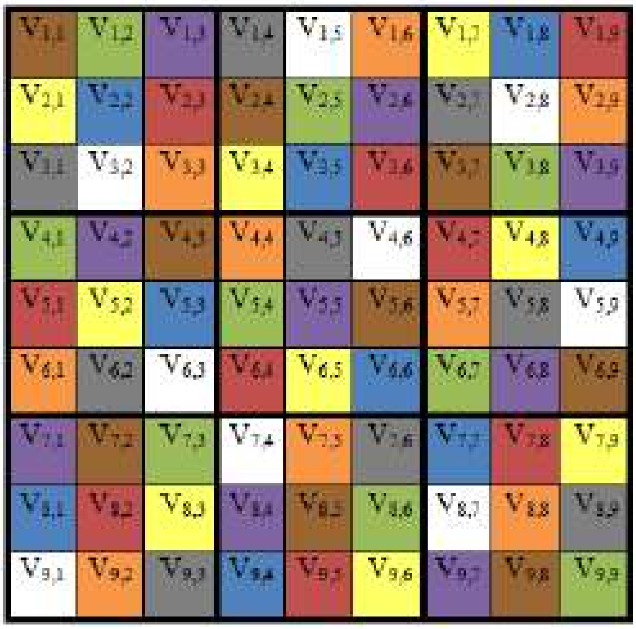


Figure 16 Gray Coloring (number 8)

Vertex v3,2, v1,5, v2,8, v6,3, v4,6, v5,9, v9,1, v7,4 and v8,7 are adjacent to vertices that have been given blue, red, green , yellow, purple, orange, brown and ash but the nine vertices are not neighboring, so they are given a pink color.

Figure 17 Pink coloring (number 9)

After all the colors are fulfilled and no neighboring vertices are the same color, then return the color to a number.

7	3	5	8	9	6	4	1	2
4	1	2	7	3	5	8	9	6
8	9	6	4	1	2	7	3	5
3	5	7	6	8	9	2	4	1
2	4	1	3	5	7	6	8	9
6	8	9	2	4	1	3	5	7
5	7	3	9	6	8	1	2	4
1	2	4	5	7	3	9	6	8
9	6	8	1	2	4	5	7	3

Figure 18 Final Sudoku

V. CONCLUSION

Based on the discussion that has been described, it can be concluded that the graph coloring technique can be used as one of the techniques to complete the Sudoku game after transforming the Sudoku matrix into a Graph.

Sudoku graph n x n is a regular graph where Sudoku n x n each vertex has the same degree and has a minimum color or chromatic number x(G)=n such that it has a chromatic polynomial $P(G, \lambda)= \lambda(\lambda-1)^{n+1} \dots (\lambda-(n-1))^{n+1}$ which shows the number of Sudoku game solutions. Sudoku 9 x 9 has a minimum of 9 colors with the number of solutions $P(G, \lambda)= \lambda(\lambda-1)^{10}(\lambda-2)^{10}(\lambda-3)^{10}(\lambda-4)^{10}(\lambda-5)^{10}(\lambda-6)^{10}(\lambda-7)^{10}(\lambda-8)^{10.}$
In general, Sudoku 9 x 9 has many problems and 1.0212×10^{47} coloring method.

VII. ACKNOWLEDGMENT

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PERNYATAAN

Dengan ini saya menyatakan bahwa makalah yang saya tulis ini adalah tulisan saya sendiri, bukan saduran, atau terjemahan dari makalah orang lain, dan bukan plagiasi.

Bandung, 8 Desember 2018

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