Graph for Traffic Control

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***Abstract*—Graph theory can be applied in traffic control for example in the control of traffic lights. By modeling the traffic flows as graph, we can create a mathematical model in order to make traffic lights can be used more efficiently to avoid long time waiting at junctions and congestions. The compatibility graph corresponding to the problem and circular arc graphs are introduced in this paper to solve the problem. The solution of this model is not unique, since it is influenced by volume that is per hour based on the rush-hour at each crossroad.**

***Keywords*—circular arc graph, clique, compatibility graph.**

I. Introduction

Traffic control is very important, especially in big cities, because the traffic of a city impacts all aspects of its citizens including economic development, traffic accidents, increase in greenhouse emissions, time spent, and health damages.

Today the growth of population in many cities is increasing more rapidly every time. Every year new road and highways are built in most of the urban areas to accommodate the growing number of vehicles. This increase in the number of vehicles in urban cities has led to the increase in time losses of traffic participants, the increase of environmental and noise pollution and also increases in the number of traffic accidents.

Traffic congestion is one of the major obstacles for the development of many cities, affecting millions of people. Constructing new roads may improve the situation, but it is very costly and in many cases it is impossible due to the existing structures. The only way to control the traffic flow in such a situation is to use the current road network more efficiently.

A method of handling city traffic in a very efficient way is by proper traffic control system instead of modifying the road infrastructure. One of the systems that regulate the flow of traffic is traffic lights. Traffic lights play the most important role on the impact of traffic. If the traffic arrangements are not optimal, then it will not only affect traffic order, but also lead to an accident.

The traffic lights are lights that control the flow of traffic and attach at a crossroads, pedestrian crossings (zebra crossing), and another traffic flow places. These lights indicate when the vehicles have to continue or stop driving alternately from different directions. The setting of traffic lights at the crossroads is intended as vehicle's movement regulation in each group of movements, so that vehicles will not interfere the existing flow. However, in reality shows that there are more solid volume in the specific crossroads rather than congested crossroads. This situation happens because the traffic control system is less effective and efficient.

In order to make traffic control system more effective and efficient, we can apply graph theory for traffic light management. In this paper we use the applications of circular arc graph and compatible graph to manage traffic lights. In this graph, the dots or vertices show the objects that will be set, and the sides indicate the compatible object. The traffic streams which can move simultaneously at an intersection without any conflict may be termed as compatible. By using compatible graph, optimal waiting time at the crossroads can be determined.

II. Basic Theory

2.1. Graph Definition.

Graph is a data structure that consists of a set of nodes (*vertices*) and a set of edges that relate the nodes to each other. The set of edges describes relationships among the vertices. A graph *G* is defined as follows:

$$G=(V,E)$$

V(G): a finite, nonempty set of vertices
E(G): a set of edges (pairs of vertices)
Each edge e in E has a set of one or two vertices associated to it which are called it's end points.

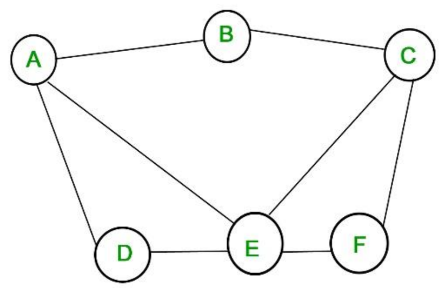


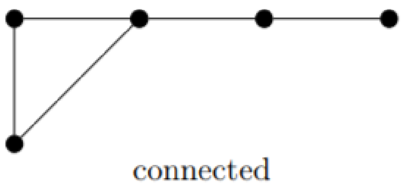
Figure 1. Example of graph
Sumber : https://www.geeksforgeeks.org/mathematics-graph-theory-basics/

2.2. Graph Terminology

Some terminologies regarding to graph are used to describe the properties, types, overall structure of a graph, as well as its characteristics.

1. Adjacent
   Two nodes are adjacent if they are connected by an edgeIncident.
2. Incident
   An edge is incident to a vertex if the vertex is connected by the edge.

3. Path
   A path with length n from a vertex v0 to a vertex vn in a graph G is a sequence v0, e1, v1, e2, …,vn-1, en, vn such that e1 = (v0, v1), e2 = (v1, v2), …, en = (vn-1, vn).
4. Cycle
   A cycle is a path in a graph G which starts and ends in the same vertex v0.
5. Outgoing Edges
   Outgoing edges of a vertex are directed edges that the vertex is the origin.
6. Incoming Edges
   Incoming edges of a vertex are directed edges that the vertex is the destination.
7. Degree
   Degree of a vertex is the quantity of edges incident to the particular vertex.
8. Out-degree
   outdeg(*v*), is the number of outgoing edges.
9. In-degree
   indeg(*v*), is the number of incoming edges.
10. Isolated Vertex
    A vertex is isolated if and only if it doesn’t have any incident edges. A vertex v is isolated if d(v) = 0.
11. Null graph
    A graph is a null graph if it contains an empty set of edges.
12. Connected Graph
    A graph is connected if there is a path from every vertex to every vertex excluding itself.



Figur 2.1 Connected graph.
Sumber : https://myalgo.wordpress.com/2010/09/04/connected-graph/

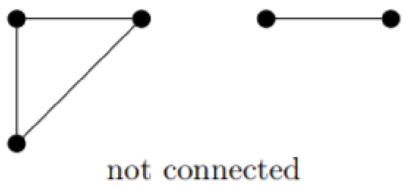


Figure 2.2 Unconnected graph.
Sumber : https://myalgo.wordpress.com/2010/09/04/connected-graph/

13. Connected Component
    Connected component is the maximal connected sub-graph of a unconnected graph.
14. Subgraph
    A graph H = (U, F) is a subgraph of a graph G = (V, E) if U ⊆ V and F ⊆ E. If U = V then H is called spanning.

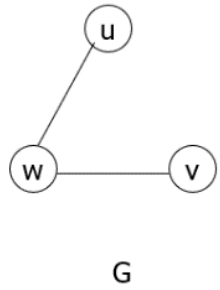


Figure 3.1. Graph G.

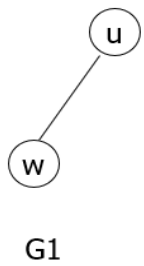


Figure 3.2. Graph G1.

G1 is subgraph of G.

15. Spanning Sub Graph
    Spanning sub graph contains all the vertices.
16. Weighted Graph
    A weighted graph is a graph in which every edge has a respective weight.

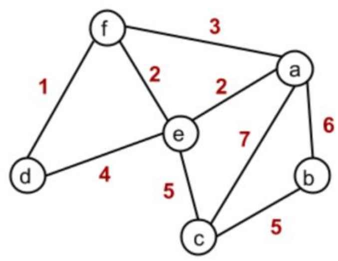


Figure 4. Weighted graph.
Sumber : https://www.chegg.com/homework-help/questions-and-answers/undirected-weighted-graph-g-given--use-prim-s-algorithm-compute-minimum-spanning-tree-weig-q21203421

2.3. Types of Graph

Graphs can be classified into many types based on their distinct properties, various numbers of its elements, or overall structures. Based on the edge’s characteristics, specifically the edge’s direction, graphs can be distinguished into two types:

1. Undirected graph
   Unordered pair of vertices. Represented as {u, v}. Disregards any sense of direction and treats both end vertices interchangeably.

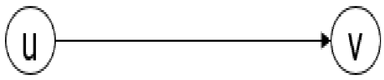


Figure 5.1. Undirected graph.

2. Directed graph
Ordered pair of vertices. Represented as (u, v) directed from vertex u to v.

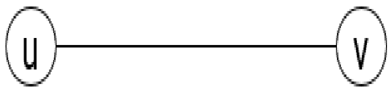


Figure 5.2. Directed graph.

Based on the edge's types, graphs can also be grouped into three types:

1. Simple Graph
   A simple graph is a graph without any loops or parallel edges. Figure 1 is an example of a simple graph.
2. Multigraph
   A graph that has multiple edges connecting the same vertices is called a multigraph.

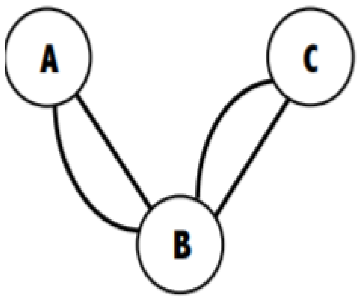


Figure 6.1. Example of Multigraph.
Sumber : https://www.mathbootcamps.com/discrete-math-guides-articles/

3. Pseudograph
   Pseudograph is a graph which has loops or multiple edges connecting the same vertices.

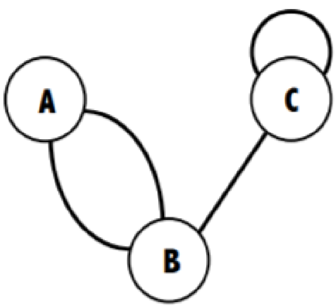


Figure 6.2. Example of Pseudograph.
Sumber : https://www.mathbootcamps.com/discrete-math-guides-articles/

Some simple graphs can also be classified based on its unique characteristics. Such graphs are called specific graph. Some of the specific graphs are:

1. Complete Graph Graphs in which each vertex is adjacent to every other vertex in the graph is called a complete graph. Such graphs are denoted by Kn, n representing the number of vertices.

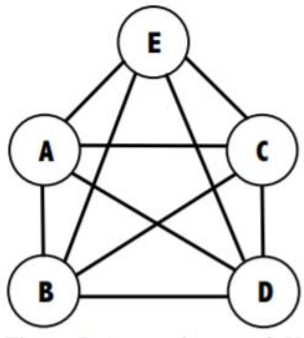


Figure 7. Example of Comple graph.
Sumber : https://access.redhat.com/documentation/en-US/Fuse_ESB_Enterprise/7.1/html/Using_Networks_of_Brokers/files/FMQNetworksTopologies.html

2. Regular Graph
   A regular graph is a graph in which every vertex has the same degree. A regular graph with n-degree for each vertex is called an n-regular graph.

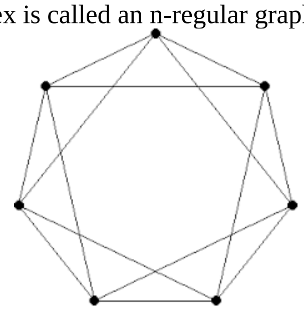


Figure 8. Example of regular graph.
Sumber : http://www.mathe2.uni-bayreuth.de/markus/REGGRAPHS/07_4_3.html

3. Cyclic Graph
   A cyclic graph is a graph that contains at least one cycle.

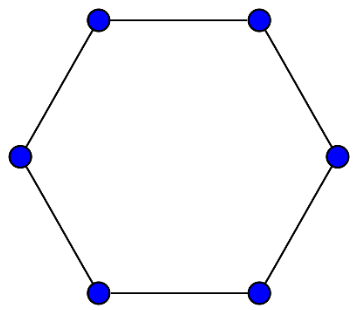


Figure 9. Example of Cyclic graph.
Sumber : https://en.wikipedia.org/wiki/Cycle_graph

2.4. circular-arc graph

In graph theory, a circular-arc graph is the intersection graph of a set of arcs on the circle. It has one vertex for each arc in the set, and an edge between every pair of vertices corresponding to arcs that intersect.

Formally, let

$$I_1, I_2, \ldots, I_n \subset C_1$$

be a set of arcs. Then the corresponding circular-arc graph is G = (V, E) where

$$V = \{I_1, I_2, \ldots, I_n\}$$

and

$$\{I_\alpha, I_\beta\} \in E \iff I_\alpha \cap I_\beta \neq \varnothing.$$

A family of arcs that corresponds to G is called an arc model.

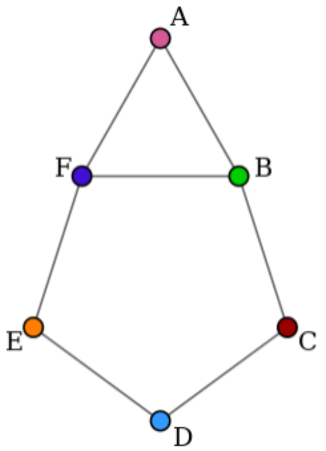


Figure 10.1. A circular-arc graph
Sumber : https://en.wikipedia.org/wiki/Circular-arc_graph

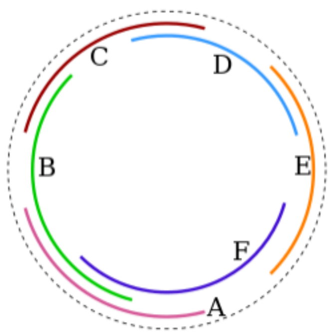


Figure 10.2. A corresponding arc model
Sumber : https://en.wikipedia.org/wiki/Circular-arc_graph

2.5. Clique

In the mathematical area of graph theory, a clique is a subset of vertices of an undirected graphsuch that every two distinct vertices in the clique are adjacent; that is, its induced subgraph is complete.

A clique, C, in an undirected graph G = (V, E) is a subset of the vertices, C ⊆ V, such that every two distinct vertices are adjacent. This is equivalent to the condition that the induced subgraph of G induced by C is a complete graph. In some cases, the term clique may also refer to the subgraph directly.

A maximal clique is a clique that cannot be extended by including one more adjacent vertex, that is, a clique which does not exist exclusively within the vertex set of a larger clique. Some authors define cliques in a way that requires them to be maximal, and use other terminology for complete subgraphs that are not maximal.

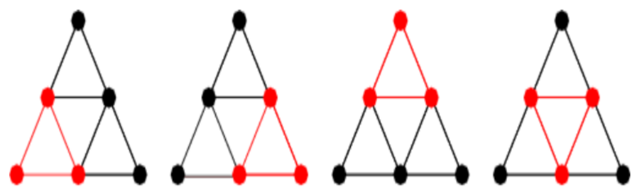


Figure 11. Example of clique.
Sumber ;
http://homepages.inf.ed.ac.uk/rbf/CVonline/LOCAL_COPIES/AV0405/ALLISON/maxCliques.html

2.6. Compatible Graph

The compatible graph of traffic flows are defined as the traffic flow which is expressed by an edge, two edges of the traffic flow are connected when the traffic flow does not cause a crashes if it flows together. In a graph form, we can model it as follows :

1. Observe many currents and directions of flows.
2. Labeling the first flow and giving dots mark (vertices).
3. Provide initial assumptions, such as whether or not to turn left to follow the light, whether or not flow must stream with the current left turn who has the same end vertices
4. Representing each flow with a vertice on the graph.
5. Connect two vertices on the graph by an edge if and only if the two flows are represented by the compatible two vertices

2.7. Crossroads

Crossroads are inseparable part of systems. Crossroad can be defined as the place one road crosses another/an crossroads of two or more roads, including roads and roadside facilities for traffic movement. The purpose of the crossroads making is to reduce the potential conflicts among drivers (including pedestrians) and to provide a great comfort and to ease movement of vehicles.

2.8. Identification of Crossroad Performance

Identification of crossroad performance is the crossroad of performance values / criteria of a junction that is used to optimize the performance of crossroad among others.

1. Saturated traffic flow

Saturated traffic flow is the maximum number of the units used are vehicles / hour. The saturation vehicles can be the base of the crossroad per hour. flow is not the same at the crossroads, because there are some things that affect the saturated flow rate as follows.

- Climbs and an instance at each crossroad
- Traffic of compositions
- Whether there is a traffic that will be turned to the right which ran into the traffic coming from the opposite direction
- Curve radius

2. Fase
   Fase is part of the cycle of traffic lights, provided for certain combinations of traffic movements. This phase will be partitioned into several parts sub set of flows, so that we know where the currents are moving and where the currents are stopped.

III. Traffic problem analysis

Traffic light systems are widely used to control the flow of vehicles through the junction of many roads. It aim to realize smooth motion of cars in the transportation routes. However, the synchronization of multiple traffic light systems at adjacent intersections is a complicated problem given the various parameters involved.

Our goal here is to install traffic lights at a road junction in such a way that traffic flows smoothly and efficiently at the junction. We will take a specific example and explain how our problem could be solved. see the figure below. it displays some traffic streams in the junction.

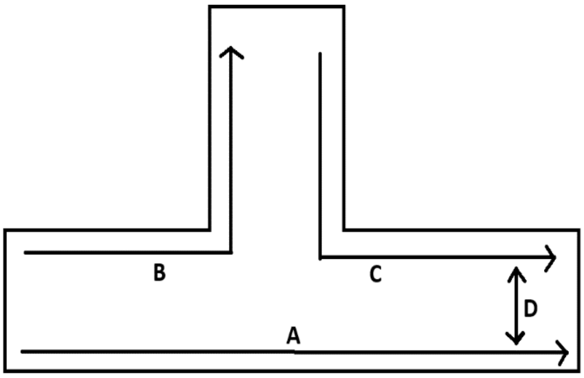


Figure 12. Sample of traffic streams.

When vehicles approach the intersection, they will do a maneuver to go straight, turn left, or turn right. The vehicles that perform this maneuver represent a flow component. Such an arrival flow component is called a traffic stream. The traffic streams which can move simultaneously at an intersection without any conflict may be termed as compatible. According to figure above, streams A and B are compatible, and also streams A with C and streams B with C are compatible. Steam D and C and also stream D and C are not compatible. The fase of lights should be such that when the green lights are on for two streams, they should be compatible. We suppose that the total time for the completion of green and red lights during one cycle is three minutes. We will form a graph G whose vertex set consist of the traffic streams in figure above and we make two vertices of G connected by an edge if, and only if, the corresponding streams are compatible. The graph we will obtain is the compatibility graph corresponding to the problem above. The compatibility graph is shown below.

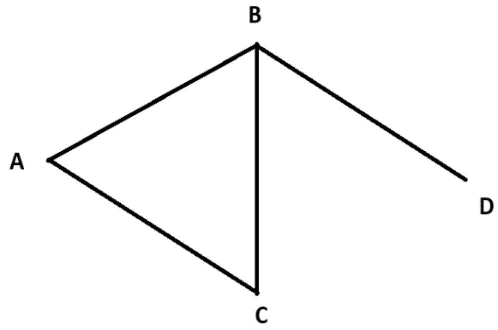


Figure 13. Graph G (Compatibility graph).

Here we will take a circuit and assume that its perimeter corresponds to the total cycle period, says it is 180 seconds. We may think that the duration when a given traffic stream gets a green light corresponds to an arc of this circle. Hence, two such arcs of the circle can overlap only if the corresponding streams are compatible. The resulting circular arc graph may not be the compatibility graph because we do not demand that two arcs intersect whenever corresponds to compatible flows. (There may be two compatible streams but they need not get a green light at the same time). However, the intersection graph H of this circular arc graph will be a spanning sub graph of the compatibility graph. So we have to take all spanning sub graph of G in to account and choose from them the spanning sub graph that has the most maximal clique. The proper graph H for the above example is shown in Figure below.

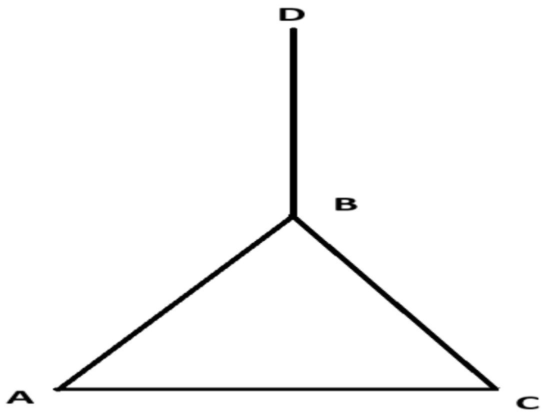


Figure 13. Graph H (Intersection graph)

Our goal now is to minimize the total red light time during a traffic cycle that is the total waiting time for all the traffic streams during a cycle. The maximal clique of graph H are K_2={A,B,C} and K_2={B,D}. Each clique Ki, $1 \leq i \leq 2$, corresponds to a fase during which all streams in that clique can receive green lights together. We assume in fase 1, traffic streams A,B and C receive a green light. In fase 2, B and D receive a green ligh. Suppose we assign to each phase Ki a duration Di. Our goal is to determine the D so that the total waiting time is minimum. Further, we may assume that the minimum green light time for any stream is 30 seconds. Traffic stream A gets a red light when the fase K_2 receive a green

light. Hence A’s total red light time is D_2 . Similarly, the total red light time of all streams C and D, respectively are D_2 and D_1. Suppose the total red light time of all streams in one cycle is Z. Now lets starts the fase. First, in fase 1, A,B and C get green lights while D get red light. Total red light is D_1.
Then we go to fase 2. In fase 2, B and D get green light while A and C get red lights. Total red light for this fase is $D_2 + D_2 = 2D_2$. Now we get the total red light time of all streams for a cycle is $Z = 2D_2 + D_1$. Our aim is to minimize Z so that total waiting time is also minimum at the traffic junction. According the analysis above, finally we can get some equations.

$$Di \geq 0,$$
$$1 \leq i \leq 2,$$
$$D_1 \geq 30,$$
$$D_2 \geq 30,$$
$$D_1 + D_2 \geq 30,$$
$$D_1 + D_2 = 180,$$
$$Z = 2D_2 + D_1$$

The optimal solustion for equations above is $D_1 = 150$, $D_2 = 30$. So we get Z = 210 as minimum value.

IV. Conclusion

Graph theory can be applied to the problem of traffic system settings at a crossroads in the following way

1. Modeling system traffic flow at crossroads into the compatible graph
2. Dividing the set vertices of the compatible graph into several sections graph such that many parts as possible
3. Make a mathematical model in the form of the function total time all the current flowing in the time intervals of flows grouping
4. Establish the conditions required e.g. minimum running time of each flow
5. Resolving mathematical model.

The calculation of the optimal cycle states traffic light at one cycle. in fact, the traffic light settings are very complicated and no single, which involves a variety of factors, and cannot adopt a suitable model to solve all problems.

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Pernyataan

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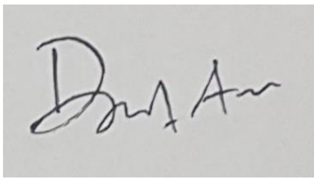
Bandung, 10 Desember 2018



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