The Application of Graph, Decision Tree and Combinatorial Theorem in The Resistance Game

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***Abstract*—The Resistance is a social deduction game in which players have secret identities, and are divided into factions which has separate goals. Players will then go on missions to be able to get a chance to achieve those goals. In order to certainly achieve those goals, players have to understand the concept of the game, and the strategies which can be involved in the game. By using Graphs, Decision Tree and Combinatorial Theorem, we can understand the game better than by playing without them.**

***Keywords*—The Resistance Game, Graph, Decision Tree, Combinatorial.**

I. INTRODUCTION

Figure 1.1. The Resistance game [3]

"The Resistance", a social deduction game in which the players have secret identities. Players are then divided into two factions that are involved in this game. One is the "Resistance", which are attempting to overthrow a malignant government. The other is the "Spies", which are attempting to thwart the "Resistance".

Players are faced with five missions in which a "Mission Team", comprised of a certain number of players, must go on the mission. The "Leader" will attempt to organize which players will join the "Mission Team". After the "Leader" attempts to organize the players which will join the "Mission Team", the players will then vote for the "Mission Team" to be approved or not. For the mission to be successfully approved, the majority of players have to approve the mission. The "Leader" is decided randomly at first, then it is changed in a clockwise rotation, if the "Mission Team" that the "Leader" organized is not approved. If five "Leaders" fail to organize an approved "Mission Team" then the "Resistance" is labeled to be unable to organize an approved "Mission Team"

After the "Mission Team" is organized and approved, players that are on the "Mission Team" will have to play a "Mission Card". The members of the "Resistance" will only be able to play a "Mission Success" card, while the members of the "Spies" will be able to play both a "Mission Success" card and a "Mission Fail" card.

The mission is declared "completed successfully" if the number of "Mission Fail" cards that are played, are lower than the required number "Mission Fail" cards needed to be played for the mission to fail.

The objective of the game is different for each faction. The "Resistance" wins if three missions are completed successfully, and the "Spies" win if three missions fail or the "Resistance" is unable to organize an approved "Mission Team".

In order to better understand "The Resistance" game, Graph, Decision Tree and Combinatorial Theorem can be used. Therefore, in this paper, I will elaborate on the applications of Graph, Decision Tree and Combinatorial Theorem in the game.

II. BASIC THEORY

A. Definition of Graph

According to reference [1], graphs are discrete structures that consist of vertices and edges that connect the vertices. Mathematically, a graph is represented as

$$G = (V, E)$$

in which G is graph, V is nonempty set of vertices (or nodes) and E is set of edges. Each edge of a graph has either one or two vertices associated with it, which is called its endpoints. Also, the edge of a graph connects its endpoints.

B. Types of Graphs

Reference [1] and [2] shows that there are multiple types of graphs. The types of graph are differentiated by multiple categories.

The types of graphs which are differentiated by the number of vertices and/or edges include:

1. Infinite graph: A type of graph in which the number of vertices and/or the number of edges is infinite.
2. Finite graph: A type of graph in which the number of vertices and the number of edges is finite.
3. Null Graph: A Graph that doesn’t contain any edge.

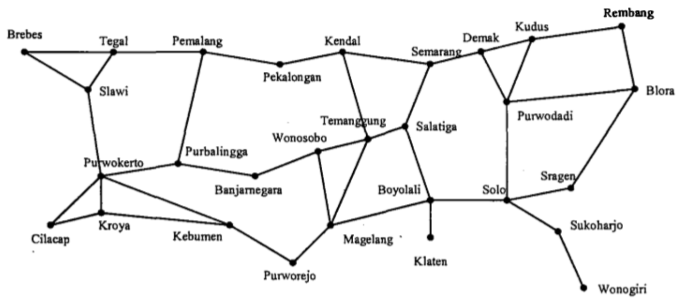

Figure 2.B.1. An example of a Finite Graph [2]

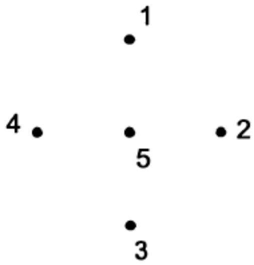

Figure 2.B.2. An example of Null Graph [2]

The types of graphs which are differentiated by the connection of its edges include:

1. Simple graph: A type of graph in which each of its edges connect two different vertices and there are no two edges that connect the same pair of vertices.
2. Multigraph: A type of graph in which some of the edges connect the same pair of vertices.
3. Pseudograph: A type of graph in which it may include loops, and/or multiple edges that connect the same pair of vertices.

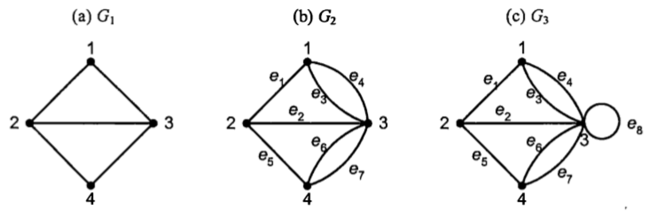

Figure 2.B.3. (a) Simple Graph, (b) Multigraph, (c) Pseudograph [2]

The types of graphs which are differentiated by the direction of its edges include:

1. Undirected graph: A type of graph in which its edges are undirected.
2. Directed graph: A type of graph in which each of its edges are directed and are associated with an ordered pair of vertices. The ordered pair of vertices (u,v) is said to start at u and end at v.
3. Multi-Directed Graph: A special type of directed graph in which it has edges that have multiple directions.
4. Mixed graph: A type of graph in which includes directed and undirected edges. Much like the directed edges in directed graphs, the directed edges in mixed graphs are also associated with an ordered pair of vertices. The ordered pair of vertices (u,v) is also said to start at u and end at v.

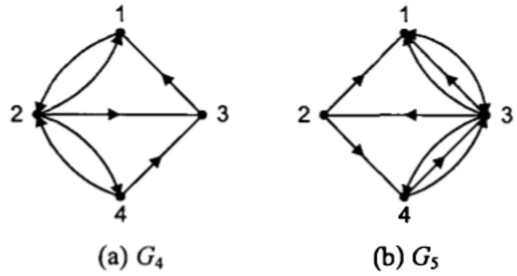

Figure 2.B.4. (a) Directed Graph, (b) Multi-Directed Graph [2]

The types of graphs which are classified based on its unique characteristics are

1. Complete Graph: A type of graph in which each of its vertices is adjacent to every other vertices in the graph. Graphs which fall into this type are denoted by K_n, in which n represents the number of vertices.

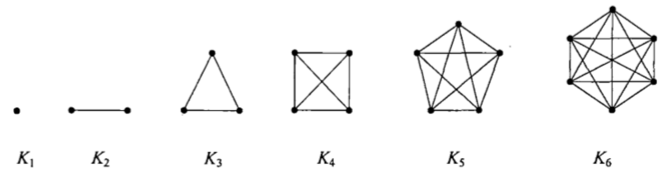

Figure 2.B.5. Complete Graphs [2]

2. Regular Graph: A type of graph in which every vertex has the same degree. A regular graph with n-degree for each vertex.

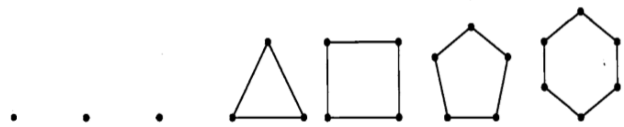

Figure 2.B.6. Regular Graph [2]

3. Cyclic Graph: A type of graph which contains at least one cycle.
4. Connected Graph: A type of graph in which every pair of vertex u and v in set of vertex V have at least one path from vertex u and v.

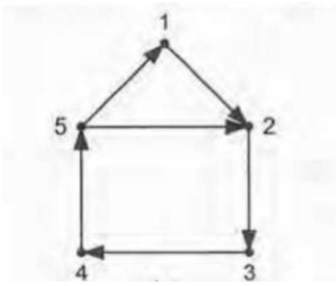

Figure 2.B.5. Connected Graph [2]

5. Weighted Graph: A type of graph in which every edge has a respective weight.

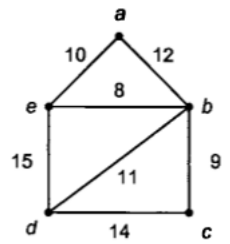

Figure 2.B.6. Weighted Graph [2]

C. Basic Terminologies in Graphs

Reference [1] and [2] shows that there are some terminologies in Graphs. These include:

1. Adjacent: Two vertices u and v in an undirected graph G are called adjacent in G if u and v are endpoints of an edge e of G.
2. Incident: An edge e in a graph G is called incident with the vertices u and v if u and v are endpoints of the edge e.
3. Connect: Two vertices u and v in a graph G connect in G if u and v are endpoints of the edge e.
4. Neighborhood: The set of all neighbors of a vertex v of G = (V,E), denoted by N(v), which is the set of all vertices in G that are adjacent to vertex v.
5. Degree: The number of edges incident with a vertex, except that a loop at a vertex contributes twice to the degree of that vertex, which is denoted by deg(v).

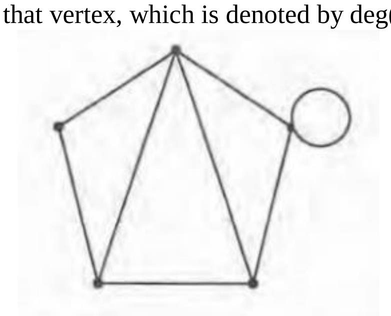

Figure 2.B.7. Graph with degrees 2,3,3,4,4 [2]

6. Isolated vertex: A vertex with a degree of zero is isolated.
7. Pendant vertex: A vertex that has a degree of one.
8. Path: A path with length n from the start, which is vertex V_0, to the destination, which is vertex V_n, in graph G is a sequence v_0, e_1, v_1, e_2, …,v_n-1, e_n, v_n such that $e_1 = (v_0, v_1)$, $e_2 = (v_1, v_2)$, …, $e_n = (v_n\text{-}1, v_n)$.
9. Cycle/Circuit: A Circuit or a cycle is a path of graph which starts and finishes in the same vertex.
10. Bridge: A bridge is an edge of a graph such that if it is deleted from the graph, it will separate the graph into two components.
11. Subgraph: A graph G = (V, E) has a subgraph $G_1 = (V_1, E_1)$ if and only if V1 ⊆ V and $E_1 \subseteq E$. A subgraph G1 is called spanning subgraph if $V = V_1$.
12. Subgraph Complement: A graph G = (V,E) which has a subgraph $G_1 = (V_1, E_1)$ has a Subgraph Complement of $G_2 = (V_2, E_2)$ such that $E_2 = E - E_1$ and V_2 are the set of vertices which are incident.

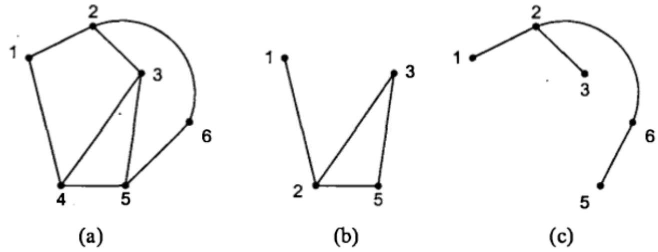

Figure 2.B.8. (a) Graph G1, (b) Subgraph of G1, (c) Complement of G1 [2]

13. Spanning Subgraph: A graph G = (V, E) which has a subgraph $G_1 = (V_1, E_1)$, has a spanning subgraph of G_1 if $V_1 = V$, or in other words, G_1 contains all vertices of G.

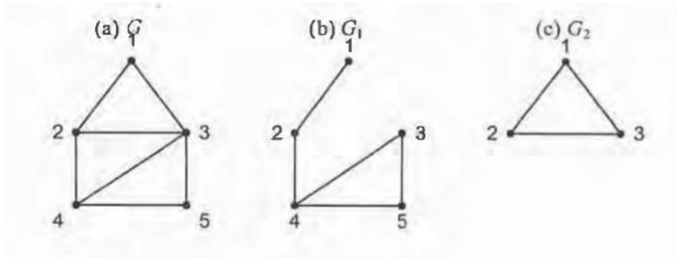

Figure 2.B.9. (a). Graph G, (b) Spanning Subgraph of G, (c) Not a Spanning Subgraph of G [2]

14. Cut-Set: A graph G has a cut-set which is the set edge of bridges.

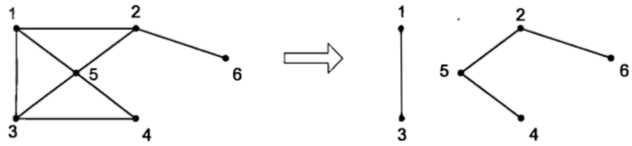

Figure 2.B.10. On the left, Graph G. On the right, Cut-set of Graph G [2]

D. Definition of Tree

According to reference [1], trees are connected undirected graphs which contain no simple circuits. More completely, graph G = (V,E) with number of nodes n is a tree if

1. It’s a simple undirected graph.
2. Every pair of nodes in G is connected with a single edge.
3. G has no simple circuits and the addition of an edge to the graph will result to the formation of only one circuit.
4. G is connected and has a number of edges of m = n -1.
5. G has no simple circuit and has a number of edges of m = n - 1.
6. G is connected, and all of its edges are bridges.

E. Types of Trees

According to reference [1] and [2], there are some types of trees. These include:

1. Rooted Tree: A type of tree in which one vertex has been designated as the root, and every edge is directed away from the root.

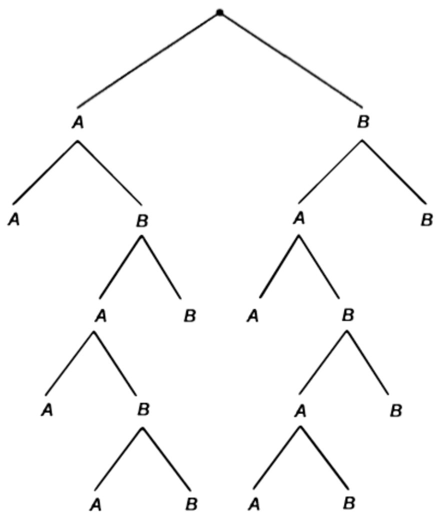

Figure 2.E.1 : An example of a rooted tree [2]

2. Ordered tree: A type of tree in which the order of its children are important.

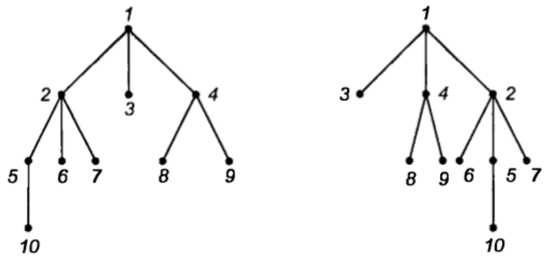

Figure 2.E.2 : Two different ordered trees [2]

3. Decision Tree: A type of tree in which it models decisions and their possible consequences. Each inside vertex denotes the decision, and each leaf denotes the solution.

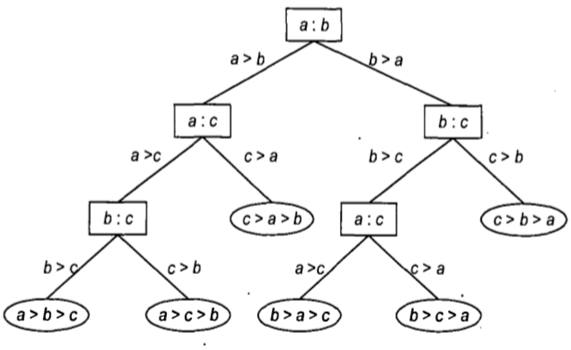

Figure 2.E.3 : A decision tree to arrange 3 elements [2]

4. M-ary tree: A type of tree in which every vertex of its branches have the most of m number of children.

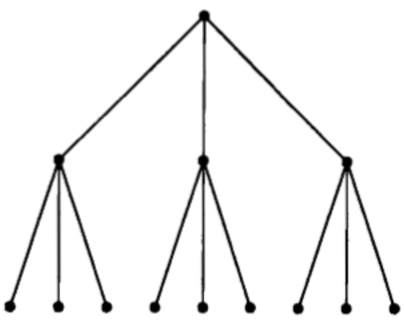

Figure 2.E.4. : An example of a 3-ary tree [2]

5. Expression tree: A type of 2-ary tree (binary tree) in which the leaf denotes the operand and internal vertices states the operator.

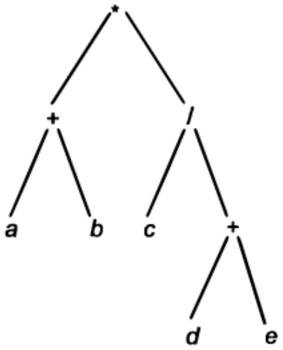

Figure 2.E.5. : Expression tree of (a+b)*(d(d+e)) [2]

6. Huffman tree: A full 2-ary tree (binary tree) in which each leaf of the tree corresponds with each symbol.

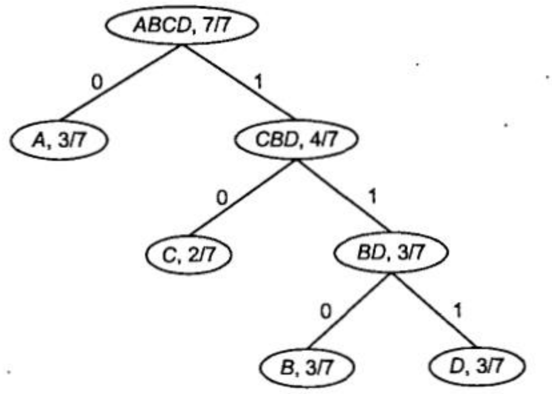

Figure 2.E.6 : Huffman tree for the message [2] ‘ABACCDA’

7. Search Tree: A type of tree used to locate specific keys from within a set. An example is binary search tree, illustrated in figure 2.E.7. In a binary search tree, all the vertices on the left subtree has a key that is smaller than Key(R), and all the vertices on the right subtree has a key that is larger than Key(R).

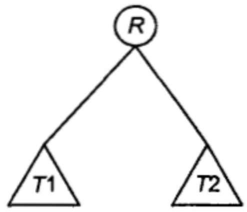

Figure 2.E.7 : Binary Search Tree [2]

F. Terminologies in Trees

According to reference [1] and [2], even though tree are also graphs, there are some terminologies which are limited to trees,.

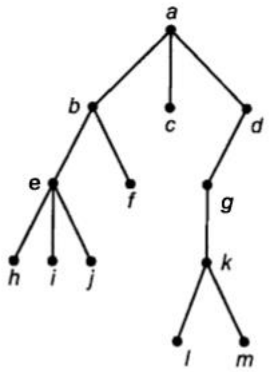

Figure 2.F.1: A tree to demonstrate terminologies in trees [2]
These include:

1. Child: Vertex v is the child of vertex x if there is an edge from vertex u to v. For example: In figure 2.F.1, b is a child of a.
2. Parent: Vertex u is the parent of vertex y if there is an edge from vertex u to v. For example: In figure 2.F.1, a is the parent of b.
3. Descendant: If there is a path from vertex u to vertex v in a tree, then vertex v is the descendant of vertex u. For example: in figure 2.F.1, h is the descendant of b.
4. Ancestor: If there is a path from vertex u to vertex v in a tree, then vertex u is the ancestor of vertex v. For example: In figure 2.F.1, b is the ascendant of h.
5. Sibling: Vertices which have the same parent is a sibling of one another. For example: in figure 2.F.1, i is the sibling of h.
6. Subtree: Assume that x is a vertex in a tree T. Subtree with x as its root is the subtree T' = (V', E') such as V' contains x and all its descendants and E' contains vertices in all paths that are originated from x. For example: T ' = (V', E') is the subtree of the tree in figure 2.F.1 with V' = {b, e, f, h, i, j}, E' = {(b, e), (b, j), (e, h), (e, i), (e, j)} and b is the root, shown in figure 2.F.2.

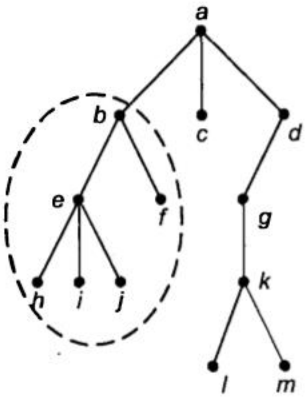

Figure 2.F.2: Subtree T' = (V, E') with b as its root. [2]

7. Degree: Degree of a vertex in a tree is the number of child on that vertex. In figure 2.F.1, the degree of e is 3, the degree of k is 2, the degree of g is 1 and the degree of m is 0.
8. Leaf: Leaf is a vertex that has a degree of zero. Vertices h, i, j, f, c, l, m are all leaves.
9. Internal vertices: Vertices which have a child or children is called internal vertices. In figure 2.F.1, vertices d, e, g and k are all internal vertices.
10. Level: The level of a vertex is 1 + the number of connections between the vertex and the root. The root has a level of zero. As a convention, we start numbering levels from zero.

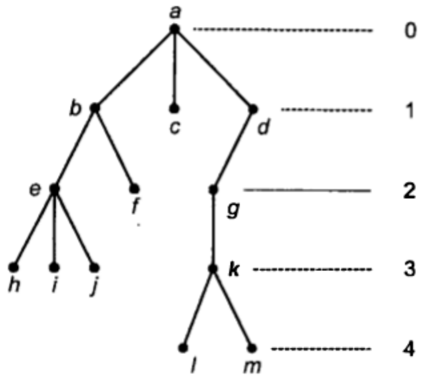

Figure 2.F.3: Defining the height each vertex in the tree [2]

11. Height or depth: Height or depth of a tree is the maximum level of that tree. Figure 2.F.3 has a height of 4.

G. Definition of Combinatorics

According to reference [2], combinatorics is a branch in mathematics that studies how objects are arranged. By using combinatorics, we can tell the possibilities of arrangement of objects. For example, we can count the possibilities of a number that is shown on the top of the dice, or the possibilities of chess moves.

H. Basic Rule in Combinatorics

According to reference [2], in combinatorics, we have to count all possibilities of arrangements. Two basics that are known are:

1. Rule of Product: If an experiment 1 has p outcomes that might happen, an experiment 2 has q outcomes that might happen, then if experiment 1 AND experiment 2 is done, then there will be p x q number of outcomes. This rule can then be expanded. If n number of experiments that doesn't depend on each other, each denoted by p_1, p_2, …, p_n, then if all experiments are done, there will be p_1 x p_2 x … x p_n number of possible outcomes.
2. Rule of Sum: If an experiment 1 has p experiment results that might happen, an experiment 2 has q experiment results that might happen, then if only 1 experiment is done, EITHER experiment 1 OR experiment 2, then there will be p + q number of experiment results. If n number of experiments that doesn't depend on each other, each denoted by p_1, p_2, …, p_n, then either one of those experiments are done, there will be $p_1 + p_2 + \ldots + p_n$ number of possible outcomes.

I. Inclusion-Exclusion Principle

According to reference [2], in combinatorics, there is a principle called inclusion-exclusion principle, which are used to count combinatorials. This principle is used when a case in which two numbers of possibilities are needed to be joined together, which may result in an inclusion of non-duplicate cases, and/or exclusion of duplicate cases. For example, to add the number of possible outcome of a binary string that ends with '11' and a string that begins with '11', we have to exclude duplicate cases which both sets already cover, and include the others. This case can be demonstrated with a Venn Diagram, which is shown below.

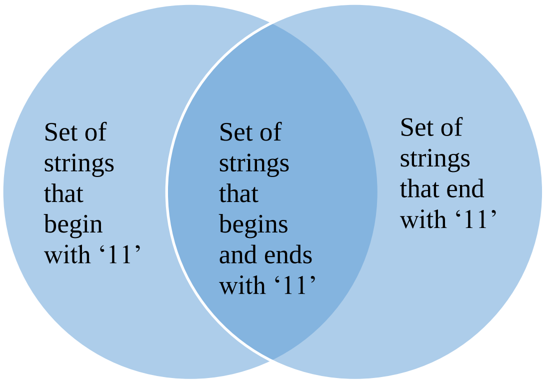

Figure 2.I.1 : Venn Diagram representation for the example above. (Author's document)

J. Permutation

According to reference [2], permutation is the number of possible combinations of the arrangement of objects. It is a special case of the Rule of Products. For instance, there is n number of objects, the first arrangement is chosen from n objects, the second arrangement is chosen from n-1 objects, the third arrangement is chosen from n-2 objects, and so on, and the last arrangement is chosen from the last object that is left. According to the Rule of Product, the permutation of n objects are:

$$\mathrm{n}(\mathrm{n}-1)(\mathrm{n}-2)\ldots(2)(1)=n!$$

Permutation r of n objects are the number of possible arrangements of r number of objects which are chosen from n number of objects, with r ≤ n, which in this case, in each possible arrangements there are no same objects. This can be demonstrated by the equation

$$\mathrm{P}(\mathrm{n},r)=n(n-1)(n-2)\ldots\big(n-(r-1)\big)=\frac{n!}{(n-r)!}$$

Circling permutation of n objects is the arrangement of objects which create a circle. The number of arrangements which create a circle is (n-1)!.

K. Combination

According to reference [2], combination is a special case of permutation. If in a permutation the order of appearance is considered, then in combination, the order of appearance is disregarded. For instance, the arrangements acb, bca and acb is considered same and are only counted as one.

Combination of r elements from n elements is the number of unordered arrangement of r elements which are taken from n number of elements. This statement can be demonstrated by the equation

$$\mathrm{C}(\mathrm{n},\mathrm{r})=\frac{n!}{r!\,(n-r)!}$$

In combination, there is a term which is called generalized combination, which is combination that regards all combination with the same item as only one. It is the same with generalized permutation. It can be demonstrated with the equation

$$\mathrm{P}(\mathrm{n};n_1,n_2,\ldots,n_k)=C(n;n_1,n_2,\ldots,n_k)=\frac{n!}{n_1!\,n_2!\ldots n_k!}$$

There is also a term which is called combination with repetition, which is combination that regards each combination with the same item differently. It can be demonstrated with the equation

$$\mathrm{C}(\mathrm{n}+\mathrm{r}-1,\mathrm{r})=C(n+r-1,n-1)$$

III. Analysis on The Resistance Game

A. The Setup of the Game

Using the information from reference [3].

The number of members of the Resistance and Spies that will be in the game changes depending on the number of players that are playing the game. The chart that will be used to determine the number of members of the Resistance and Spies that will be in the game are shown below:

Players	5	6	7	8	9	10
Resistance	3	4	4	5	6	6
Spies	2	2	3	3	3	4

Table III.A.I. The number of members of the Resistance and Spies that will be in the game. [3]

The number of players that must be on the Mission Team also changes depending on the number of players that are playing the game, and depending on the order of Missions that are currently played. The chart that will be used to determine the number of players that will be on the Mission Team is shown below

Players	5	6	7	8	9	10
1st Mission Team	2	2	2	3	3	3
2nd Mission Team	3	3	3	4	4	4
3rd Mission Team	2	4	3	4	4	4
4th Mission Team	3	3	4	5	5	5
5th Mission Team	3	4	4	5	5	5

Table III.A.II. The number of players that must be on the Mission Team [3]

Spies know the other members of their team, while the members of the Resistance do not.

B. The Optimal Mission Team

The Optimal Mission Team, for players that are members of the Resistance players, is a Mission Team which consists of less number of Spies than the required number of mission fail cards needed to be played for the mission to fail, because even if the Spies that are in the Mission Team plays a Mission Fail card, the Mission will still be a success. Therefore, it is impossible to fail a Mission if the Captain organizes the Mission Team in such a way.

Let's assume that there are 5 players, 2 of them are Spies and 3 of them are members of the Resistance. If they are playing Mission 1, in which 2 players have to form the Mission Team, then with combinatorics theory, we know that there will be C(5,2) numbers of missions that can be made.

For that mission to be successful, there must be no Mission Fail cards that are played. Therefore, there must be no Spies in the Mission Team for it to be optimal. To make it easier to calculate, we can differentiate the types of cases that we will find into 3 types, which are

1. There are no spies on the Mission Team, or in another way, all the players that are on the Mission Team are members of the Resistance.
2. There is 1 spy and 1 member of the Resistance on the team.
3. There are 2 spies on the Mission Team, or in another way, there are no members of the Resistance on the Team.

With the combinatorics theory, there are two ways that we can use to calculate the number of missions that will certainly be successful. One is by calculating the missions that have less number of Spies than the required number of mission fail cards needed to be played for the mission to fail, which is demonstrated below:

$$C(3,2) = 3$$

Another way to calculate is by subtracting the missions that have equal number or more Spies than the required number of mission fail cards needed to be played, which is demonstrated below:

$$C(5,2) - \big(C(2,1) * C(3,1) + C(3,2)\big) = 3$$

By dividing the number of Optimal Missions that we can organize with the total number of Mission Teams that we can organize, we can calculate the chance in which optimal missions will be produced.

$$\frac{C(3,2)}{C(5,2)} = \frac{C(5,2) - \big(C(2,1) \times C(3,1) + C(3,2)\big)}{C(5,2)} = \frac{3}{10}$$

Therefore, in this case, there is a $\frac{3}{10}$ chance that an optimal Mission Team for the members of the Resistance will be organized, if the members of the Mission Team are chosen randomly.

However, there is a better strategy. If you are a member of the Resistance, you can increase the chance of optimal Mission Team that will organized by putting yourself in the Mission Team. We can illustrate this using graphs.

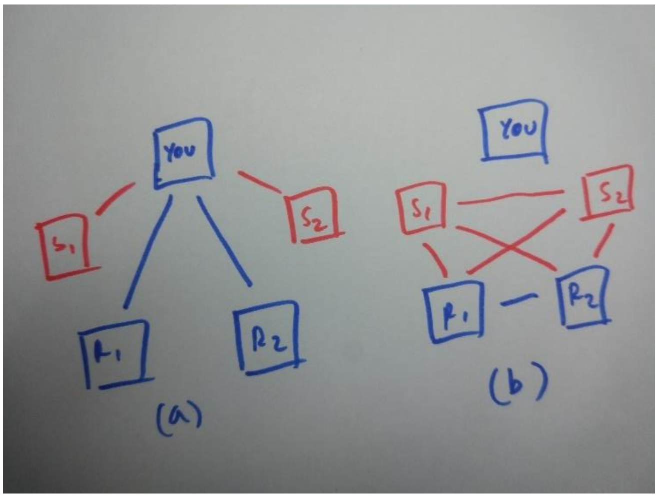

Figure 3.B.1. Illustration of the Mission Team. (a) includes you on the team, (b) doesn't include you on the team. The blue square denotes members of the Resistance, the red square denotes Spies. The blue line denotes the edge that only has members of the Resistance as its endpoints, and the red line denotes the edge that has spies on either of its endpoints.

(Author's Documents)

Using those graphs, we can see that there are 2 Spies and 2 members of the Resistance left that can be chosen, either if you are put on the team, or not. If you are put on the Mission Team, the captain only needs to choose 1 other player to be put in the Mission Team. However, if you are not put on the Mission Team, the captain needs to choose 2 other players to be put on the team. We can also see that, if we create a path of 2 players (Spies or members of the Resistance) on Figure 3.B.1. (a), there are 2 out of 4 possible paths that include less number of spies than the number of mission fail cards needed for the mission to fail, and that, if we create a path of 2 players (Spies or members of the Resistance), on Figure 3.B.2. (b), there is only 1 out of 6 possible paths that include less number of spies than the number of mission fail cards needed for the mission to fail. Therefore, in this case, there are more compositions of Optimal Mission Teams if you are on the team.

We can also demonstrate this by using Combinatorics, like before, we can calculate the chance that an optimal Mission Team for the members of the Resistance will be organized. If you are on the Mission Team, the calculation will be as follows:

$$\frac{C(2,1) \times C(2,0)}{C(4,1)} = \frac{1}{2}$$

or

$$1 - \frac{C(2,1) \times C(2,0)}{C(4,1)} = \frac{1}{2}$$

If you are not on the Mission Team, the calculation will be as follows:

$$\frac{C(2,2) \times C(2,0)}{C(4,2)} = \frac{1}{6}$$

or

$$1 - \frac{C(2,1) \times C(3,1)}{C(4,2)} - \frac{C(2,2) \times C(3,0)}{C(4,2)} = \frac{1}{6}$$

From those two calculations, in this case, we can see that it is better for you to be on the Mission Team to create the most Optimal Mission Team for the Resistance, rather than not being on the Mission Team, if you are a member of the Resistance, since it is more likely for the organized Mission Team to be an Optimal Team rather than not.

Let's look at another case. Assume that there are 5 players, 2 of them are Spies and 3 of them are members of the Resistance. But this time, they are playing Mission 2, in which 3 players must form the Mission Team. With combinatorics theory, we know that there will be C(5,3) numbers of missions that can be made.

For that mission to be successful, there must be no Mission Fail cards that are played. Therefore, there must be no Spies in the Mission Team for it to be optimal. To make it easier to calculate, we can differentiate the types of cases that we will find into 3 types, which are:

1. There are no spies on the Mission Team, or in another way, all the players that are on the Mission Team are members of the Resistance.
2. There are 2 members of the Resistance and 1 Spy on the Mission Team.
3. There are 1 member of the Resistance and 2 Spies on the

Mission Team.

With the combinatorics theory, just like in the previous case, there are two ways that we can use to calculate the number of missions that will certainly be successful, which is by calculating the missions that have less number of Spies than the required number of mission fail cards needed to be played for the mission to fail:

$$C(3,3) \times C(3,0) = 1$$

Another way to calculate is by subtracting the missions that have equal number or more Spies than the required number of mission fail cards needed to be played from the total number of Mission Teams that can be organized, which is demonstrated below:

$$C(5,3) - (C(2,1) \times C(3,2) + C(2,2) \times C(3,1)) = 1$$

By dividing the number of Optimal Missions that we can organize with the total number of Mission Teams that we can organize, we can calculate the chance in which optimal missions will be produced.

$$\frac{C(3,3) \times C(3,0)}{C(5,3)} = \frac{1}{10}$$

or

$$\frac{C(5,3) - (C(2,1) \times C(3,2) + C(2,2) \times C(3,1))}{C(5,3)} = \frac{1}{10}$$

Therefore, in this case, there is a $\frac{1}{10}$ chance that an optimal Mission Team for the members of the Resistance will be organized, if the members of the Mission Team are chosen randomly.

We can also find a better strategy than randomly choosing the members of the Mission Team. If you are a member of the Resistance, just like in the previous case, you can increase the chance of optimal Mission Team that will organized by putting yourself in the Mission Team. We can illustrate this using graphs.

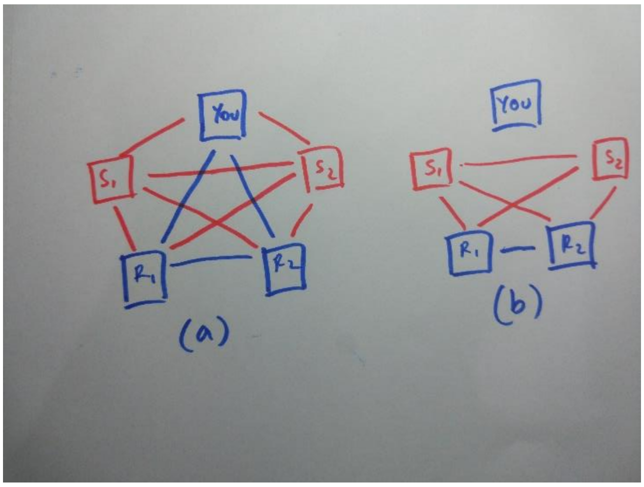

Figure 3.B.2. Illustration of the Mission Team. (a) includes you on the team, (b) doesn't include you on the team. The blue square denotes members of the Resistance, the red square denotes Spies. The blue line denotes the edge that only has members of the Resistance as its endpoints, and the red line denotes the edge that has spies on either of its endpoints. (Author's Documents)

Using those graphs, we can see that there are 2 Spies and 2 members of the Resistance left that can be chosen, either if you are put on the team, or not. If you are put on the Mission Team, the captain only needs to choose 2 other players to be put in the Mission Team. However, if you are not put on the Mission Team, the captain needs to choose 3 other players to be put on the team. We can also see that, if we create a path of 3 players (Spies or members of the Resistance) on Figure 3.B.2. (a), there are 1 out of 6 possible paths that include less number of spies than the number of mission fail cards needed for the mission to fail, and that, if we create a path of 3 players (Spies or members of the Resistance), on Figure 3.B.2. (b), there are no possible paths that include less number of spies than the number of mission fail cards needed for the mission to fail. Therefore, in this case, there are also more compositions of Optimal Mission Teams if you are on the team.

Using those graphs, we can see that there are only 2 Spies and 2 members of the Resistance left that can be chosen, either if you are put on the team, or not. If you are put on the Mission Team, the captain only needs to choose 2 other players to be put in the Mission Team. However, if you are not put on the Mission Team, the captain needs to choose 3 other players to be put on the team.

In this case, by using Combinatorics, like before, we can calculate the chance that an optimal Mission Team for the members of the Resistance will be organized. If you are on the Mission Team, the calculation will be as follows:

$$\frac{C(2,2) \times C(2,0)}{C(4,2)} = \frac{1}{6}$$

or

$$1 - \frac{C(2,1) \times C(2,1)}{C(4,1)} - \frac{C(2,2) \times C(2,0)}{C(4,2)} = \frac{1}{6}$$

If you are not on the Mission Team, the calculation is a bit interesting, since there are less number of agents as the number of players needed to be put on the Mission Team. In this case, you cannot calculate the missions that have less number of Spies than the required number of mission fail cards needed to be played for the mission to fail. The method will be demonstrated below:

$$\frac{C(2,3) \times C(2,0)}{C(4,2)} = Invalid$$

The calculation is invalid because C(2,3) is invalid. C(2,3) is invalid because in this case, $n < r$ for C(n, r). It fails to fulfill the requirement of combinations which is $n \geq r$ for C(n, r).

On the other hand, the other method, which is subtracting the missions that have equal number or more Spies than the required number of mission fail cards needed to be played from the total number of Mission Teams that can be organized, is valid. The method is demonstrated below:

$$1 - \frac{C(2,1) \times C(2,1)}{C(4,2)} - \frac{C(2,2) \times C(3,0)}{C(4,2)} = 0$$

Therefore, only 1 method can be used to calculate the chance of organizing an Optimal Mission Team in every case of the Resistance game, which is subtracting the missions that have equal number or more Spies than the required number of mission fail cards needed to be played from the total number of Mission Teams that can be organized.

From those calculations, in this case, we can also see that it is better for you to be on the Mission Team to create the most Optimal Mission Team for the Resistance, rather than not being on the Mission Team, if you are a member of the Resistance, since it is more likely for the organized Mission Team to be an Optimal Team rather than not.

By looking at the above calculations, we can generally assume that if there are w number of spies, x number of agents, y number of players in the Mission Team, and it takes z mission fail cards to be played for the mission to fail, the calculation will be as follows.

$$1-\frac{C(w,z)\times C(x,y-z)}{C(w+x,y)}-\frac{C(w,z+1)\times C\big(x,y-(z+1)\big)}{C(w+x,y)}-\cdots-\frac{C(w,y)\times C(x,y-y)}{C(w+x,y)}$$

$$=1-(\frac{C(w,z)\times C(x,y-z)}{C(w+x,y)}+\frac{C(w,z+1)\times C\big(x,y-(z+1)\big)}{C(w+x,y)}+\cdots+\frac{C(w,y)\times C(x,y-y)}{C(w+x,y)})$$

$$=1-(\sum_{v=z}^{y}\frac{C(w,v)\times C(x,y-v)}{C(w+x,y)})$$

Also, we can generally assume that it is better for you to be on the Mission Team to create the most Optimal Mission Team for the Resistance, rather than not being on the Mission Team, if you are a member of the Resistance, since it is more likely for the organized Mission Team to be an Optimal Team rather than not.

The Optimal Mission team for the Spies are having equal number or more Spies than the required number of mission fail cards needed to be played from the total number of Mission Teams that can be organized.

The complement of the equation for the Optimal Mission Team for members of the Resistance, produces the chance that an Optimal Mission Team for the opposition of the Resistance, which are the Spies, will be organized. The calculation is shown below:

$$\frac{C(w,z)\times C(x,y-z)}{C(w+x,y)}+\frac{C(w,z+1)\times C\big(x,y-(z+1)\big)}{C(w+x,y)}+\cdots+\frac{C(w,y)\times C(x,y-y)}{C(w+x,y)}$$

$$=\sum_{v=z}^{y}\frac{C(w,v)\times C(x,y-v)}{C(w+x,y)}$$

Since the Spies know the identities of the other Spies, Spies can approve the Mission Team if the team is optimal, with any Spies are on the Mission Team.

C. Combinatorics on Nonoptimal Mission Teams

In the case of the organized Mission Team is nonoptimal for players that are members of the Resistance players, which means that the Mission Team consist of more number of Spies than the required number of mission fail cards needed to be played for the mission to fail, there is still a chance that the mission will still be successful.

Using combinatorics, we can calculate the chance for the mission to be successful, and for the mission to fail.

If there are at least 1 mission fail cards that are needed to be played for the members of the Resistance to lose, and there are 1 spy in the mission, we can calculate the chance for the missions that succeed, shown below:

$$\frac{C(1,1)}{\big(C(2,1)\big)^1}=\frac{1}{2}$$

Or calculate the number for the missions that fail, then subtract 1 with that number, shown below:

$$1-\frac{C(1,1)}{C(2,1)}=\frac{1}{2}$$

Since the spy have two choices, which is to play a mission success card or play a mission fail card, and for the mission to be successful, the Spy must play the mission success card, which is one out of two cards, therefore the chance of the mission being successful is $\frac{1}{2}$.

Let's look at another case. If there are at least 2 Mission Fail cards that are needed to be played for the members of the Resistance to lose, and there are 3 spies in the mission, we can calculate the chance for the missions that are able to succeed, there are 2 combinations in which we need to have for the mission to be successful, which are

1. All spies play the mission success card.
2. Only 1 spy plays the mission fail card.

To calculate the chance, we have to add the combinations for these cases and divide them by the total number of possible combinations, which is shown below

$$\frac{C(3,0)+\ C(3,1)}{\big(C(2,1)\big)^3}=\frac{4}{8}=\frac{1}{2}$$

We can also calculate this by adding the combinations for the cases in which the mission ends up in a failure and subtract 1 with that sum.

$$1-\left(\frac{C(3,2)+\ C(3,3)}{\big(C(2,1)\big)^3}\right)=1-\frac{4}{8}=\frac{4}{8}=\frac{1}{2}$$

By looking at the above calculations, we can generally assume that if there are w number of spies in the mission, and it takes z number of mission fail cards to be played for the mission to fail, the calculation for the chance that the mission will be successful is shown below:

$$\frac{C(w,0)+\ C(w,1)+\cdots+\ C(w,z-1)}{\big(C(2,1)\big)^w}$$

$$=\sum_{v=0}^{z-1}\frac{C(w,v)}{\big(C(2,1)\big)^w}$$

or

$$1-\frac{C(w,z)+C(w,z+1)+\cdots+C(3,w)}{\big(C(2,1)\big)^w}$$

$$=1-\sum_{v=z}^{w}\frac{C(w,v)}{\big(C(2,1)\big)^w}$$

The complement of this is the chance that the mission will fail. The calculation is shown below:

$$1-\frac{C(w,0)+\ C(w,1)+\cdots+\ C(w,z-1)}{\big(C(2,1)\big)^w}$$

$$=1-\sum_{v=0}^{z-1}\frac{C(w,v)}{\big(C(2,1)\big)^w}$$

or

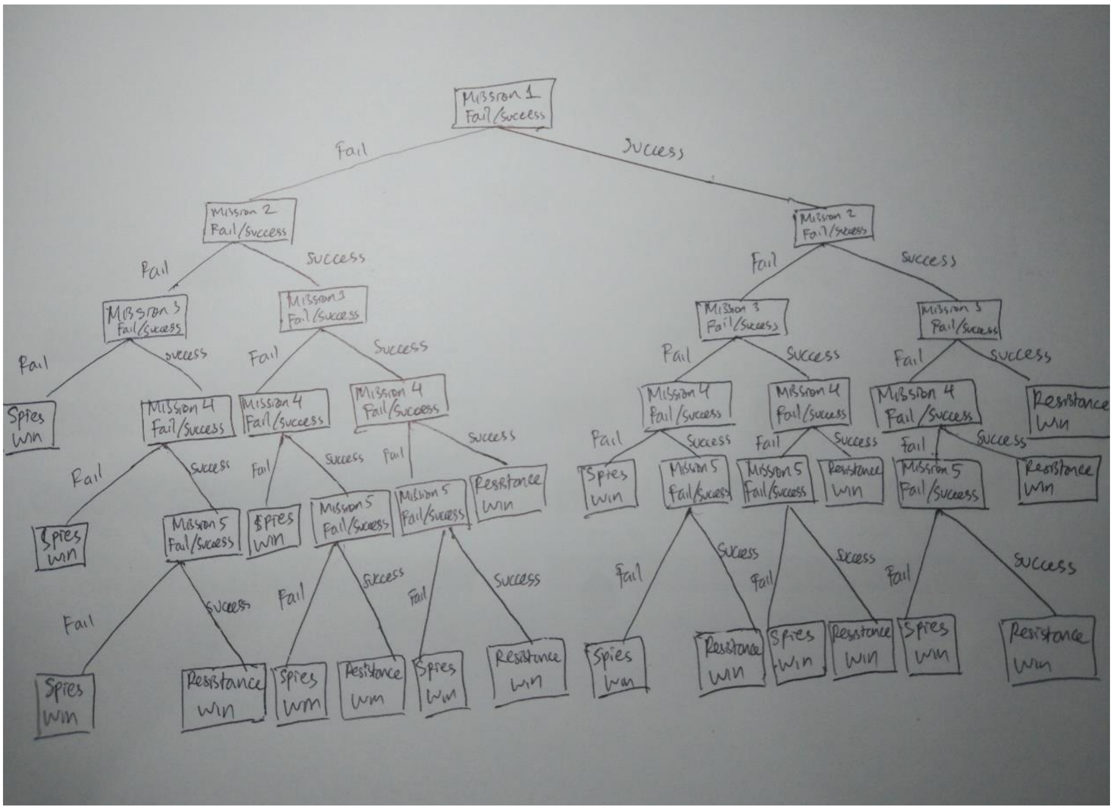

Figure 3.D.1. Decision Tree for the Result of the Game (Author’s Documents)

$$\frac{C(w,z) + C(w,z+1) + \cdots + C(3,w)}{\left(C(2,1)\right)^w} = \sum_{v=z}^{w} \frac{C(w,v)}{\left(C(2,1)\right)^w}$$

D. Decision Tree for the Result of the Game

There are five missions in the game. The “Resistance” wins if three missions are completed successfully, and the “Spies” win if three missions fail or the “Resistance” is unable to organize an approved “Mission Team”. In this case, I am going to elaborate on the “Resistance” winning if three missions are completed successfully, and the “Spies” winning three missions fail.

By using the decision tree, we can decide which of the faction that won the game. The decision tree that can be illustrated is shown on Figure 3.D.1.

IV. Conclusion

In conclusion, we can use Graph, Decision Tree and Combinatorics to understand the Resistance game. Graph and Combinatorics can be used to find optimal Mission Teams, calculate chances of winning for optimal and nonoptimal Mission Teams, and Decision Tree can be used to determine the result of the game.

V. Acknowledgment

The author is grateful to The One Almighty God for the blessing that has been given so that this paper can be finished successfully. The author is also grateful to Mrs. Harlili, M.Sc, Mr. Dr. Ir. Rinaldi Munir, MT. and Mr. Dr. Judhi Santoso. our lecturers in Discrete Mathematics Course, for the knowledge and the time which are shared with the college students attending the course. The author is also grateful for the support from the author’s family and friends.

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Pernyataan

Dengan ini saya menyatakan bahwa makalah yang saya tulis ini adalah tulisan saya sendiri, bukan saduran, atau terjemahan dari makalah orang lain, dan bukan plagiasi.

Bandung, 10 Desember 2018

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