The Recurrence of Golden Ratio in the Universe

Jeremy Arden Hartono 13517101
Program Studi Teknik Informatika
Sekolah Teknik Elektro dan Informatika
Institut Teknologi Bandung, Jl. Ganesha 10 Bandung 40132, Indonesia
13517101@std.stei.itb.ac.id

***Abstract*—Golden Mean, or widely known as the Golden Ratio, is an irrational number represented by the Greek letter *Phi (φ)* in lowercase, which has the value of approximately 1.6180339887. The recurrence of Golden Ratio has appeared throughout everything we ever know and will ever know. Nature, man-made objects, even outer space contains this set of unique number. Because of its visually appealing and aesthetically beautiful properties, Golden Ratio is used by artists, photographers and designers to create stunning imagery for others to gaze their thoughts in. In this vast universe of ours, Golden Ratio exists in every inch of its corner.**

***Keywords*—golden ratio, golden mean, recurrence, phi, fibonacci.**

I. Introduction

Unknown exactly when it was first discovered, it is assumed that it was discovered and rediscovered for several times, which explains why it has so many names. The Greeks were one of the first to have studied and applied the Golden Ratio in their lives. Phidias (500 BC − 432 BC), a Greek sculptor and mathematician, studied *phi* and applied it to the design of the sculptures of Parthenon. It appears that the Great Pyramid of Giza was designed using Golden Ratio too, pushing the discovery date even further back to the Egyptian era.

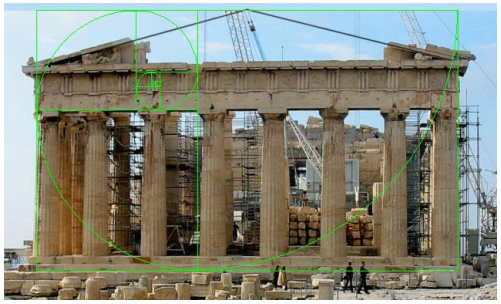


***Fig. 1.1** The construction of Parthenon using Golden Ratio (Source: https://www.goldennumber.net/parthenon-phi-golden-ratio/)*

Throughout medieval and modern history, the usage of Golden Ratio has created some number of masterpieces. For instances, the greatest masterpiece of Leonardo da Vinci, *Mona Lisa*, was created using divine proportion, or *divina proportione* as da Vinci called it, another name for Golden Ratio. Perhaps the most famous application of Golden Ratio is the conception of Fibonacci sequence by Leonardo Fibonacci, an Italian born in 1175. Fibonacci sequence is a sequence where every number after the first two is a sum of the two preceding ones. The sequence may start either with 0 and 1 or 1 and 1, depending on the chosen starting point.

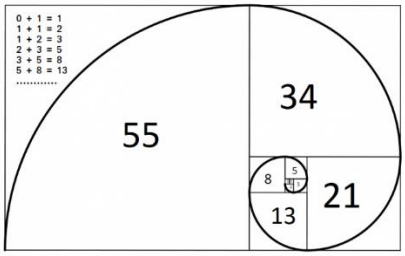


***Fig. 1.2** Fibonacci sequence in Golden Ratio. (Source: https://www.geeksforgeeks.org/program-to-print-first-n-fibonacci-numbers/)*

Not only on Earth, our very own solar system does have a Golden Ratio occurrence in it. Planets' orbital periods in the Solar System are found to be fit within the Fibonacci sequence, and therefore also fit within the Golden Ratio. The human body does have a fair share of Golden Ratio within itself. The ratio of arm to forearm is approximately 1.618, and each bone in the index finger is approximately 1.618 than the bone preceding it. The human face is also based on the Golden Ratio, and popular beliefs state that the attractiveness of someone is based on the ratio itself. The relation between the Golden Ratio and nature can be found easily on the pattern of a sunflower seed, where it forms a golden spiral, a spiral based on the Golden Ratio, on the center of the flower.

In modern day, the application Golden Ratio can be found everywhere. For example, a standard-sized credit card is 54mm x 86mm, creating a ratio of 0.628, which is less than a millimeter off from a perfect Golden Ratio of 0.618, the reciprocal of 1.618. The Apple iPod Classic, designed by Jonathan Ive, was designed with perfection in mind. The dimension of the iPod Classic itself is 1:1.67, not even a millimeter off to the Golden Ratio. Designers, architects and engineers often use the ratio to create and manufacture objects that is also beautiful and fabulous, not only functional.

II. Golden Ratio, Fibonacci Sequence, and Recurrence

This section will explain about the basics of the Golden Ratio, Fibonacci sequence and recurrence relation, and how to use them.

A. Golden Ratio

Mathematically, the definition of Golden Ratio is if there is a line divided into two parts, and the ratio of the longer and shorter parts is the same as the ratio of the whole line and the longer part.

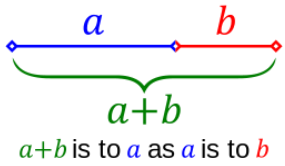


Fig. 2.1 *Line segments in the Golden Ratio*
(Source: https://en.wikipedia.org/wiki/Golden_ratio)

Using algebra, we can define the Golden Ratio as:

$$\frac{a+b}{a} = \frac{a}{b} = \varphi = \frac{1+\sqrt{5}}{2} = 1.6180339887\ldots$$

In the making of pentagrams, Golden Ratio plays a very important role. To create the perfect pentagram, the ratio of each segment must be equal to *phi*, or else you will have an irregular pentagram.

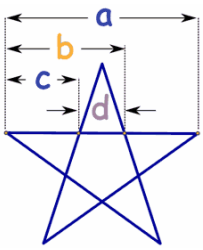


Fig 2.2 *The perfect pentagram*
(Source: https://www.mathsisfun.com/geometry/pentagram.html)

As you can see on Figure 2.2, the scale of a and b, b and c, and c and d form a perfect Golden Ratio of 1.618. On 3-dimensional shapes, dodecahedron is a perfect example of Golden Ratio. A dodecahedron is a polyhedron with 12 flat faces. A convex regular dodecahedron has pentagons as its faces, making it has Golden Ratio all over the faces. Pentagon also serves as the base of the drawing of pentagram.

A Golden Rectangle is a rectangle which sides has the ratio of 1:1.618. Such example of Golden Rectangle has already been mentioned before, a standard-sized credit card. The construction of the Golden Spiral requires Golden Rectangle as its base.

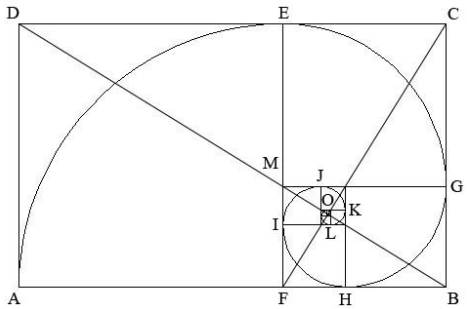


Fig 2.3 *Golden Rectangle ABCD and Golden Spiral AEGHIJKLO*
(Source: Geometrical Substantiation of Phi, the Golden Ratio and the Baroque of Nature, Architecture, Design and Engineering, by Md. Akhtaruzzaman, Amir A. Shafie)

B. Fibonacci Sequence

Fibonacci sequence, introduced by Leonardo Fibonacci (1175 – 1250), is a sequence of numbers starting from 0 and 1, such that each number is the sum of the two preceding ones. Fibonacci numbers are strongly related to the Golden Ratio, this is because that after the fourth number, 2, the ratio of one number and the next approximates *phi*, 1.618. After the 40th number in the sequence, the ratio is accurate to 15 decimal places.

The sequence created from Fibonacci numbers also creates what we call as recurrence. Recurrence, by general definition, is an act to return to the previous condition. In recurrence function, we need to have basis defined. The basis for Fibonacci sequence is 0 and 1.

$$F_0 = 0, F_1 = 1$$

Fibonacci numbers can be defined as:

$$F_n = F_{n-1} + F_{n-2}$$

The above equation proves that Fibonacci sequence indeed creates a recurrence. Therefore, the result of the Fibonacci sequence will be 0,1,1,2,3,5,8,13,… and so on. There is also an alternative where the basis is 1 and 1, but the rest of the sequence will still be the same. Fibonacci sequence can be expressed as:

$$F_n = \begin{cases} 0, & n = 0 \\ 1, & n = 1 \\ F_{n-1} + F_{n-2}, & n \geq 2 \end{cases}$$

When using 1 and 1 as basis, do remember to change n in previous equation to 1, 2 and 3.

C. Recurrence Relation

A recurrence relation is an equation which is defined in terms of itself. In mathematics, one of the proper examples of recurrence relation is the Fibonacci sequence which was explained in the previous part. To create recurrence relation, you need to have basis or initial conditions. Without it, you cannot create a function. Commonly used basis is 0 or 1, but everything depends on what kind of relation or function you want to create.

Another example of a recurrence relation is factorial equation. Factorial equation, denoted by $n!$, is an equation that multiplies itself and subtracting it by 1, until the value of n becomes 1. The basis used in factorial equation is $n = 0$, resulting in 0! = 1.

$$n! = n.(n-1)!$$

From the above equation, we can see that the equation defines itself by calling another factorial function. Keep in mind that in $n = 0$, it doesn't subtract the 0 and creates a factorial function

of -1.

III. The Recurrence of Golden Ratio in Nature

As the previous section has explained about the basics of the Golden Ratio, Fibonacci sequence and recurrence relation, this section will further explain about the recurrence of Golden Ratio and its appearance in real life.

A. Recurrence in Nature

In the wild nature, some things may look random and just happened spontaneously. But, if we take a deeper look at it, we may find some intriguing details. The Golden Ratio always find its way to manifests into many places, Mother Nature is no exception, from the molecule of DNA to structures and bodies in outer space.

When we look at flowers, the most noticeable thing about it is its petals. A lot of flowers have petals that are the same as Fibonacci numbers. Flowers like lilies or iris have 3 petals, which 3 is a Fibonacci number. A wild rose has 5 petals, which is also a Fibonacci number. Not only the petals, as the first section has told us, a sunflower construct its seeds in the center according to Fibonacci sequence, which is also Golden Ratio.

Fig 3.1 *Golden spiral on the seeds of sunflower*
(Source: https://design.zemniimages.info/golder-ratio/)

Moving on to the animal kingdom, perhaps the most blatant Golden shape is found on a nautilus, where its shell is shaped the same as a Golden Spiral. Although it isn't on a perfect Golden Ratio, it is still amazing how nature can create something perfectly in order like that. The similar shape can also be found on a garden snail and some mollusks, where their shells are also in the shape of a Golden Spiral. Not only sea creatures, insects and birds too have the Golden Ratio on their body. For example, ants and butterflies. Ants' body proportion scales correctly to the Golden Ratio, and a butterfly's wings scales correctly too to the Golden Ratio. Even the molecules of our DNA consist of the Golden Ratio. A molecule of DNA, measures 34 Å long and 21 Å wide for each full cycle of its double helix spiral, which both numbers are Fibonacci numbers.

Fig 3.2 *The shell of a nautilus*
(Source: http://www.barrierisman.com/blog/2016/12/8/the-nautilus-shell-as-a-symbol-for-the-evolution-of-yoga-practice-part-1)

B. Recurrence in Outer Space

Going outside of this small Earth, we can find Golden Ratio relatively easy on our Solar System. Previously mentioned in the first section, the orbital periods of the planets of Solar System aligns correctly to Fibonacci sequence, and not only that. The relative measurements and distance between each planets average at 1.61874, if the measurements start from Mercury and the asteroid belt counted as a part of the Solar System. That number hits too close to the value of *phi*, 1.61803. The spiral formed in the center of the galaxy also follows the rule of the Golden Ratio, where it forms a lot of Golden Spiral.

Fig 3.3 *The galaxy compared to the Golden Ratio*
(Source: https://www.cnet.com/pictures/natures-patterns-golden-spirals-and-branching-fractals/)

C. Recurrence in Human Body

Going back from the vast universe of ours to this mortal human body, our human body has a lot of things going on. Not only the arm to forearm ratio mentioned in the previous section, the ear does resemble the shape of a Golden Spiral. The eye position also has the Golden Ratio which measured from the length of one eye compared to the distance from one eye to the corner of the other. *L'Uomo Vitruviano,* or known popularly as *The Vitruvian Man*, is a drawing made by Leonardo da Vinci. The drawing depicts a man with arms and legs extended apart inscribed in a circle and a square, with guidelines that perfectly aligns with the Golden Ratio. The Golden Ratio is used repeatedly to draw the scale from hairline to navel, the collarbone, the pectoral nipples, and even the base of the hand. The drawing is based on the ideal geometry of human body proportions described by Vitruvius, an ancient Roman architect.

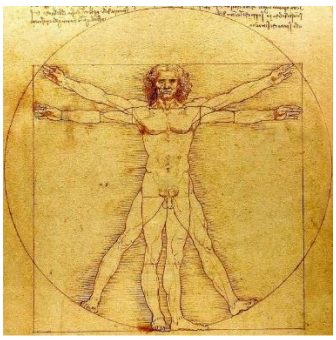


Fig 3.4 *The Vitruvian Man*
(Source: https://www.fineartone.com/shop/collections/most-famous-masterpieces/vitruvian-man-2)

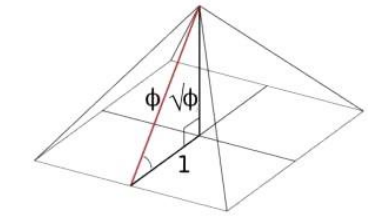
IV. The Recurrence and Application of Golden Ratio and Fibonacci Sequence in Design and Engineering

In daily life, we can see the example of Golden Ratio applied in our everyday-life tools. The easiest example would be a soccer ball, where its faces are the shape of a pentagon. The iPod Classic, previously mentioned on the second section, also follows the Golden Ratio rule in its design, creating an almost perfect ratio of 1:1.67, close to 1:1.618. The usage of the Golden Ratio is not carelessly and obviously not just because the designer wants to. Most designers believe that the Golden Ratio really represents beauty and perfection, where everything fits just perfectly in it. People may not believe in it instantly, but Apple has some proof to back it up. By the time the iPod Classic went out in 2006, their sales doubled from before. Unconsciously, we sometimes prefer things that are in this ratio rather than the other. Renaissance-era artists and architects used *phi* in their design to create something that people unconsciously admire and marvel at. Fibonacci sequence also appears in many aspects of arts. One of the obvious examples is music notes. A music notes consists of 8 notes, and the word *octave* comes from the Latin word *octo*, which means eight.

The Great Pyramid of Giza's dimensions add up to become the equation to find the value of *phi*:

$$1 + \varphi = \varphi^2$$

It is unknown until today whether the construction of Great Pyramid really used the Golden Ratio or is it just a coincidence. Turns out that it's not only the dimensions that resemble the Golden Ratio, but The Great Sphinx too has a ratio of 1:1.618 by comparing head-to-torso to the whole body.



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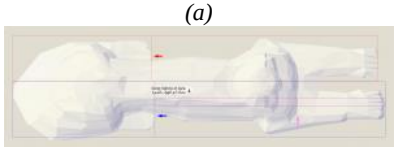

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Fig 4.1 *(a) The Great Pyramid's dimensions, (b) The Great Sphinx (Source: https://www.goldennumber.net/great-pyramid-giza-complex-golden-ratio/)*

The sculptures of Parthenon in Athens, Greek, was designed by Phidias, whose first letter of his name became a symbol of the Golden Ratio, *phi*, and used the Golden Ratio in his designs. The architects of Parthenon themselves was deduced to had used the Golden Ratio as the foundation design of the Parthenon, because you can find the Golden Rio everywhere, on every corner of the temple. The width and length of the Parthenon was based on a Golden Rectangle, the pillars and façade too. It is assumed that by this era, the Greeks had been aware of the Golden Ratio and had been using it in their designs. The statues of Greek Gods too have been using the ratio in its design. The face of the statue of Athena has several Golden Ratio found on it.

In the painting of Leonardo da Vinci, *The Last Supper*, da Vinci used the Golden Section to implement his painting. The placement of the table, the disciples' position at the table, the position of the windows behind Jesus Christ, everything aligns perfectly with the Golden Section. Salvador Dali followed the Golden Section that da Vinci left behind and created another painting called *The Sacrament of the Last Supper*.

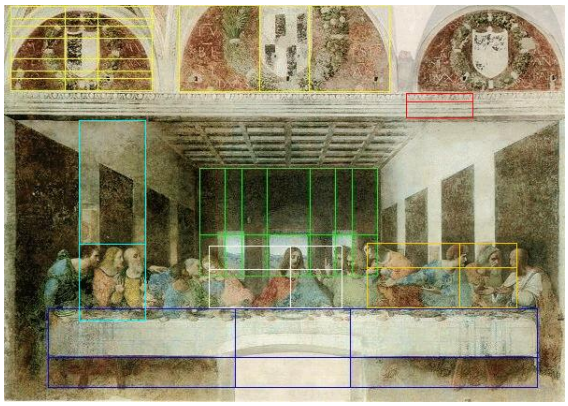


Fig 4.2 *The Last Supper by Leonardo da Vinci (Source: https://www.goldennumber.net/art-composition-design/)*

Another work from the Renaissance era, *The Creation of Adam* by Michelangelo, also is an example of the Golden Ratio in the creation of art. The finger of God touches the finger of Adam at the exact point of the Golden Ratio. This is indeed not a result of coincidence, because it is done so perfectly that whether you look at it from vertical or horizontal point of view, they both still touches the point perfectly. There are a few more of Golden Ratio found in the painting, but the most popular one is the touch.

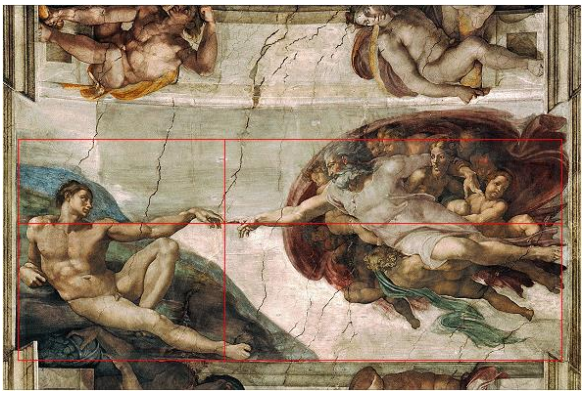


Fig 4.3 *The Creation of Adam by Michelangelo (Source: : https://www.goldennumber.net/art-composition-design/)*

Various automobile company has implemented the Golden Ratio in their design. In the design of VW Beetle, a Golden Ratio can be found on its body. The body of the Beetle matches with

the Golden Ellipse, and the windows also matches with the Golden Ellipse, creating a recurrence. On the Toyota Supra, the measurements fit with the Golden Ratio, even from its tire to its whole body.

The Great Mosque of Kaioruan, built by Uqba ibn Nafi around 670 AD, consists of the Golden Ratio all over the building, from its prayer room, court, and minaret. The Borobudur temple located in Indonesia, has the base with a perfect Golden Ratio of 1.618:1. Another example of modern application of the Golden Ratio can be found in company logos. One of the biggest tech companies in the world, *Apple*, implements the use of Golden Ratio in its logo of *iCloud*, a cloud-based storage.

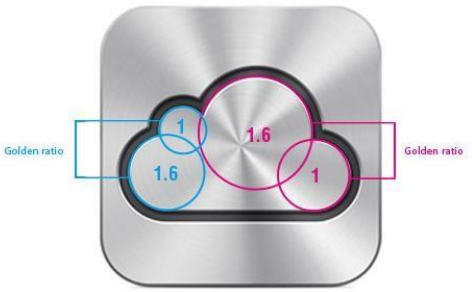
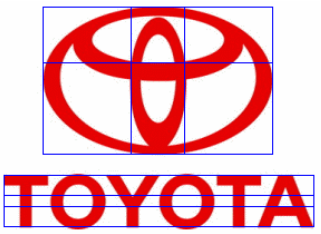


Fig 4.4 *iCloud logo*
(Source: https://www.pinterest.com/pin/197314027401043486/)

Another example is from the logo of *Toyota* and *Pepsi*. In the logo of *Toyota*, the ratio of the height is based on the Golden Ratio, and not only the height but the width too. There are multiple appearances of Golden Ratio in their logo. *Pepsi*'s logo was created using three circles and a few lines, that perfectly scales to the Golden Ratio too. One of the world's leading social media, *Twitter*, also uses the Golden Ratio in its logo, creating a logo of a bird from a few circles with the scale of the Golden Ratio.



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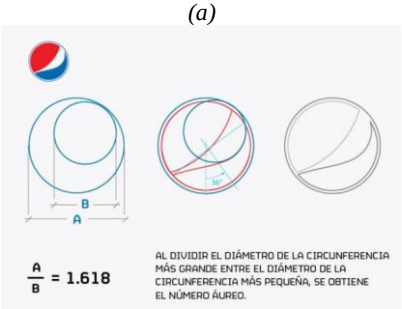

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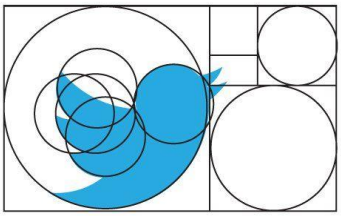


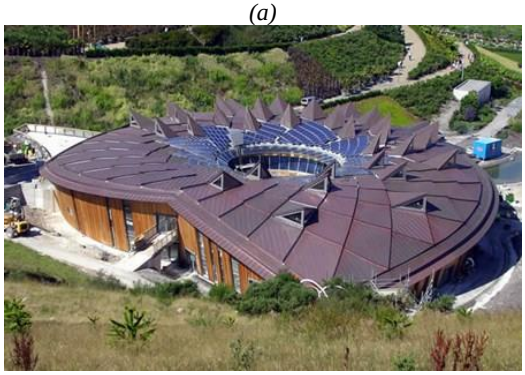
(c)

Fig 4.5 *The logo of Toyota, Pepsi and Twitter*
(Source: https://www.goldennumber.net/car-auto-golden-ratio-logo-design/ , https://www.pinterest.com/pin/279293614374879482/ , https://www.canva.com/learn/what-is-the-golden-ratio/)

Ludwig Mies van der Rohe was the director of the School of Architecture at the Illinois Institute of Technology. He designed and created a variety of buildings and even the entire campus. Perhaps the most famous work of his is the Chapel. The Chapel is an example of the Golden Ratio applied in modern infrastructure. The Chapel is in scale with the Golden Ratio of 1:1.618, and divided into 5 columns by the Golden Rectangles, which is a recurrence of itself.

Eden Project is a popular tourist attraction in Cornwall, United Kingdom, and they have an education center called The Core, which is designed with Fibonacci numbers and has spiraling plants and vines. Perhaps the most intriguing part of this center is the roof, where it displays the pattern of a Golden Spiral, even if you can only see it when you are airborne.

(a)



(b)

Fig 4.6 *(a) The Chapel, (b) The Core*
(Source: https://en.wikiarquitectura.com/building/robert-f-carr-memorial-chapel-of-st-savior/ , https://www.sensorytrust.org.uk/projects/eden/eden-core-inclusive-design.html)

V. CONCLUSION

Nobody knows if the recurrence of the Golden Ratio throughout everything we know of is a coincidence or not. However, the discovery of the Golden Ratio and everything related to it has helped us a lot in advancing towards a better future. Golden Ratio is another way to implement science and art together to create something marvelous and timeless that mankind can call as masterpiece.

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PERNYATAAN

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Bandung, 10 Desember 2018

Nama dan NIM
Jeremy Arden Hartono 13517101