Graph Theory Application

Bali's Train Transportation System Design

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This paper will examine how to build trans-island train route system for Bali's transportation. Somet possible solution scheme will be analyzed to meet the problem requiremens. The task is to make train routes (solution) that cover all the requirements. This problem is a sort of Graph application problem. Bali's map will be simplified and represented as weighted graph. Then, we will examine those representation graph to make the solution.

***Index Terms*—Bali map, Circuit, Graph Theory, Path, Train Routes.**

I. INTRODUCTION

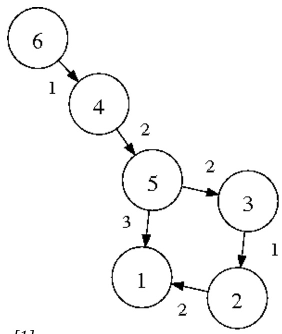
Bali, a well known island for its beauty is visited many tourists from various country everyday. Only Bali's south area beauty that has been explored so far, because Bali's airport is located at its south. Other areas beauty remain unexplored. If Bali has its own train transportation system across the island, it will be beneficial for its tourism and citizen.

One of the descrete mathematic topics is Graph theory. Graph theory is frequently used to solve route system. This paper approach Bali's map as an weighted graph to design efective train transportation system accross the island.

II. RELATED THEORM

GRAPH

A graph is used to represent discrete objects and their relation to each other, commonly used for representing a map, analyzing network, finding shortest path , molecules in chemistry, biology and etc. [5]A *graph G = (V ,E)* consists of *V* , a nonempty set of *vertices* (or *nodes*) and *E,* a set of *edges*. Each edge has either one or two vertices associated with it, called its *endpoints*. An edge is said to *connect* its endpoints.



[1]Fig 1. Graph Example

Based on presence of ring or double edge in a graph, a graph can be classified into :

1. *Simple Graph*
   There is no ring or double edge in this kind of graph.
2. *Unsimple Graph*
   There is at least a ring or double edge in this kind of graph.

Based on its vertices, a graph can be classified into:

1. *Limited Graph*
   Its quantity of vertices is limited.
2. *Unlimited Graph*
   Its quantity of vertices is unlimited.

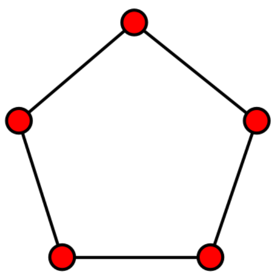
Based on edge orientation, a graph can be classified into :

1. *Undirected Graph*
   A graph that has no orientation on its edge. A pair of vertices (u,v) denoted same edge as (v,u).
2. *Directed Graph*
   A graph that has orientation on its edge. A pair of vertices (u,v) denoted different edge to (v,u).

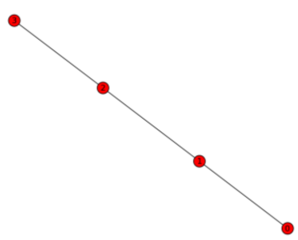
There are some other important graph terminology :

1. *Adjacent*
   2 vertices in undirected graph G is said to be adjacent if they are interconneced with an edge.
2. *Incident*
   For a random edge e = (v1, v2). It is said that the edge e is incident to vertice v1 and vertice v2.

3. *Isolated Vertex*
   Isolated vertex is a vertex that has no edge incident to it.
4. *Subgraph*
   Supposed a graph G = (V,E). A graph G^1 = (V^1,E^1) is its subgraph if V^1 is subset of V and E^1 is subset of E.
5. *Weighted Graph.*
   *I*s a graph that has value on its each edge.
6. *Path and Circuit*
   A path is a sequence of edged that connected sequence of vertices. If a path started and ended in a same vertices, its called as circuit.
7. *Connected*
   2 vertices (X,Y) is said connected if there is at least a path from X to Y.
8. *Euler Circuit*
   [6]An euler circuit of a graph G is a simple circuit that contains every edge of G and have to end up at the begining.
9. *Eurler Path*
   [6]An euler path of a graph G is a simple path that contains every edge of G but don't have to end up at the begining.
10. *Hamilton Circuit*
    [6]An hamilton circuit of a graph G is a simple circuit that visit every vertices of G and have to end up at the begining.
11. *Hamilton Path*
    [6]An hamilton path of a graph G is a simple path that contains visit every vertices of G but don't have to end up at the begining.



[8] *Fig. Circuit*



[9]*Fig. Path*

TRANSPORTATION IN TOURISM

[7] Transportation is the key element in the tourism industry. The demand in international and even national transport infrastructures implies a very large number of people who wants to move in an efficient, fast and inexpensive manner. Well organized terminals and intelligently planned schedules are essential in promoting effective transportation facilities for tourists, notably since the industry is growing at a fast rate.

Improved facility will stimulate the tourism, including transportation system. Rail transport was a dominant mean to move a mass number of people.

III. PROBLEM DEFINITION

Supposed that Bali's goverment want to build Bali's trans-island train system, but there are many requirements:

- The goverment has limited budget for purchasing the train (supposed they can only purchase up to 8 trains which speed are 200KMph for entire island).
- Because of the first problem, the goverment decide the 2 places that separated no futher than 20 KM (time travel < 6 minutes) don't nessecarily connected by trans-island train, but if 2 places separated futher than 20 Km then they are nessecarily connected by trans-island train.
- The government don't want 2 routes intersecting (they have no common edge) for effectiveness. A route is maxsimumly operated by 2 trains (vice versa start)
- The citizens want the train frequence in coming to a particular station is no more than 120 minutes each coming, each coming doesn't necessarily the same train as the last train came. The last drop off and loading time is not counted for a circuit or vice versa traversal path, but the first loading time in the start vertice counted (example, from seririt to singaraja then back to seririt again needs 10 + 6.78 + 10 + 6.78 minutes). The first and the last loading time will be counted for linier path (example, Seririt – Singaraja – Tejakula weighted 10 + 6.78 + 10 + 3.6 + 10 in minutes)
- Supposed the train stopped 10 minutes for each station to dropoff and load passengers.

This paper will examine the solution based on all asumptions that have been stated.

IV. PROBLEM SOLVING

[2]*Fig 2 Bali Island Tourism Map*

We will examine the solution options based on the help of theorm about graph that we have learned (the key theorms is in the **RELATED THEORM**).

First, we must represent the map as an weighted undirected graph (simplified map, Fig 3 and Fig 4 in **Graph Image**). Its weight represent the length (time travel for Fig 4) between 2 vertices. We can see there are 21 vertices (the cities/towns) and 25 edges (roads). This routing problem is related to making Euler path and/or Hamilton path and/or Hamilton curcuit and/or Hamilton path from subgraph of Fig 3 (or Fig 4). The smallest length is from Singaraja to Kubutambahan (12 KM) or vice versa, the longest length is from Seririt to Gilimanuk (66.7 KM) or vice versa.

Supposed every edges weighted 12 KM. The maximum vertices connected by a train route possible

$$120 = ((12 * (v-1))/200) * 60 + (v * 10)$$
$$120 = (36v – 36)/10 + (v*10)$$
$$1200 = 136v – 36$$
$$1236 = 136v$$
$$9 = v \text{ \{rounded down\}}$$

Supposed every edges weighted 66.7 KM. The minimum vertices connected by a long train route (> 100 minutes) possible

$$100 = ((66.7 * (v-1))/200) * 60 + (v * 10)$$
$$100 = (200.1v – 200.1)/10 + (v*10)$$
$$1000 = 300.1v – 200.1$$
$$1200 = 300.1v$$
$$3 = v \text{ \{rounded down\}}$$

This facts help us in picking vertices number for a route. Now we should find X subgraph (X is the number of train that can affordable by the goverment) that satisfy the requirements in segment III (PROBLEM DEFINITION). To build the routes, we need to make partition of map graph, then make best route for each partitions that still satisfy the requirements, after that, we merge the partition solution into global solution which every partitions solution are interconnected to each other by common vertices (but not common edge). We have 2 options in building a route :

1. Build a path
   Supposed a, b and c are vertices.
   The train will travel from a then to b then to c then to b then to a and so on (making inverse).
2. Build a circuit
   Supposed a, b and c are vertices.
   The train will travel from a then to b then to c then to b and so on (making loop). This option is same as building an euler/ hamilton circuit from the subgraph.

Building Circuit Routes Only

It is impossible to build circuit route only because there will be some intersecting cycles, that condition is not permitted.

Building Path Routes Only

We have 2 option in building path route. First we make the longest possible long path (> 100 minutes) starts from vertex X and end in vertex Y, then first train starts travel from vertex X to Y (then vice versa) picking at least 3 vertices connected. On the other hand, second train starts travel from vertex Y to X (then vice versa), to satisfy the frequency in comoing requirement. Second option is to make a path which length made it's possible to be traveled by a single train vice versa.

We can create 5 path routes operationg 8 trains :

- Pupuan – Pekutaan – Negara – Gilimanuk – Seririt that take 104.12 minutes travel. We operate 2 trains for this route, the first train starts from Pupuan and the second train starts from Serirt (vice versa start).
- Seririt – Pupuan – Antasari – Mengwi – Denpasar – Uluwati that take 101.13 minutes travel. We operate 2 trains for this route, the first train start from Seririt while the other starts from Uluwatu.
- Seririt – Singaraja – Bedugul – Mengwi that take 112.68 minutes travel started from seririt until the train come back to seririt again (vice versa traveling, last drop off and loading time is not counted). This route only needs one train for this route.
- Bangli – Kintamani – Tejakula – Amlampura – Padang Bai – Gianyar – Denpasar that take 107.84 minutes travel. We operate 2 trains for this route, the first train start from Bangli while the other starts from Denpasar (vice versa start).
- Kintamani – Tampaksiring – Gianyar that take 64.06 minutes to travel from Kintamani until it come to Kintamani again (vice versa traveling, last drop off and loading time is not counted). We only needs one train for this route.

Becase Pekukutan – Antasri, Singaraja – Kubutambahan, Kubutambahan – Tejakula, Denpasar – Nusa dua, Nusa Dua – Uluwatu (or vice versa) edges length < 20KM, they don't necessarily connected by train route. Those 5 routes have no common edges, take 8 trains operation and fulfill all other requiremens. This option when unioned make a graph of 18 connected vertices using 20 edges. All long route (> 100 minutes) connect 3 <= vertices <= 9. Those all the optimal solution possible from examination for this option.

Building Circuit and Path Routes

Type 1

For effectiveness of train number, we will make as many loop as possible, because a loop only needs 1 train for its operation to satisfy the time requirement. First, we make as long as possible circuit from Fig.3 subgraph, they are :

- Seririt – Gilimanuk – Negara – Pekutatan – Pupuan – Seririt – looping, that take 113.6 minutes. . This route only needs to operate 1 train.
- Tejakula - Amlampura – Padang Bai – Gianyar – Tampaksiring – Kintamani – Tejakula – looping that take 115.59 minutes. This route only needs to operate 1 train.

There's 1 more circuit which share no common edges with those 2 above (Denpasar – Nusa Dua – Uluwatu – Denpasar – looping), but adding it only will be a waste because its a very short route, we will examine it too later. Adding 1 more circuit (except Denpasar – Nusa Dua – Uluwatu – Denpasar – looping) will cause 2 routes share common edges, so we must make path for the rest.

We can make some path route (excluding some edges that weighted < 20KM) :

- Seririt – Singaraja – Bedugul – Mengwi – Denpasar – Uluwatu – Nusa Dua that weighted 116.32 minutes. We need 2 trains for this route started from vice versa point (first train from seririt, the other train from Nusa Dua to satisfy frequency requirement)
- Pupuan – Antasari – Mengwi that take 74.92 minutes to travel from Pupuan until the train come to Pupuan again (vice versa traveling). This route only needs to operate one train.
- Kintamani – Bangli – Gianyar – Denpasar that take 98.64 minutes to travel from Kintamani until the train come to Kintamani again (vice versa traveling). This route only needs to operate one train.

Those routes are the best possible solution from examination for this case option 1). This solution make a graph of **19 connected vertices using 22 edges**. This solution will needs to operate **6 trains** for all routes and meet at the requirements.

Type 2

Let us examine adding Denpasar – Uluwatu – Nusa Dua loop in the circuit solution, it will cause 1 more train needed (7 trains) to cover the solution, these are the routes available :

- Seririt – Gilimanuk – Negara – Pekutatan – Pupuan – Seririt – looping, that take 113.6 minutes. . This route only needs to operate 1 train.
- Tejakula - Amlampura – Padang Bai – Gianyar – Tampaksiring – Kintamani – Tejakula – looping that length 115.59 minutes. This route only needs to operate 1 train.
- Denpasar – Uluwatu – Nusa Dua looping that take 49.05 in minutes. This route only needs to operate 1 train.

Then these are the path routes route (excluding some edges that weighted < 20KM) :

- Seririt – Singaraja – Bedugul – Mengwi – Denpasar that weighted 83.21 minutes. We need 2 trains for this route started from vice versa point (first train from seririt, the other train from Nusa Dua to satisfy frequency requirement)
- Pupuan – Antasari – Mengwi that take 74.92 minutes to travel from Pupuan until the train come to Pupuan again (vice versa traveling). This route only needs to operate one train.
- Kintamani – Bangli – Gianyar – Denpasar that take 98.64 minutes to travel from Kintamani until the train come to Kintamani again (vice versa traveling). This route only needs to operate one train.

Those routes are the best possible solution from examination for this case (option 2). This solution make a graph of **19 connected vertices using 22 edges**. This solution will needs to operate **7 trains** for all routes and meet at the requirements.

From all solutions option stated above ("Building path routes only" and "Building circuit and path routes" (Type 1 and 2)). We can conclude that, the best solution is **Building circuit and path routes Type 1**.

V. CONCLUSION

Building circuit and path routes will be better to just building path routes only. Its solution graph contains more vertices and edges. Not only have larger subgraph from Fig.3, but also "building circuit and path route Type 1" only needs to operate 6 trains for all route. This solution is the best solution possible in term of effectiveness and eficiency. You can see the best solution graph in Fig.5 in **Graph Image.**

REFERENCES

[1] http://www.cafed.sssup.it/~giulio/img/graph.png. 4 December 2013 (08:13PM).

[2] http://www.indonesia-tourism.com/bali/map/bali-map-high.png. 4 December 2013 (08:19PM).

[3] https://maps.google.com/. 4 December 2013 (09:05PM)

[4] Munir, Rinaldi. 2006. Diktat Kuliah IF 2120 Matematika Diskrit 4th ed. Bandung : Program Studi Teknik Informatika STEI ITB.

[5] Rosen, Keneth H. 2012. Discrete Mathematics and Its Application 7th ed. New York : The McGraw-Hill.

[6] http://www.cs.sfu.ca/~ggbaker/zju/math/euler-ham.html. 4 December 2013 (09:48PM).

[7] http://people.hofstra.edu/geotrans/eng/ch7en/appl7en/ch7a3en.html. 5 December 2013 (08:03PM).

[8] http://upload.wikimedia.org/wikipedia/commons/b/b5/Cycle_graph_C5.png. 5 December 2013 (08:52PM)

[9] http://www.python-course.eu/images/path_graph1.png. 5 December 2013 (08:53PM)

GRAPH IMAGE

[3]*Fig 3 (Bali map representation as undirected weighted graph)*
Weighting is based on google maps service and my knowledge as Bali citizen.

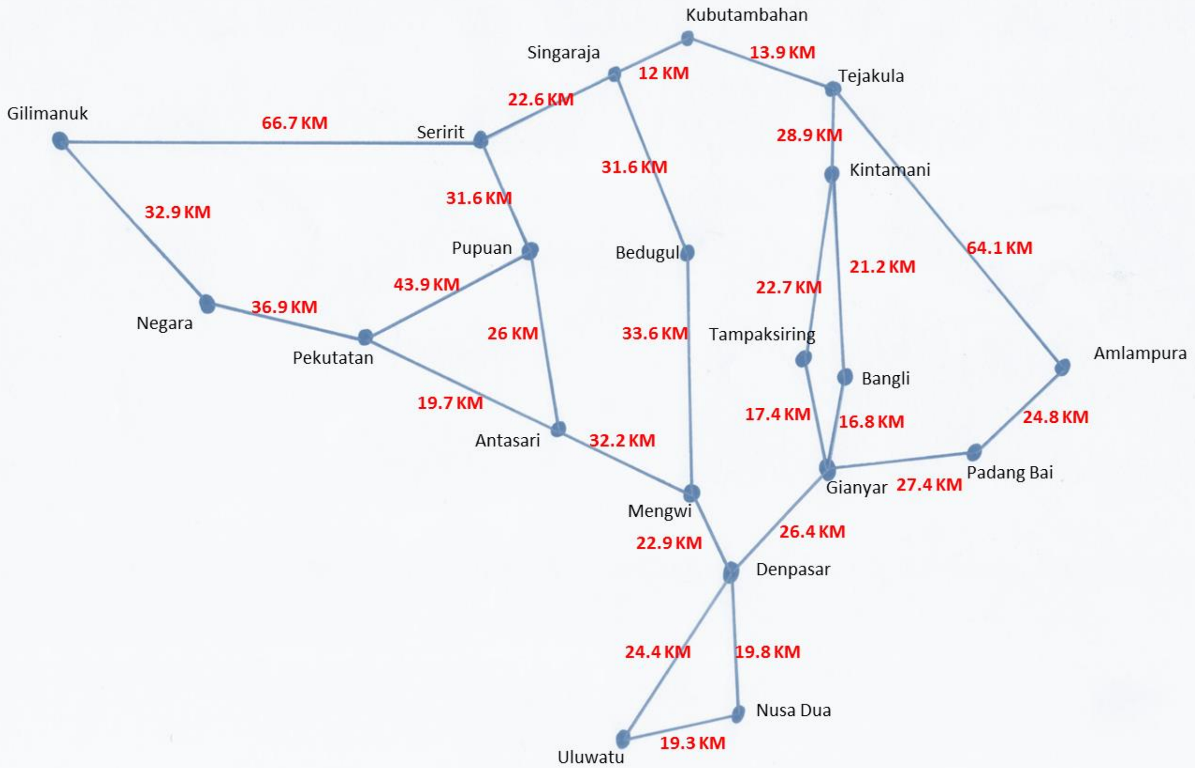


Fig 4 (Bali map representation as undirected weighted graph)
Weighting is based on time travel needed

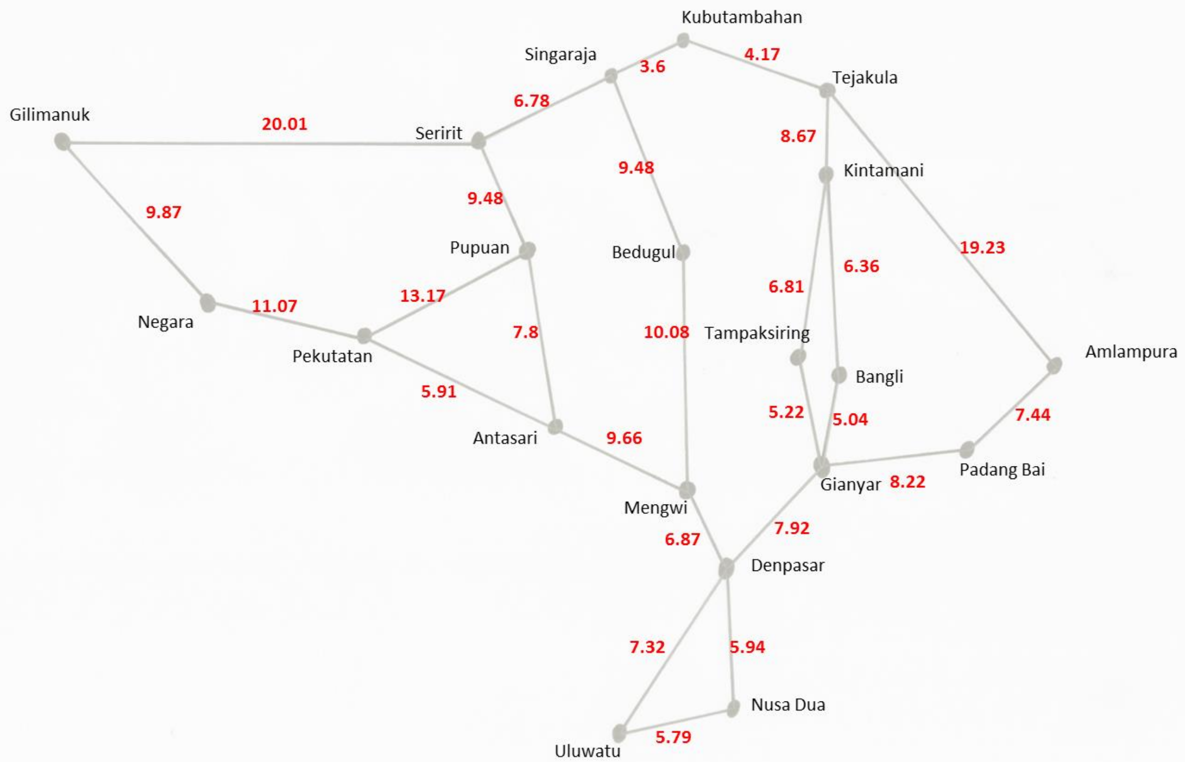
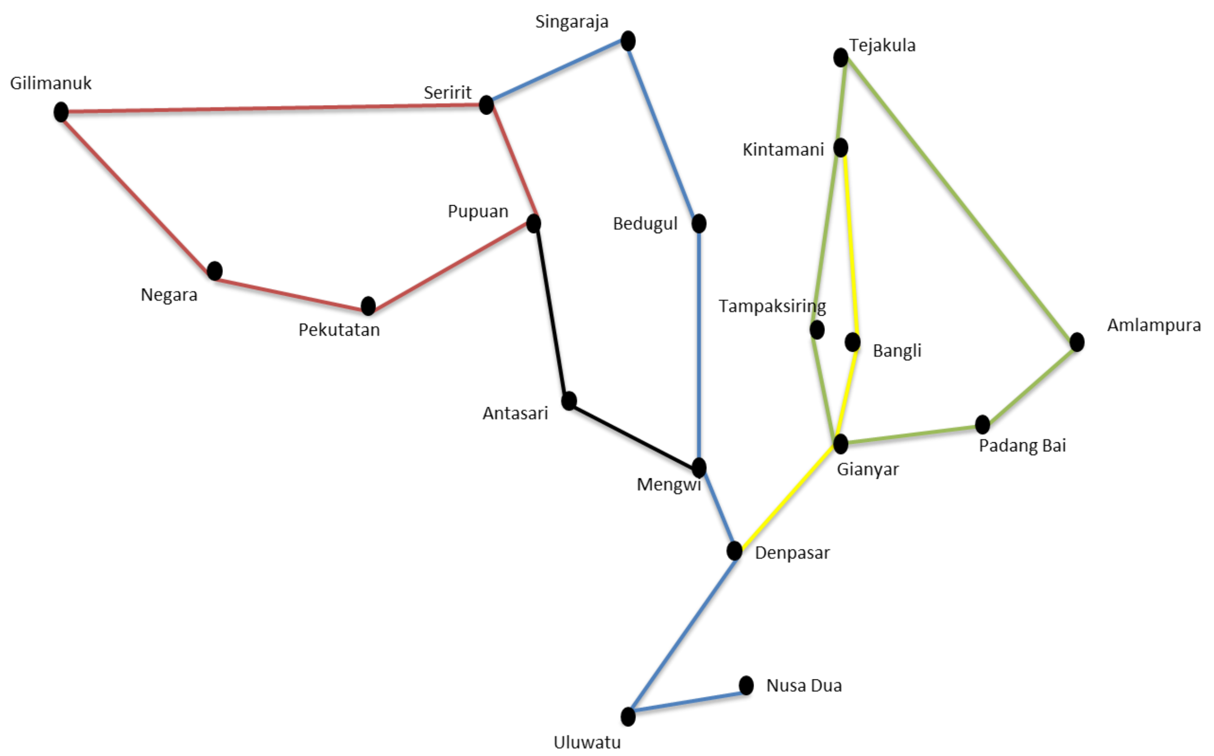


Fig 5 Best solution graph
Same edges color mean they are in the same route
19 vertices are connected



STATEMENT

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Bandung, 17 December 2013

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