Implementation of Prim's Algorithm in Optimizing Drinking Water's Pipe Distribution Network in South-Central Bandung

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***Abstract*— Optimization is one of the most important aspects in both engineering and sciences fields. There are many major problems happened regarding to the optimization problems. The implementation of the optimization process will result the most optimal solution. The best optimized solution will have minimum or maximum values among set of possible solutions. In drinking water distribution network, optimization problem also occurred on time delay of the distribution for certain districts in South-Central Bandung and extravagant pipe usage in the pipe installation process. This problematic issue inflicts higher operating costs and disadvantages the citizens. Therefore, this issue has to be highlighted and must implement efficient algorithm to find the minimum length of pipe required and its path within a network. Thus, the drinking water distribution can be optimized even in hard condition such as in dry season. This paper will focus on the application of Prim's algorithm in finding the most efficient drinking water's pipe path by representing it as a weighted graph in minimum spanning tree form. The running time of this algorithm is O(E log V).**

***Index Terms*— drinking water's pipe, graph, minimum spanning tree, optimization**

I. Introduction

Nowadays, there are numerous problems can be modeled by using graph. Graph is one of the most common data structures that can be represented in a set of vertices and edges. This data structure can be applied for finding the minimum spanning tree within a graph. One of the applications of spanning tree that would be discussed inside this paper is about drinking water's pipe distribution network.

Today, water is the essential source of life for every living thing and its demand in Indonesia has rapidly increased due to increment of citizens. But, the demand has not completely fulfilled due to unstable distribution of water throughout each house. For instance in dry season 2012, in most of districts around South-Central of Bandung have lower water discharged rate and this fact was also agreed by the Director of Perusahaan Daerah Air Minum (PDAM) Kota Bandung.[6] This is occurred because the water capacity in Cisangkuy River decreased in dry season as well as the water evaporates more in that season. Thus, in Badak Singa storage has lower amount of drinking water than in wet season. That condition must be one of the factors that caused the instability of the water distribution itself. When the water capacity of the storage decreased, the discharge rate also lowered. Therefore, the company must find an efficient way to reduce the time delays of the drinking water distributions by using efficient algorithm such as Prim's algorithm. So, even in hard condition such as in dry season, the drinking water distribution can be optimized to maximum. This condition affects the citizen's satisfaction upon the PDAM services. In other hand, this algorithm will support the company to optimize the usage of water pipe in long-term period. This beneficial fact must also be highlighted in order to reduce the operational cost of PDAM Bandung. Thus, this paper will introduce a way to find the minimum spanning tree from a drinking water's pipe distribution network in South-Central Bandung by implementing Prim's Algorithm. Thus, the costs of the operation and goods of the pipe installation and the time delay of the distribution to certain household could be less and can avoid instability of the distribution service. In this paper, the writer would not discuss the constraints that affecting the water's distribution such as the pressure of land and pipes, height of the ground, humidity, and temperature and assuming that the discharge rate of water is equal while the distribution of water is in progress.

II. Fundamental Theories

2.1 Definition of Graph

Graphs are discrete structures consisting of vertices and edges that connect these vertices.[1] There are different types of graphs, regarding whether edges have direction, same pair of vertices, or loops are allowed. We use this structure to model acquaintanceships between things for instance organizations, people, species, etc. Mathematically, graph is defined as G = (V,E) where

- V = a nonempty set of vertices (or nodes)
- E = a set of edges. Each edge has either one or two vertices associated with it, called its endpoints. An edge is said to connect its endpoints.

Here is the example of graph:

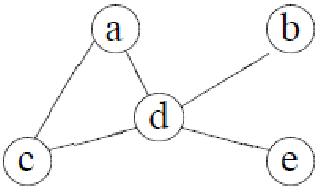


Figure 2.1 A graph with 5 edges and 5 vertices/nodes.

The graph above is undirected graph that has 5 edged and 5 vertices. The vertices are a,b,c,d, and e.

2.2 Weighted Graph

A weight graph is a graph G in which each edge e has been assigned a real number w(e), called weight (or length) of e.[1] This weight of each edge represents the distance between two vertex/node. Here is the example:

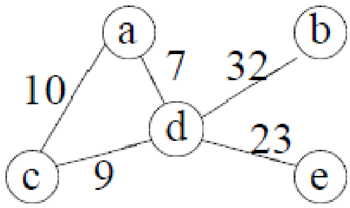
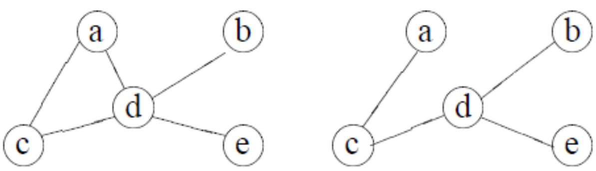


Figure 2.2 Weighted Graph

According to Figure 2.2, every edge between two vertices has weight that represents the distance between them.

2.3 Spanning Tree

A spanning tree in a graph G is a minimal subgraph connecting all the vertices of G.[3] The spanning tree is represented as an undirected graph. If we consider that the graph is a weighted graph, then the weight of a spanning tree of G is defined as the sum of weights of all branches.



(a) (b)

Figure 2.3 (a) A graph with 5 edges and 5 vertices/nodes. (b) Spanning Tree

2.4 Minimum Spanning Tree

A minimum spanning tree in an undirected connected weighted graph is a spanning tree of minimum weight. The minimum weight is the term given to a spanning tree which the sum of its edges is minimal. For instance, take a look at Figure 2.4.

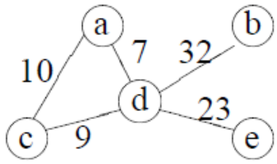


Figure 2.4 Weighted Graph

Figure 2.4 above is a weighted graph. The graph has a minimum spanning tree and illustrated in Figure 2.5.

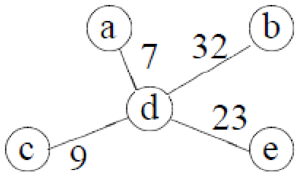


Figure 2.5 Minimum Spanning Tree from Graph in Figure 2.3

The minimum spanning tree obtained by using Prim's Algorithm or Kauskal's Algorithm. In Figure 2.5, we can discover that the minimum value of the sum of its weight is 71.

2.5 Prim's Algorithm

Prim's algorithm is a greedy algorithm which searches for a minimum spanning tree for a connected weighted undirected graph. This algorithm will find the shortest path within a weighted graph in effective and efficient way and forms a tree that includes every vertex, where the total weight of all the edges in the tree is minimized. In order to apply this algorithm effectively and efficiently, we need a fast way to select the edge in forming the tree. [5]

Now, take a look at pseudo-code for Prim's algorithm in Figure 2.6. [5]

```
MST-PRIM(G, w, r)
for each u ∈ G.V
    u.key = ∞
    u.π = NIL
r.key = 0
Q = G.V
while Q ≠ ∅
    u = EXTRACT-MIN(Q)
    for each v ∈ G.Adj[u]
        if v ∈ Q and w(u, v) < v.key
            v.π = u
            v.key = w(u, v)
```

Figure 2.6 Pseudo code for Minimum Spanning Tree by using Prim's algorithm

Figure 2.6 shows how the Prim's Algorithm works. Lines 1-5, show the key of each vertex ∞ except the root r. The key for root is set to 0. Then, the priority queue Q is initialized. Then, in lines 6-10 there are iterations, each vertices already placed into the minimum spanning tree are those in V – Q. For all vertices v, which is subset of Q, if v.π ≠ NIL, then v.key < ∞ and v.key is representing the weight of a light edge (v,v,π) connecting v to some vertex already placed into the minimum spanning tree.

Here is the execution of Prim's Algorithm by taking graph in Fig 2.3 as the example:

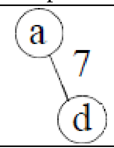
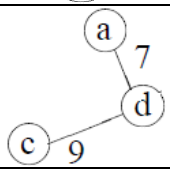
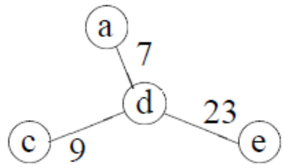
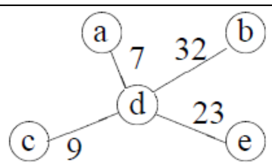
Steps	Edges	Weight	Graph
1	(a,d)	7	
2	(d,c)	9	
3	(d,e)	23	
4	(d,b)	32	

Table I. Execution of Prim's Algorithm for graph in Figure 2.3

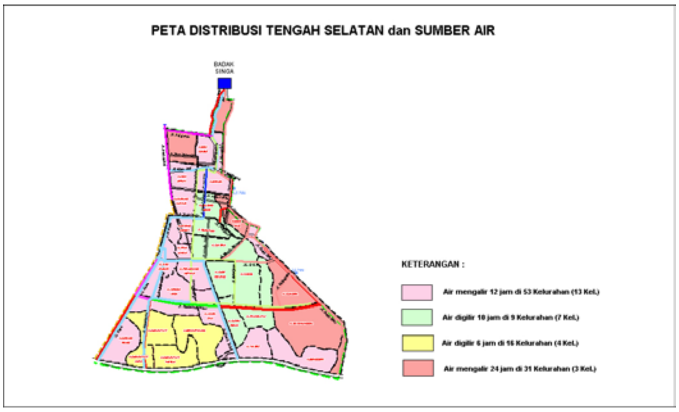
According to Table I above, the Prim's Algorithm did 4 steps. The first step, we choose edge (a,d) with weight 7. Next, we choose edge (c,d) because it has the lowest weight (9). Then, we choose edge (d,e) with weight 23 because there is no vertex after vertex c and edge(d,e) has the lowest weight(23). At last, we choose (d,b) with weight 32.

Now, let's calculate the running time of Prim's Algorithm. In order to calculate the running time, we should analyze each line including the loops. The running time of this algorithm depends on how we implement the minimum priority queue Q. In case, we implement them as a binary min-heap, we can use the Build-Min-Heap procedure to perform the first 5 lines in O(V) time. Each of the EXTRACT-MIN operation takes O(log V) time. The for loops in lines 8-11 will execute the O(E) times. Thus, the total time for Prim's Algorithm is O(V log V + E log V) = O(E log V), where E is the edge and V is the vertex.

III. Drinking Water Distribution Design System Analysis

3.1 System Description

The current drinking water distribution design system in South-Central Bandung can be showed in Figure 3.1 (taken from the website of PDAM Bandung). The design below is divided into 4 parts, according to the color within the map.



Source: http://www.pambdg.co.id/new/images/stories/peta-tengsel.jpg

Figure 3.1 Current Drinking Water Distribution in South-Central Bandung

In Figure 3.1, we noticed that the water source is come only from one water source which is Badak Singa Storage. This water originates from Cisangkuy River and flows to Badak Singa as the distribution storage. Then, PDAM divides the distribution into 3 parts and it flows through all the districts. There are 4 groups of district according to data collected from PDAM Bandung's website:

No	Color	Number of Villages / *Kelurahan*	Distribution
1		13	Water flows every 12 hours
2		7	Rotated every 10 hours
3		4	Rotated every 6 hours
4		3	Water flows 24 hours

Table II. The Groups of Drinking Water Distribution

According to Table II, there are groups with different water distribution rate and some of them are rotated in a period of time. The first group represents with pink and covered 13 villages. The second group represents with cyan and covered 7 villages. The third group represents with yellow and covered 4 villages. The fourth group represents with orange and covered 3 villages.

3.2 Problem Analysis

[6]In 2012, a problem arises when the discharge rate of drinking water reduced significantly because the water capacity in Cisangkuy River (water source) decreased. This issue happened in every dry season of the year and disadvantages the residents. The residents argued that this issue would make them have insufficient amount of drinking water for daily lives. This situation forced them to buy extra drinking water that much more expensive while they were still paying for the PDAM's service. Moreover, the residents were losing their reliance to PDAM due to this issue. This shows that PDAM has to pay attention to this situation and find out the solution to finish the problem.

Therefore, the writer would suggest implementing the Prim's Algorithm to the system. The Prim's Algorithm would reduce the time of water distribution delay for each household by finding the shortest spanning tree as the main canal that distributed the water to the district. The reduction of time would optimize the discharge rate of drinking water, so each household would receive the maximum discharge rate even in dry season that could reduce the capacity of available drinking water.

According to Figure 3.1, the map can be transformed into graph. By transforming the map into graph form, will effectively help to simplify the path and implementing the Prim's Algorithm. Here is the graph that representing the map in Figure 3.1

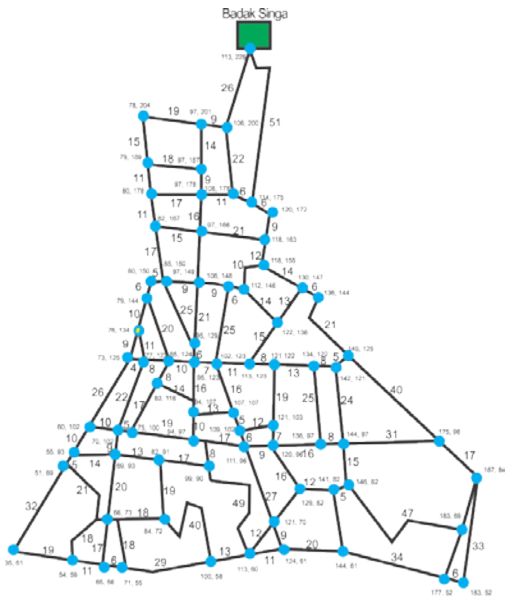


Figure 3.2 Weighted Graph Representation of Current Drinking Water Distribution in South-Central Bandung

Based on above figure, the vertices represent the intersection of each district and the edges represent the distance between each vertex. In every vertex is given coordinate as the representation of each location in real map and in each of edge is given weight that represents the length between each vertex within the scale of the map in millimeter. Mathematically, the length of edge is calculated by following equation:

$$length = \sqrt{(x1 - x2)^2 + (y1 - y2)^2} \quad (1)$$

x1 = x coordinate of vertex 1
x2 = x coordinate of vertex 2
y1 = y coordinate of vertex 1
y2 = y coordinate of vertex 2

This equation (1) will be used to plot the length of every edge. Every edge stands for length between two nodes/vertices.

IV. Implementation of Prim's Algorithm in Drinking Water Distribution Design

The Prim's Algorithm would be implemented in optimizing the distance path between vertices. The expectation of this implementation is lying on the minimum sum of the length of all edges. Let's define the graph in Figure 4.1 as the drinking water distribution graph.

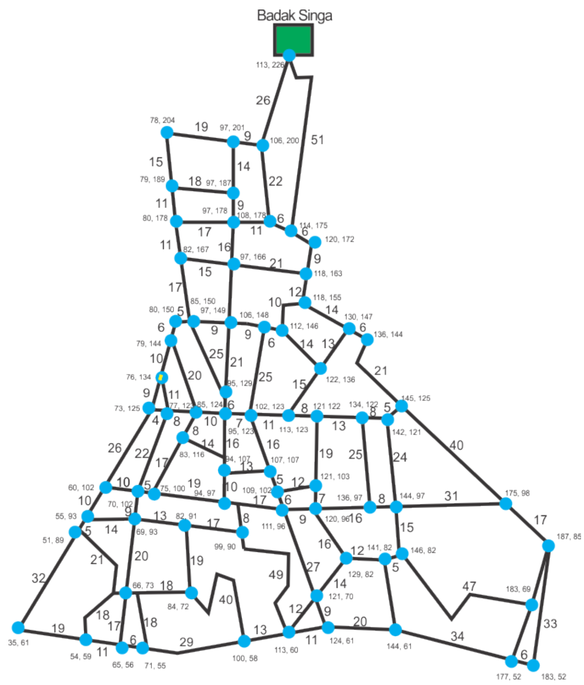


Figure 4.1 Graph Representation of Water Distribution in South-Central Bandung

Let the graph divided into 3 parts in 2 sections for better analyzing and implementation of Prim's Algorithm. Each part will work as the main canal of each district. This is done to stabilize the rate of water in all districts. The Central Bandung will be a single part and the Southern Bandung will be divided into two parts due to large areas covered. Here is the graph for section one:

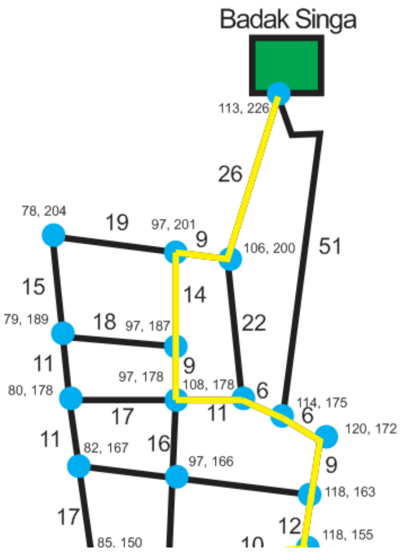


Figure 4.2 Section One

The figure above shows the start location of the water distribution which is Badak Singa (113,226) and it divides to 2 other nodes (106, 200) and (114, 175). Firstly, we can apply the algorithm for choosing the path with starting point Badak Singa node. Then, we jumped to node (106, 200). The node is chosen because its length is much less that node (114, 175). Then, we continue to choose node (97, 201), node (97, 187), node (97, 178), node (108, 178), node (114, 175), node (120, 172), node (118,163), and node (118,155) respectively. The path can also be seen in yellow in Figure 4.2. Here is the table for showing the steps more clearly:

Steps	Current Vertex	Destination Vertex	Weight
1	Badak Singa (113, 226)	(106, 200)	26
2	(106, 200)	(97, 201)	9
3	(97, 201)	(97, 187)	14
4	(97, 187)	(97, 178)	9
5	(97, 178)	(108, 178)	11
6	(108, 178)	(114, 175)	6
7	(114, 175)	(120, 172)	6
8	(120, 172)	(118, 163)	9
9	(118, 163)	(118, 155)	12

Table III. The Path for Section One by using Prim's Algorithm

From the Table III the path is created based on the minimum weight chosen by implementing the Prim's Algorithm to the graph (see Figure 4.2). The current vertex shows the starting point and the destination vertex shows the end point. We choose the lowest weight in path or the end of each node. Then, we join the vertex into edges with yellow color as seen on Figure 4.2. Next, insert the weight in every edge between the vertices. And, finally we already formed the minimum path from Badak Singa to vertex (118, 155) with total weight 102.

Now, let's continue to the next section of the map. Here is the graph for section two that shows the implementation of the Prim's Algorithm:

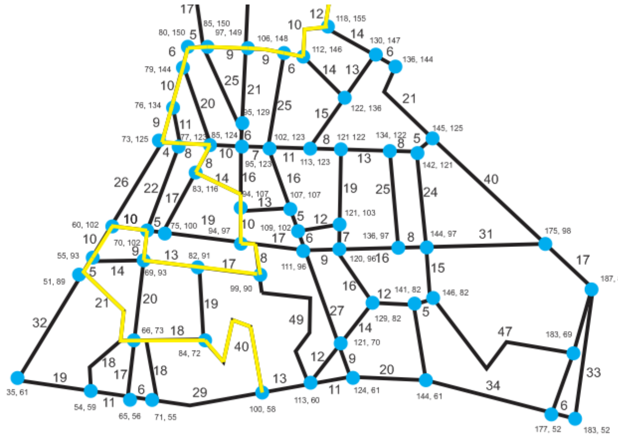


Figure 4.3 Section Two

The figure above is continuous with the previous section. It starts from node (118, 155) and by using Prim's Algorithm to continue the path from the section one (see Figure 4.2 for reference). This is showed in Figure 4.3 with yellow color. From the Table III below, we could able to calculate that the minimum sum of weight is 259. This is the shortest way to go to the bottom (end) of the map.

Here is the table for showing the steps:

Steps	Current Vertex	Destination Vertex	Weight
1	118, 155	112, 146	10
2	112, 146	106, 148	6
3	106, 148	97, 149	9
4	97, 149	85, 150	9
5	85, 150	80, 150	5
6	80, 150	79, 144	6
7	79, 144	76, 134	10
8	76, 134	73, 125	9
9	73, 125	77, 123	4
10	77, 123	85, 124	8
11	85, 124	83, 116	8
12	83, 116	94, 107	14
13	94, 107	94, 97	10
14	94, 97	99, 90	8
15	99, 90	82, 91	17
16	82, 91	69, 93	13
17	69, 93	70, 102	9
18	70, 102	60, 102	10
19	60, 102	55, 93	10
20	55, 93	51, 89	5

21	51, 89	66, 73	21
22	66, 73	84, 72	18
23	84, 72	100, 58	40

Table IV. the Path for Section Two by using Prim's Algorithm

After the Prim's Algorithm applied to both two sections, the shortest spanning tree is found (in yellow color). With the same steps in drawing the graph from section one, we did the same things to the section two. Then, we are connecting the path from the section one to section two and the joined spanning tree formed (see Figure 4.5 for the reference).

This minimum spanning tree will work as the first main canal of drinking water distribution. Then, continue to mark the edge with other colors for another two parts. The other two parts will go the path where hasn't been passed by the first main canal path. So, we could able avoid path with multiple pipes. Both two parts are using the Prim's Algorithm and marked with orange and blue color.

Then, for each location that hasn't been marked as the path will be marked as the undirected graph of each adjacent node with pink and green color. The pink and green paths are used to distribute the drinking water to each household that aren't passed by the main canal.

Here is the description for each path:

No	Color	Distribution
1		First main canal
2		Second main canal
3		Third main canal
4		Distribution from second main canal
5		Distribution from third main canal

Figure 4.4 Description of each path

By using the description above, we draw the graph that has been marked with color of each drinking water distribution. The first main canal will distribute the water to the Central Bandung. The second main canal will distribute the water to East Southern Bandung and lastly, the third main canal will distribute the water to West Southern Bandung. Then, the pink and pale green color paths will distribution each drinking water to the surroundings.

Here is the new drinking water distribution system design:

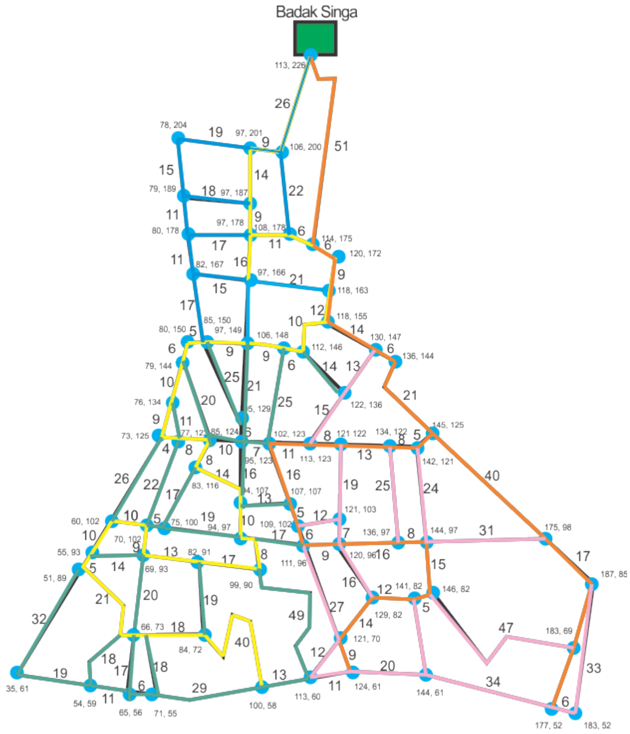


Figure 4.5 Marked Graph with Paths

Figure 4.5 shows the new designed graph with paths. The graph consists of 3 main canals that distributing the drinking water to the 3 big divisions with the minimum distance from the source of the storage (Badak Singa).

The new distribution paths design will be more efficient in distributing the drinking water because the sum of length of the path is lower than previous design due to the implementation of Prim's Algorithm in finding the shortest weighted subgraphs. It assures that every household will definitely receive better discharge of water with this optimized design.

V. Conclusion

Graph theories can be applied in representing drinking water distribution's pipe map of certain areas. The vertex/node of the graph represents the intersection of each location and the edge represents the length between the vertices. In drinking water pipe's system is using the weight graph and each edge is given weight that represents the length between the vertices by using the formula for finding the length between two nodes.

The implementation of Prim's Algorithm is very useful in optimizing the drinking water's pipe distribution path. This algorithm would able to find the shortest spanning tree within the map that represented by graph. This would effectively reduce the pipe's usage and optimizing the water's discharge rate through all districts/locations. The spanning tree obtained from this algorithm would be used as the main canals of the distribution. The optimized graph would optimize the speed of the drinking water distribution even in dry condition. This new drinking water system in South-Central Bandung is divided into 3 groups and shows in 3 different colors and also the distributions from the main canals using different colors. Then, after the main canals are formed, we created the distribution from each main canal to their surroundings.

VI. Acknowledgment

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Pernyataan

Dengan ini saya menyatakan bahwa makalah yang saya tulis ini adalah tulisan saya sendiri, bukan saduran, atau terjemahan dari makalah orang lain, dan bukan plagiasi.

Bandung, 18 Desember 2012

Genta Indra Winata