Application of Logic Theorem within Logic-Based Puzzle

Focusing on Knights and Knaves Puzzle and Einstein's Riddle Puzzle

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This paper was made in order of fulfilling the requirement of subject Discrete Structure (IF 2091) for Informatics student 2011/2012. The theme of this paper is logic, and it takes its title as "Application of Logic Theorem within Logic-Based Puzzle – Focusing on Knights and Knaves Puzzle and Einstein's Riddle Puzzle". This paper will covered some logic theorm used in the puzzle, examples of the puzzle, and the conclusion.

***Index Terms*: einstein's riddle puzzle, knights and knaves puzzle, logic, puzzle**

I. Introduction

This chapter will covered the background of author's decision in making this paper. Puzzle have been people's choice on filling up their free time, thus it has been considered as some game to refresh people's mind. It can be proven by todays famous puzzle game such as Profesor Layton series on Nintendo NDS platform, Ace Attorney, and even through many websites which provide some online logic puzzles including its solution, some of the examples are http://www.brainden.com and http://www.brainbashers.com . These are some examples which shown us, the application of logic theorems are widely found in our daily life, even in our games. In order to solve a logic puzzle, we have to – even not within our consciousness – use the logic theorems we learned on high school or college. Therefore, this is the reason why author took this theme and title. The subtitle, however, "Focusing on Knights and Knaves Puzzle and Einstein Riddle", used to show author's intention in focusing on these area of interests. Knights and Knaves have been a really classic puzzle, being well-knowned since a long time ago, and it still become people's choice for puzzle in modern days. The Einstein Riddle, is a quite new type of puzzle, focusing on using combination table from some data given, thus using it to make some conclusion. Taken from the grand scientist Albert Einstein, it is said that he made this puzzle himself on his early teenage era. This too, has been a famous type of logic puzzle this day, and author will covered this two types of puzzle on this paper.

II. Term and Definition

Before proceeding any further, in order to avoid the misunderstanding of the definitions used within this paper, author will define some term and definitions which used in this paper.

The first one is the definition of the logic puzzle itself. What is a logic puzzle? According to [9], logic puzzle is a puzzle deriving from the mathematics field of deduction.

On the other side, puzzle itself is a problem or enigma that tests the ingenuity of the solver, according to [11]. Puzzles are often contrived as a form of entertainment, as being listed before, but they can also stem from serious mathematical or logistical problems. Solutions to puzzles may require recognizing patterns and creating a particular order. It is said that people with a high inductive reasoning aptitude may be better to solve puzzles than other.

In this part author will covered a brief history of logic puzzle. The earliest version of logic puzzle was made by Charles Lutwidge Dodgson [7], better known as Lewis Carroll, the author of Alice's Adventures in Wonderland. He introduced a game to solve problems, such as you were given two premises: "Some grayhounds run well" and "No fat creatures run well". You have to make a conclusion based on these premises. By using syllogism – we will covered this on later part – we can deduced a conclusion of "Some grayhounds are not fat". Later on, characters such as mathematician Raymond M. Smullyan [12] has continued and expanded the branch of logic puzzles through his books. He was the one who introducing the knights-and-knaves puzzles.

Although logic puzzles are mostly knowned by its verbal approachment, there are also logic puzzles that are completely non-verbal, such as Sudoku, which involves using deduction in placing numbers in a grid, and logic mazes, which involve using deduction to figure out the rules of a maze.

Another popular form of logic puzzle is a logic-grid puzzle. This puzzle consist of a grid-table and some conditions and clues, where we were asked to fill the table according to the informations given. The most famous example is the Einstein's Riddle or also knowned by Zebra Puzzle [13], which will also be covered on later part. There are many variations of logic puzzle, but these are some common examples of it.

Following the previous part, author will list some definitions on the knights-and-knaves puzzle. As being listed beforehand, this type of puzzle was originated by Raymond Smullyan. On a fictional place – usually an isolated island – there are only 2 types of people. The one is knights, or honestants, or whatever it is called, the one to be noted is they will only tell truth, in any conditions. The opposite is the knaves, or swindlecats, or its variant, which will only tell lies, in any conditions. There is also some kind of limitations that we, the player who playing as a visitor to the island, may only ask yes-no questions and such. In some variations, such as the puzzle known as 'the hardest logic puzzle ever' [13], there also be someone who played as an alternator, which talk randomly, they can say either a truth or lie. The other complication is the inhabitants may only answer yes/no in their own native languages, therefore we do not comprehend which one is yes or no.

'The hardest logic puzzle ever' itself is a puzzle introduced by American philosopher and logician George Boolos (which also introduced the algebra which later on will be known as boolean algebra, a memoration of him) in an article published by The Harvard Review of Philosophy in 1996. The earlier version, however, was publised in Italian's La Repubblica, under the title "*L'indovinello piu difficile del mondo*". The story is such:

"Three gods A, B, and C are called, in no particular order, True, False, and Random. True always speaks truly, False always speaks falsely, but whether Random speaks truly or falsely is a completely random matter. Your task is to determine the identities of A, B, and C by asking three yes-no questions; each question must be put to exactly one god. The gods understand English, but will answer all questions in their own language, in which the words for *yes* and *no* are *da* and *ja*, in some order. You do not know which word means which.

There is also some additional rule:

- It could be that some god gets asked more than one question (and hence that some god is not asked any question at all).
- What the second question is, and to which god it is put, may depend on the answer to the first question. (And of course similarly for the third question.)
- Whether Random speaks truly or not should be thought of as depending on the flip of a coin hidden in his brain: if the coin comes down heads, he speaks truly; if tails, falsely.
- Random will answer *da* or *ja* when asked any yes-no question.

The solution to this problem, however, is very complicated and out of this paper coverage. If you interested, though, you can simply visit [13] in order getting full information about this puzzle.

Talking about the knights-and-knaves puzzle, one of the most important part on deducing the conclusion is using the liar paradox. In philosophy and logic, the term liar paradox [8] (pseudomenon in Ancient Greek) is the statement "this sentence is false". If that sentence is true, then the sentence value is false, which would in turn mean that it is actually true, which makes it is false, and so on uninfinitely. And the same does occurred if the sentence is false. What is the function of this paradox? Well, by knowing this liar paradox, we have an information that a knaves will never tell "I am a knave", or they will make this liar paradox. There are many possible resolutions and applications of liar paradox, but it is beyond this topic.

The other definitions author would like to define is the Einstein's Riddle. This is a grid-table puzzle, and there are several version of this puzzle. Known as the first publication is by Life International magazine on December 17, 1962. Although it is named by the famous Albert Einstein, there is no proved evidence for Einstein's authorship, which can be shown that the problems published earlier mention a brand of cigarette, Kools, which did not exist during Einstein's boyhood. The problem itself consist of 5 variables, such as house color, nationality, cigarette, cocktails, and pet. Given some informations of who-drink-what or who-lived-in-where, we are going to solve the puzzle by filling the informations in a grid-table. After the table are completely done, we can deduce a conclusion of the puzzle.

Those are some definitions the author thought are needly to be noticed on this paper, and the 3rd part will listed the logic theorem which most usually used in solving a logic puzzle.

III. LOGIC THEOREM

A. Proposition and Logic Law

Proposition refers to either the content or meaning of a meaningful declarative sentence, or the pattern of symbols, marks, or sounds that make up a meaningful declarative sentence [10]. In this lecture, proposition is defined as a sentence which its value is exact, either true or false, can not be both. Proposition is used in logical sentence, and sometimes, there is no connection at all between two propositions within a sentence, even when a proposition value is valid, the sentence is valid. Proposition is such "My car is blue", or "I kept the money in the bank". Even in this weird sentence: "If I have a blue car, I will keep my money in the bank", as long as we know whether the premises are either correct or wrong, the proposition is valid.

Logic Law is a series of law of boolean algebra, it looks simple on each law, but it is very essential on making some conclusion or building implication and arguments. Some of the logic laws are:

Identity Law : p OR F = p ; p AND T = p
Null Law : p AND F = F ; p OR T = T
Negation Law : p OR p' = T ; p AND p' = F
Idempotent Law : p OR p = p ; p AND p = p

Involution Law : ~(p') = p
Absorption Law : p OR (p AND q) = p
p AND (p OR q) = p
Commutative Law : p OR q = q OR p
p AND q = q AND p
Associative Law:
p OR (q OR r) = (p OR q) OR r
p AND (q AND r) = (p AND q) AND r
Distributive Law :
p OR (q AND r) = (p OR q) AND (p OR r)
p AND (q OR r) = (p AND q) OR (p AND r)
De Morgans' Law :
~(p AND q) = p' OR q'
~(p OR q) = p' AND q'

Where p, q, and r are the premises, p' or ~q shows the negation of the propositions, and either OR or AND as the boolean operator.

B. Implication and Bi-Implication

Implication [4] in a logical terms is a sentence which built up by two clauses, the condition (protasis) and the consequence (apodosis) [4]. Thus, the form of implication on its simplest form is If [condition], then [consequence]. Once again, perhaps the condition and consequence do not have any similiarity in any way, but as long as it is valid, then so be it. One of the function of using implication is to describe some logical situation which you have to show in a sequence matter, something followed after something. The truth table for implication is:

Table 3.1 Implication's Truth Table

P	q	p → q
T	T	T
T	F	F
F	T	T
F	F	T

On the other side, there is also a term called bi-implication. People often noted this with 'if and only if' [5] or 'iff'. In its simplest form, the sentence will be "p if and only if q", with p and q are the sentence premises. The function of bi-implication is to note something that will return the value of true if only both conditions are either correct or uncorrect. It will return false if either one of the premises is different to the other. Bi-implication is one of the most powerful weapon used in knights and knaves puzzle, when we are faced with some complicated condition, we could ask some bi-implication yes/no questions to the inhabitants of the island, which we are going to know, only if the conditions are both correct or both false, it will return true.

The truth table for bi-implication is:

Table 3.2 Truth Table for Bi-implication

p	q	p ↔ q
T	T	T
T	F	F
F	T	F
F	F	T

C. Arguments

Argument [3] is a list of proposition which consist of 2 parts, the list of hypothesis (or premises) and the conclusion. Arguments can be either valid or invalid. By a definition taught in this lecture, arguments is valid if the conclusion is true for all of the hypothesis are true, other possibilities lead to invalid arguments.

Arguments can also be a good choice in logic puzzle game, by getting some informations and gather them together, we can deduce a solid conclusion. In this paper, argument may be used within the scope of knights and knaves puzzle, where the questions asked to the island inhabitants can make several patterns and lead us to make the conclusion from argument.

There are several argument which been scientifically proven valid:

Modus Ponen
Hyp1 : p → q
Hyp2 : p
Concl : q

Modus Tollen
Hyp1 : p → q
Hyp2 : q'
Concl : p'

Syllogism
Hyp1 : p → q
Hyp2 : q → r
Concl : p → r

Disjunctive Syllogism
Hyp1 : p OR q
Hyp2 : p'
Concl : q

Those are some arguments which been proven valid, thus we can simply use them in order to make some conclusion during the logic puzzle solving phase.

IV. EXAMPLES AND SOLUTION

A. The Knights and Knaves Puzzle

For the example of the logic application within this puzzle type, let us have a look on these examples:

Question 1 : Someone says : "We are both knaves"

Solution : There is no possibilities that someone will tell "I am a knaves" (look on the liar paradox section). This deliver us to make a conclusion that the one talking is a knaves and the other one must be a knights.

Question 2 : John : "We are the same kind"
Bill : "We are different"

Solution : By looking on both statements, we could find that both if them are making a contradictory situation, and the one seems legit is the Bill's words that say that they are different. Assume that Bill is right, then John's word will be wrong. This means that Bill is a knight and John is a knave.

To take a better look on this problem, there are some very good examples of this puzzle which arranged to make a story.

Part I :

There are two kinds of people on a mysterious island. There are so-called Honestants who speak always the truth, and the others are Swindlecants who always lie.
Three fellows (A, B and C) are having a quarrel at the market. A gringo goes by and asks the A fellow: "Are you an Honestant or a Swindlecant?" The answer is incomprehensible so the gringo asks B: "What did A say?" B answers: "A said that he is a Swindlecant." And to that says the fellow C: "Do not believe B, he is lying!"
Who is B and C?

Explanation of Part I :

As we have been discussed beforehand, there is impossible for a villager to say that he/she is a swindlecat, therefore, B answer is false, making his as a bad guy. On the other hand, assuming that B is lying, than C's statement is correct, leads him to be the honestant. How about A? Due to the information given, we can not decide exactly which side A is.

Part II :

Afterwards he meets another two aborigines. One says: "I am a Swindlecant or the other one is an Honestant."
Who are they?

Solution for Part II :

In this puzzle, we learnt something called exclusive disjunction or 'exclusive or' (XOR). This means, the statement will be true if either the first or the second one is true, but not for both. Even when both statements are correct, the entire statement will be false.

Thus, from the first part of his words, "I am a swindlecat", once again we are made sure that this must be wrong (due to the liar paradox), making A is a honestant. If A is a honestant, however, the sentence must be valid, and either the first part or the second part of his statement must be true. Since we already knew that the first part is wrong, then the second one ("B is an honestant") must be true. This means that both of the aborigines is a honestant.

Part III :

Our gringo displeased the sovereign with his intrusive questions and was condemned to death. But there was also a chance to save himself by solving the following logic problem. The gringo was shown two doors - one leading to a scaffold and the second one to freedom (both doors were the same) and only the door guards knew what was behind the doors. The sovereign let the gringo put one question to one guard. And because the sovereign was an honest man he warned that one guard is a Swindlecant.
What question can save the gringo's life?

Solution for Part III :

We could simpy asking this question: "He you, does an honestant stand at the door to freedom?". We will expecting 2 answers, yes or no. The first case if when the answer is yes. When asked to a honestant and he stand on the right door, he will answer yes, however if a swindlecat stand at the right door, he too will say yes. Thus, making the door is safe to enter. On the other side, if the answer is no, when a honestant stand on the wrong door, he will say no, however, when a swindlecat stand on the wrong door, and the honestant stand on the right one, when we ask this to the swindlecat he will also answer no – because he always lying. Therefore, the right one was the other door.

Part IV :

Our gringo was lucky and survived. On his way to the pub he met three aborigines. One made this statement: "We are all Swindlecants." The second one concluded: "Just one of us is an honest man."
Who are they?

Solution for Part IV :

Again, the first one must be untrue, because no one can say that he/she is a swindlecat [8]. Thus, this man must be a swindlecat. Now we take a look on the other true. First case, if the second one is lying, then the third is an honestant, but it will make his own sentence true, so the right combination is the second one is an honestant and the third one is a swindlecat.

There are many variations of this knights and knaves puzzle, one of them is this:

Pandora Box :

Once upon a time, there was a girl named Pandora, who wanted a bright groom so she made up a few logic problems for the wannabe. This is one of them.
Based upon the inscriptions on the boxes (none or just one of them is true), choose one box where the wedding ring is hidden.

Gold	Silver	Lead
"The ring is in this box"	"The ring is not in this box""	"The ring is not in the golden box""

Solution to the Pandora Box problem:

To answer this puzzle, we may use 3 cases, the first one, let us take a look when the description on the gold box is true, than the ring is on the golden box. This will make the description on the lead box false and the silver one correct. On the specification however, there is no more than 1 true description, so this combination is invalid.

The second case, when the description on the silver is correct, the possibilities of the coin lies within the gold or the lead box. When the coin is placed on the gold, it will make two true descriptions, just like the first case, so it is unlikely. On the other hand, if the ring is on the lead box, the description on the lead box will be correct, thus we again find the two correct description, so this case is not sufficient.

The third one, when the description on the lead box is correct, which mean the coin is not on the golden box, which lead the probabilities that the coin lies within silver or lead box. When it comes on the lead box, however, there will be another two correct descriptions, so the ring must be inside the silver box, making the only correct description is the one on the lead box.

One last example of this puzzle is the coin problem.

Coins :

Imagine there are 3 coins on the table: gold, silver, and copper. If you make a truthful statement, you will get one coin. If you make a false statement, you will get nothing. What sentence can guarantee you getting the gold coin?

Solution to the Coins problem:

We have to say this line to the problem giver:

“You will give me neither copper nor silver coin”

If the sentence we made is true, than he has to give us the golden coin, when it is incorrect, however, the sentence will be “You will give me either copper or silver coin”, which guarantees us in having at least one coin, a contradiction to the specification which stated that we will get nothing if the sentence is false, therefore, the sentence must be true and lead us in getting the gold coin.

B. The Einstein’s Riddle

The most important part of this riddle is analyzing the information given. Let us say that the information given is as such:

1. The British person lives in the red house.
2. The Swede keeps dogs as pets.
3. The Dane drinks tea.
4. The green house is on the left of the white house.
5. The green homeowner drinks coffee.
6. The man who smokes Pall Mall keeps birds.
7. The owner of the yellow house smokes Dunhill.
8. The man living in the center house drinks milk.
9. The Norwegian lives in the first house.
10. The man who smokes Blend lives next to the one who keeps cats.
11. The man who keeps the horse lives next to the man who smokes Dunhill.
12. The man who smokes Bluemaster drinks beer.
13. The German smokes Prince.
14. The Norwegian lives next to the blue house.
15. The man who smokes Blend has a neighbor who drinks water.

The question is: Who owns the fish?

From these informations, let us make a grid table

House	1	2	3	4	5
Color					
Nationality					
Drink					
Smoke					
Pet					

There are several informations that had to be placed firsthand. From the first step we will getting this table:

House	1	2	3	4	5
Color	**Yellow**	**Blue**			
Nationality	**Norway**				
Drink			**Milk**		
Smoke					
Pet					

The bold text mean that this is the first step taken, so it must be likely correct. Then move on to the second step:

House	1	2	3	4	5
Color	**Yellow**	**Blue**			
Nationality	**Norway**				
Drink			**Milk**		
Smoke					
Pet					

From this position, look at the color section, the combination is the green house must be located on the left of the white house, while the green house owner also drinks coffe, which means the only possible place to be the green house is number (4). Followed by this, the number (5) must be white and the last one, the (3) must be red. Fill in the number (3) nationality with British because the Brit lives in the red house. Then look at the smoke section. We have the information that the one in yellow house smokes Dunhill, fill the smoke section on number (1) with Dunhill. For the drink of number (1), let’s see the possibilities. Is it Tea? No, it is for the Dane. Is it Milk? It is already occupied on (3). Coffee then? No, the house color must be green. How about Beer? The one who drinks beer must also smokes BlueMaster, therefore the only possible choice is the Norway drinks water. Followed by this answer, the number (2) must be smokes Blend (information number 15). Now let us fill the tables first.

House	1	2	3	4	5
Color	**Yellow**	**Blue**	Red	Green	White
Nationality	**Norway**		British		
Drink	Water		**Milk**	Coffee	
Smoke	Dunhill	Blend			
Pet					

Now let us take a look on the drink section. The one who drinks beer must also smokes BlueMaster, therefore the most possible combination is number (5) drinks beer and smokes BlueMaster while the number (2) drinks tea, which followed that the number (2) is the Dane. Now focusing on the cigarette section. We would like to place the one who smokes Prince. The available spots is the (3) and (4), but, oh! The one who smokes Prince is a Deutschman, so the most proper position is the number (4). Filling the rest of the cigarette section, the number (3) is PallMall. For the nationality, the one who smokes Prince is a Deutchman, so number (4) is German. Filling out the rest, the number (5) is Sweden. Fill the table now

House	1	2	3	4	5
Color	**Yellow**	**Blue**	Red	Green	White
Nat	**Norway**	Dane	British	German	Swede
Drink	Water	Tea	**Milk**	Coffee	Beer
Smoke	Dunhill	Blend	PM	Prince	BM
Pet					

Now, for the last part, the only part that needs our attention is the pet section. The Swede keeps dogs, so number (5) is dogs. The one who smokes PallMall keeps birds, so number (3) is birds. The one who keeps horse live beside the one who smokes Dunhill, which means the number (2) is horse. The one who smokes Blend, the Dane, has a neighbour who keeps cats, but since the number (3) has been occupied with the birds, the cats go to number (1). Filling up the table,

House	1	2	3	4	5
Color	**Yellow**	**Blue**	Red	Green	White
Nat	**Norway**	Dane	British	German	Swede
Drink	Water	Tea	**Milk**	Coffee	Beer
Smoke	Dunhill	Blend	PM	Prince	BM
Pet	Cats	Horses	Birds		Dogs

Now, it is very simple to determine who owns the fish, it is the German person who keeps fish as his pet.

This type of logic puzzle requires several combination, and without even realizing it, there are many implication used in fulfilling the table. By correctly analyzing the information given, we can make the table and left only the missing part. There are also many variations on this type of puzzle, but that was beyond this paper scope.

V. Conclusion

Logic is an essential component within a logic-based puzzle. The examples of this type of puzzle are the knights and knaves puzzle and Einstein's Riddle. We may use some logic theorem in solving those puzzle, with Einstein's Riddle is focusing on gathering information and possibilities, then placing them on the right spot on the grid table. Knights and Knaves puzzle focusing on gathering informations by asking a yes/no question, may contain implication/bi-implication, and we can take the conclusion by using argument.

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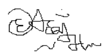

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