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# 27 - Image Warping dan Image Morphing

Bahan Kuliah IF4073 Interpretasi dan Pengolahan Citra

Oleh: Rinaldi Munir

Program Studi Teknik Informatika  
Sekolah Teknik Elektro dan Informatika  
ITB

# SUMBER (REFERENSI):

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1. Alexei Efros, *Image Warping, 15-463: Computational Photography*, CMU, Fall 2008
2. Connelly Barnes, *Image Warping / Morphing, Computational Photography*.
3. Yao Wang, *EL512 Image Processing, Geometric Transformations: Warping, Registration, Morphing*, Polytechnic University, Brooklyn

# Image Warping

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<http://www.jeffrey-martin.com>

15-463: Computational Photography  
Alexei Efros, CMU, Fall 2008

## Image warping example (Sumber: Wikipedia)

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# Image Warping / Morphing

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[Wolberg 1996, Recent Advances in Image Morphing]

## Computational Photography

Connelly Barnes

# EL512 --- Image Processing

## Geometric Transformations: Warping, Registration, Morphing

Yao Wang

Polytechnic University, Brooklyn, NY 11201

With contribution from Zhu Liu, Onur Guleryuz, and

Partly based on

A. K. Jain, Fundamentals of Digital Image Processing

# What is Geometric Transformation?

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- So far, the image processing operations we have discussed modify the **color values** of pixels in a given image
- With geometric transformation, we modify the **positions** of pixels in a image, but keep their colors unchanged
  - To create special effects
  - To register two images taken of the same scene at different times
  - To morph one image to another

# Image Transformations

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image filtering: change **range** of image

$$g(x) = T(f(x))$$

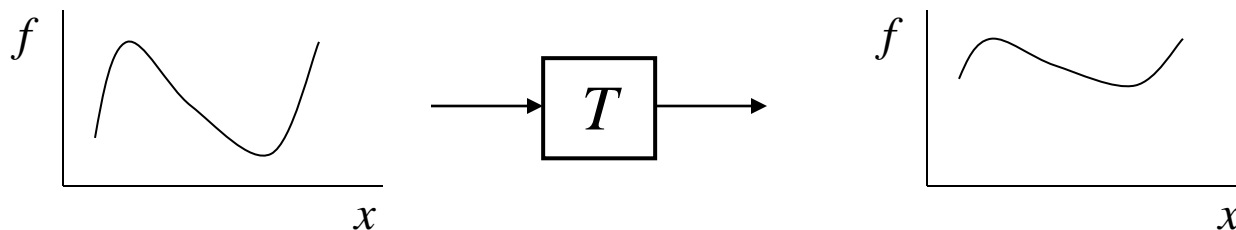
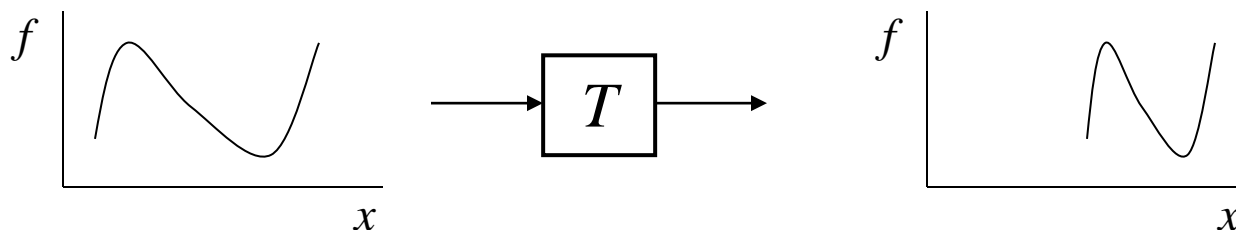


image warping: change **domain** of image

$$g(x) = f(T(x))$$





# Image Transformations

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image filtering: change **range** of image

$$g(x) = T(f(x))$$

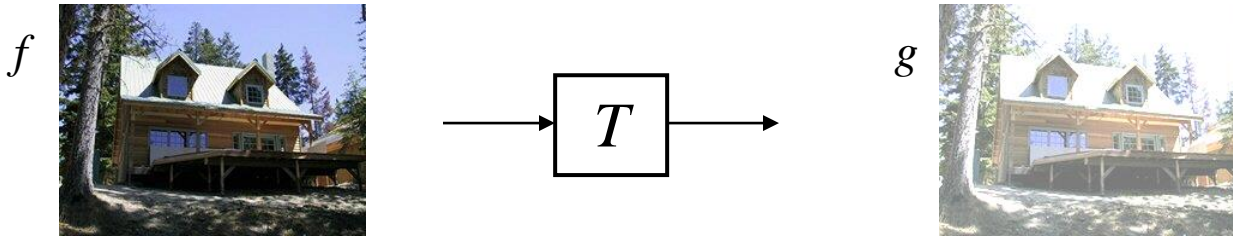
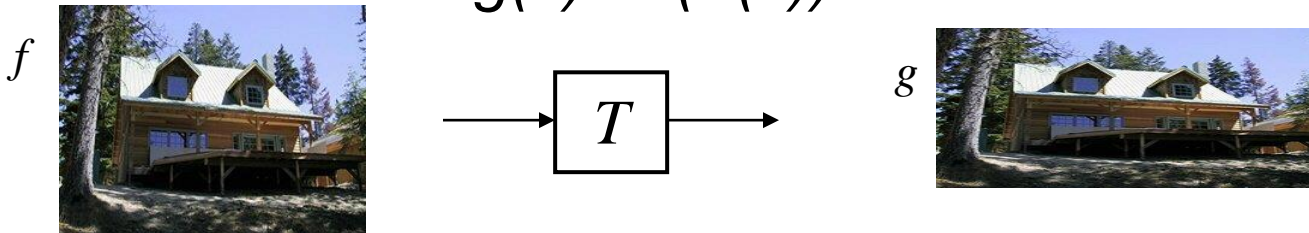


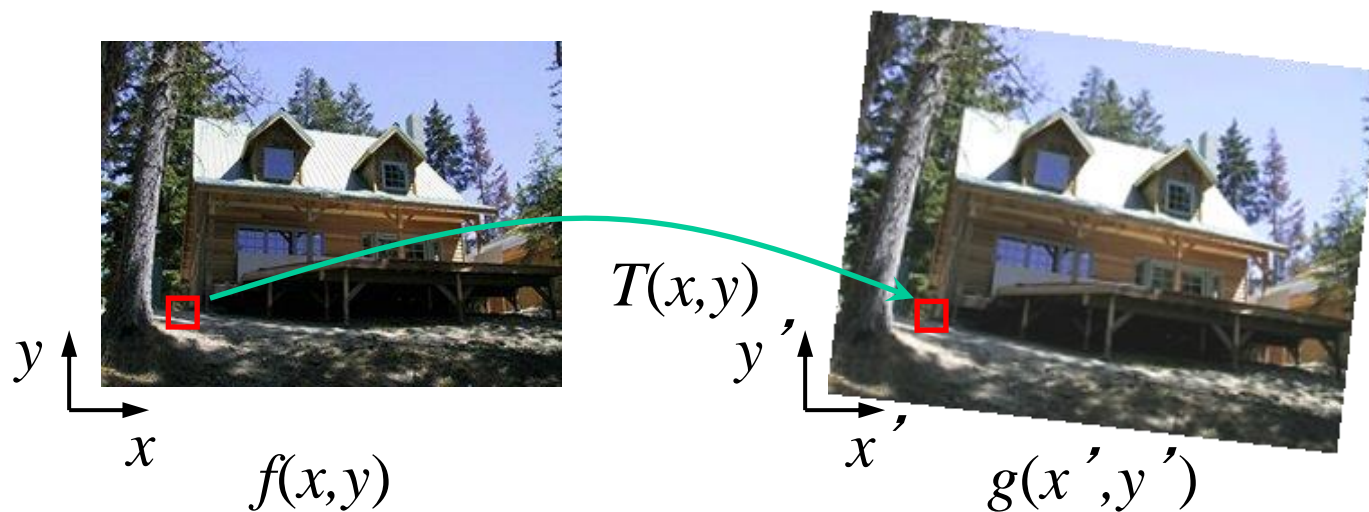
image warping: change **domain** of image

$$g(x) = f(T(x))$$



# Image Warping

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Given a coordinate transform  $(x',y') = T(x,y)$  and a source image  $f(x,y)$ , how do we compute a transformed image  $g(x',y') = f(T(x,y))$ ?

# Parametric (global) warping

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Examples of parametric warps:



translation



rotation



aspect



affine



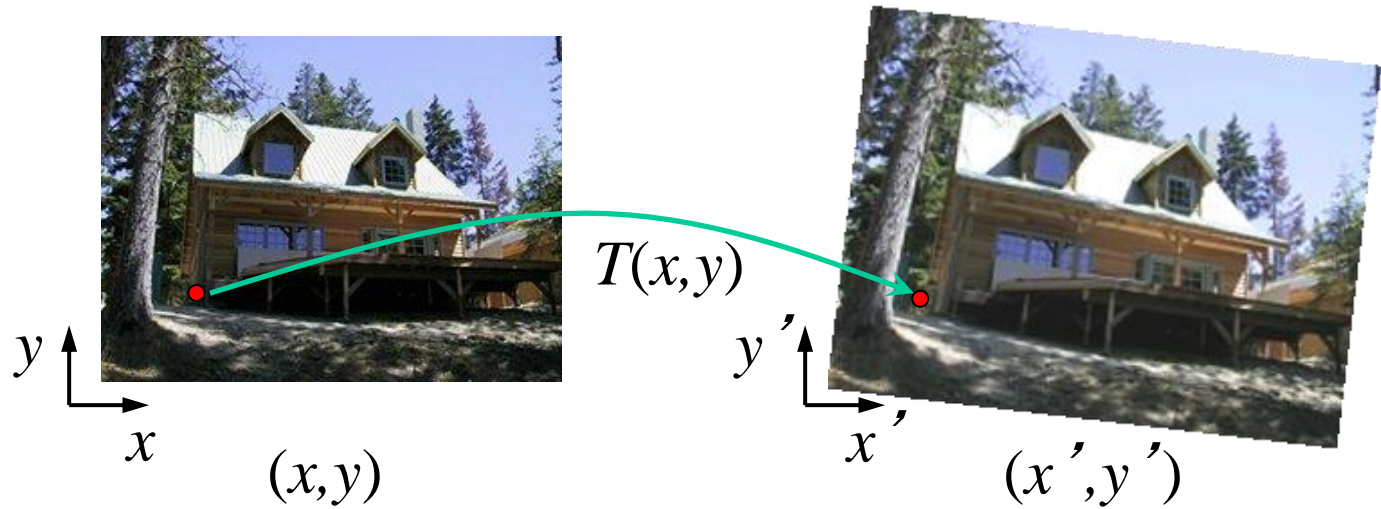
perspective



cylindrical

# Forward warping

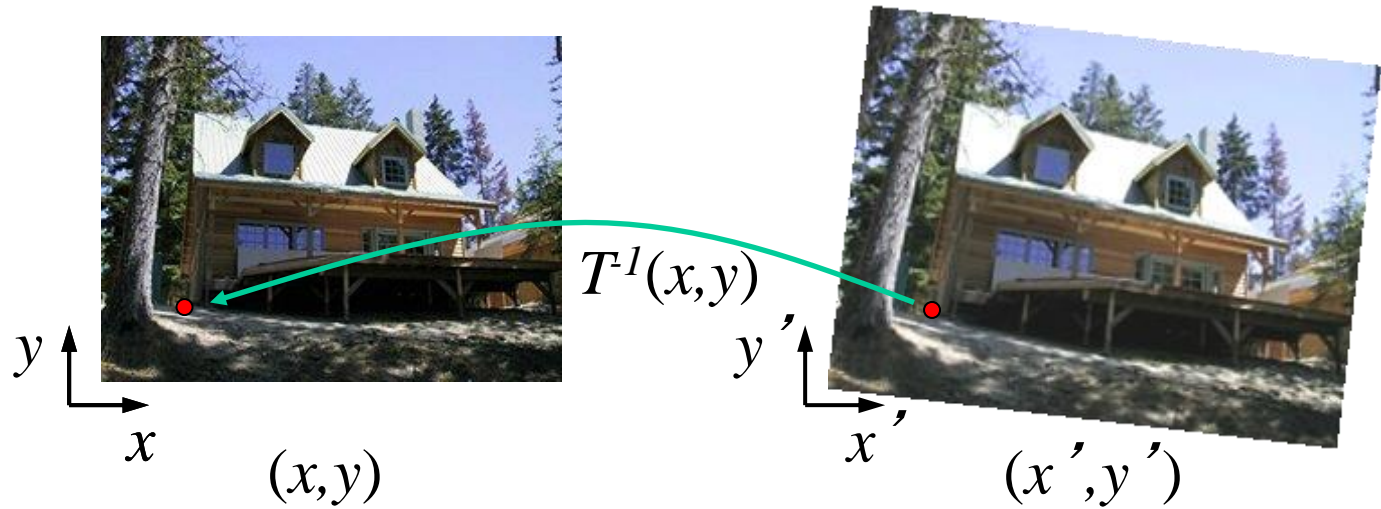
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Send each pixel  $(x, y)$  to its corresponding location  
 $(x', y') = T(x, y)$  in the second image

# Inverse warping

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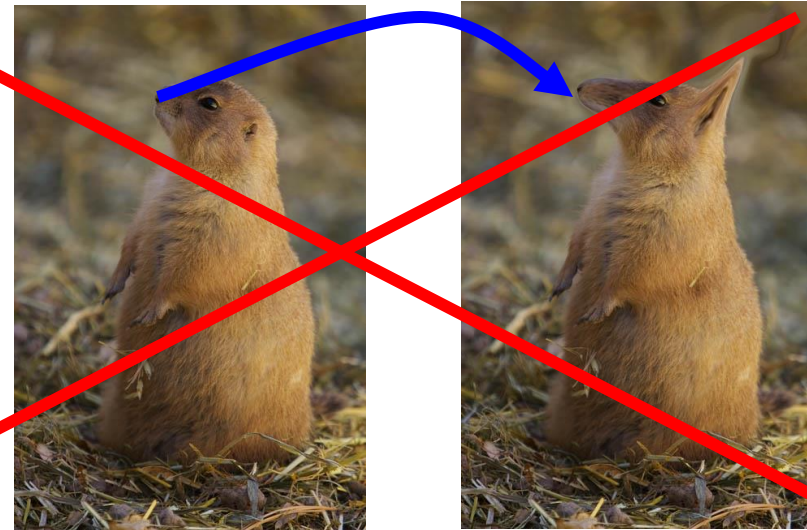
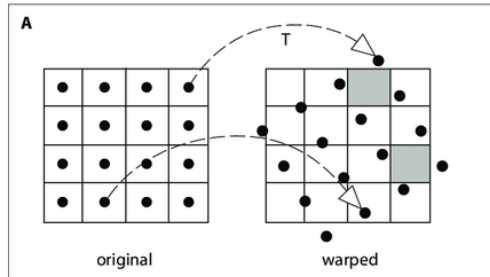
Get each pixel color  $g(x', y')$  from its corresponding location

$$(x, y) = T^{-1}(x', y') \text{ in the first image}$$

# Applying a warp: use inverse

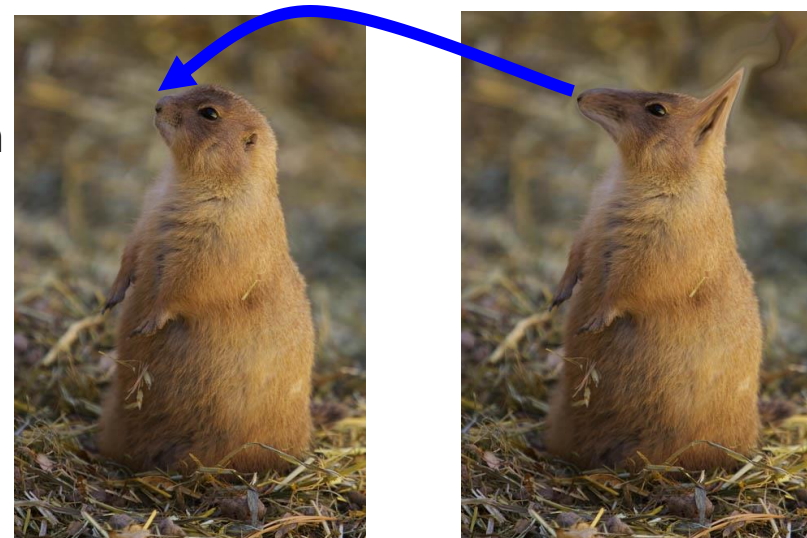
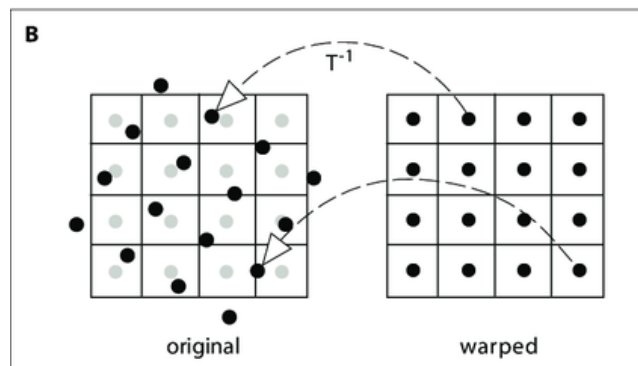
## Forward warp:

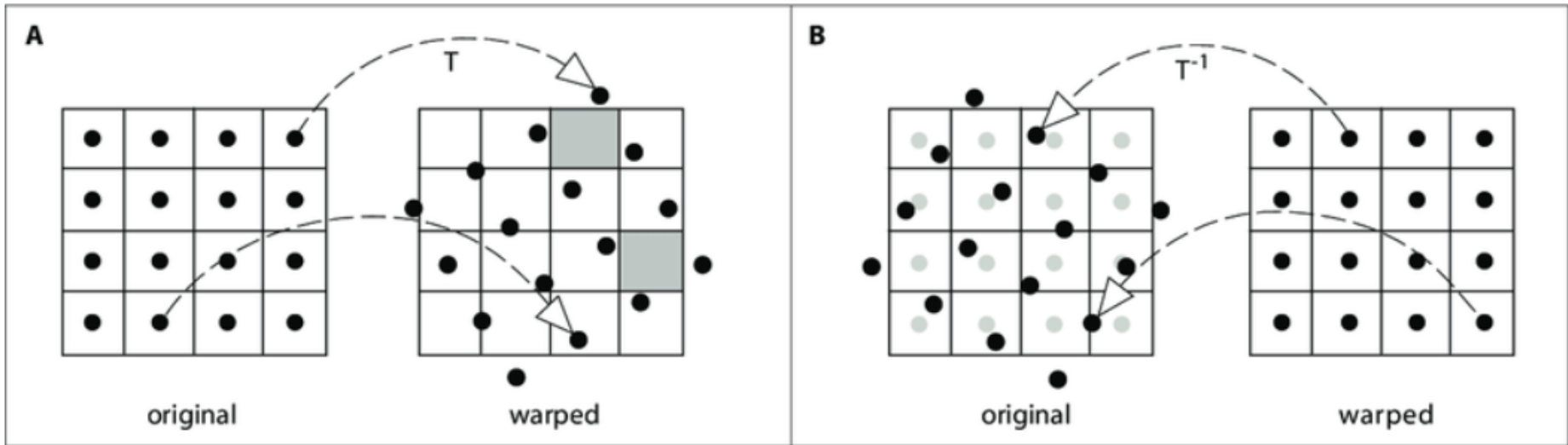
- For each pixel in **input** image
  - Paste color **to warped** location in output
- Problem: gaps



## Inverse warp

- For each pixel in **output** image
  - Lookup color **from inverse-warped** location





2.: Forward and backward image warping. In the case of forward warping (A), holes can occur in the warped image, marked in gray. Backward warping (B) eliminates this problem since intensities at locations that do not coincide with pixel coordinates can be obtained from the original image using an interpolation scheme.

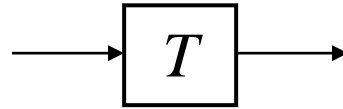
Sumber: Non-rigid Registration Using Free-form Deformations  
 Authors: Loren Arthur Schwarz

# Parametric (global) warping

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$$\mathbf{p} = (x, y)$$



$$\mathbf{p}' = (x', y')$$

Transformation  $T$  is a coordinate-changing machine:

$$\mathbf{p}' = T(\mathbf{p})$$

What does it mean that  $T$  is global?

- Is the same for any point  $\mathbf{p}$
- can be described by just a few numbers (parameters)

Let's represent  $T$  as a matrix:

$$\mathbf{p}' = \mathbf{M}\mathbf{p}$$
$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \mathbf{M} \begin{bmatrix} x \\ y \end{bmatrix}$$



# Translation

- **Translation** is defined by the following mapping functions:

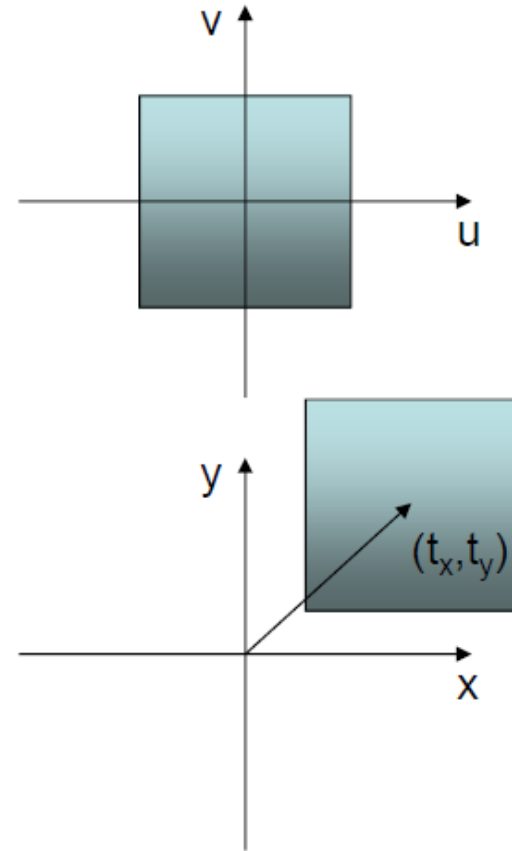
$$\begin{cases} x = u + t_x \\ y = v + t_y \end{cases} \quad \text{and} \quad \begin{cases} u = x - t_x \\ v = y - t_y \end{cases}$$

- In matrix notation

$$\mathbf{x} = \mathbf{u} + \mathbf{t}, \quad \mathbf{u} = \mathbf{x} - \mathbf{t}$$

where

$$\mathbf{x} = \begin{bmatrix} x \\ y \end{bmatrix}, \quad \mathbf{u} = \begin{bmatrix} u \\ v \end{bmatrix}, \quad \mathbf{t} = \begin{bmatrix} t_x \\ t_y \end{bmatrix}.$$

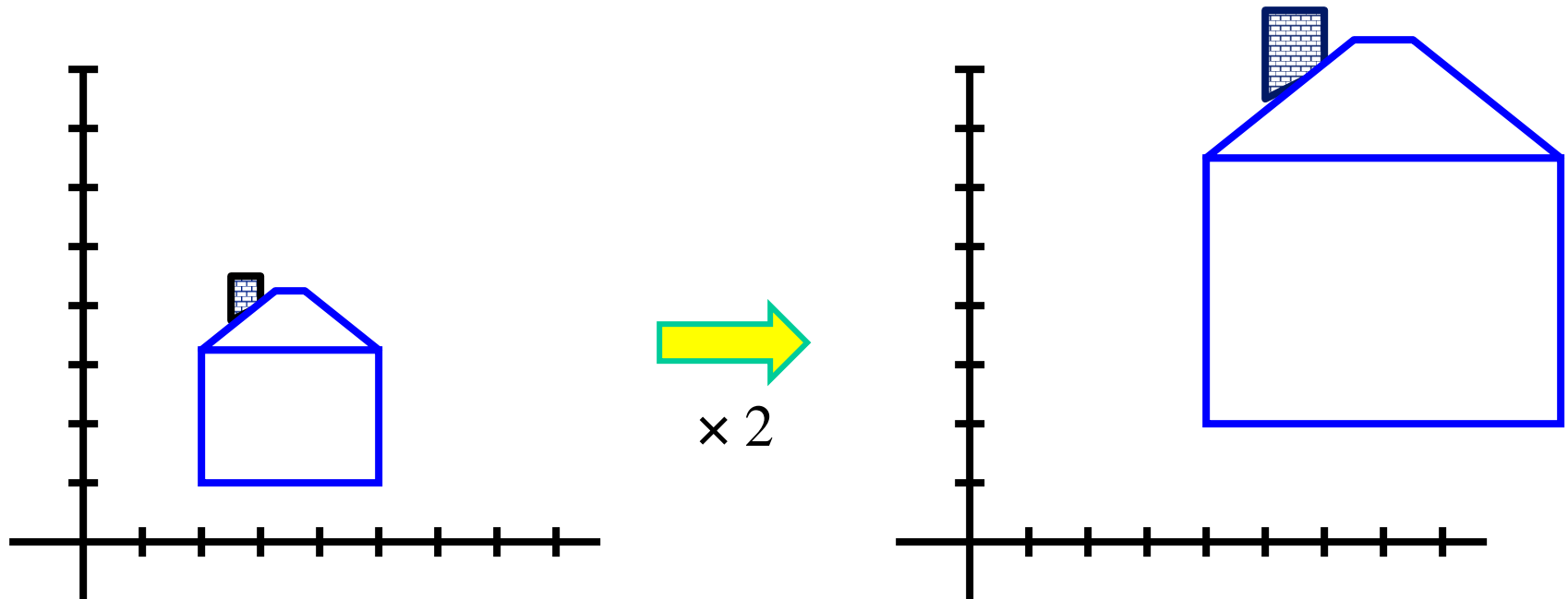


# Scaling

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*Scaling* a coordinate means multiplying each of its components by a scalar

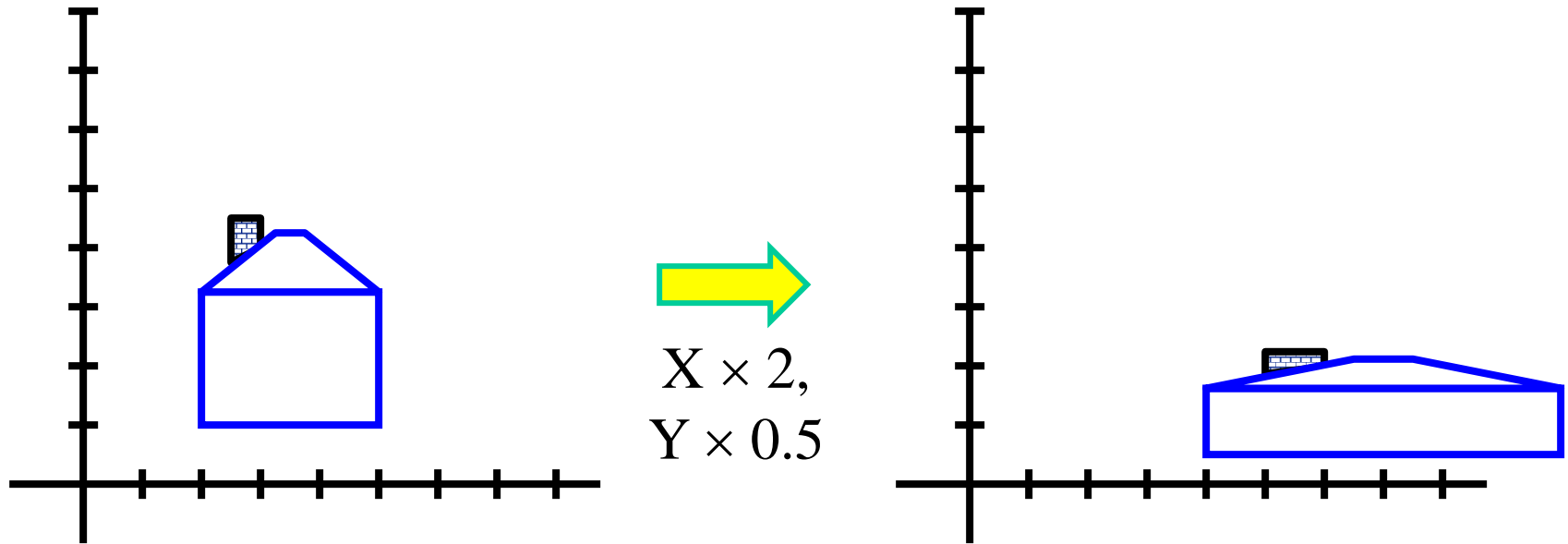
*Uniform scaling* means this scalar is the same for all components:



# Scaling

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*Non-uniform scaling*: different scalars per component:



# Scaling

- **Scaling** is defined by

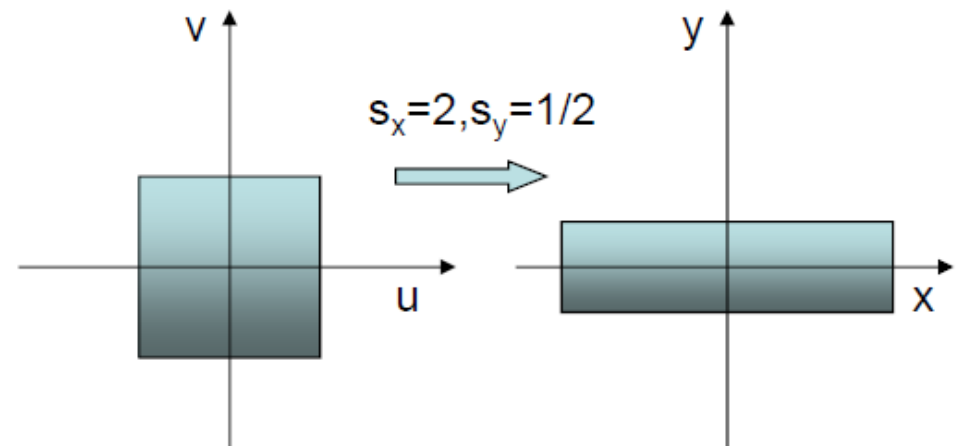
$$\begin{cases} x = s_x u \\ y = s_y v \end{cases} \quad \text{and} \quad \begin{cases} u = x / s_x \\ v = y / s_y \end{cases}$$

- **Matrix notation**

$$\mathbf{x} = \mathbf{S}\mathbf{u}, \quad \mathbf{u} = \mathbf{S}^{-1}\mathbf{x}$$

where

$$\mathbf{S} = \begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix}$$



- If  $s_x < 1$  and  $s_y < 1$ , this represents a minification or **shrinking**, if  $s_x > 1$  and  $s_y > 1$ , it represents a magnification or **zoom**.

# Scaling

---

Scaling operation:

$$x' = ax$$

$$y' = by$$

Or, in matrix form:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \underbrace{\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}}_{\text{scaling matrix } S} \begin{bmatrix} x \\ y \end{bmatrix}$$

What's inverse of S?

# Rotation

- Rotation by an angle of  $\theta$  is defined by

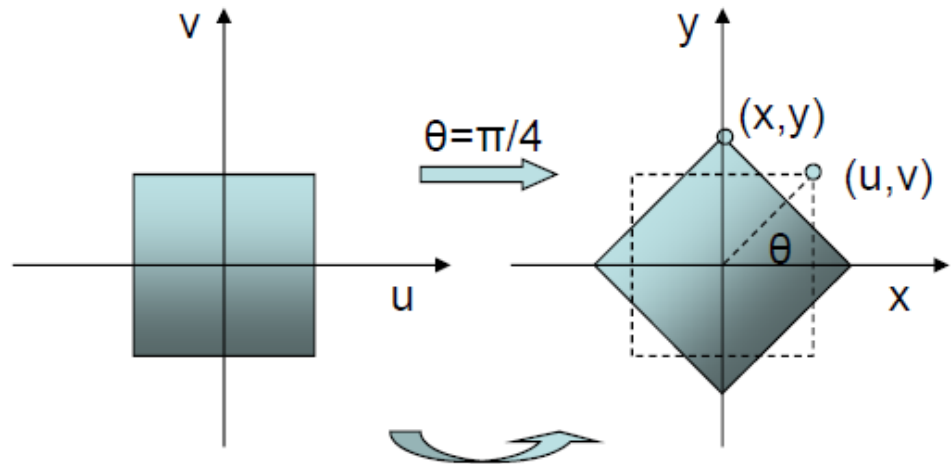
$$\begin{cases} x = u \cos \theta - v \sin \theta \\ y = u \sin \theta + v \cos \theta \end{cases} \quad \text{and} \quad \begin{cases} u = x \cos \theta + y \sin \theta \\ v = -x \sin \theta + y \cos \theta \end{cases}$$

- In matrix format

$$\mathbf{x} = \mathbf{R}\mathbf{u}, \quad \mathbf{u} = \mathbf{R}^T \mathbf{x}$$

where

$$\mathbf{R} = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$$



- $\mathbf{R}$  is a unitary matrix:  $\mathbf{R}^{-1} = \mathbf{R}^T$

B translation



B rotation



Translation:  $x(k, l) = k + 50; y(k, l) = l;$

Rotation:  $x(k, l) = (k - x_0)\cos(\theta) + (l - y_0)\sin(\theta) + x_0;$   
 $y(k, l) = -(k - x_0)\sin(\theta) + (l - y_0)\cos(\theta) + y_0;$

$x_0 = y_0 = 256.5$  the center of the image  $\mathbf{A}$ ,  $\theta = \pi/6$

By Onur Guleyuz

# 2x2 Matrices

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What types of transformations can be represented with a 2x2 matrix?

2D Identity?

$$\begin{aligned}x' &= x \\ y' &= y\end{aligned} \quad \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

2D Scale around (0,0)?

$$\begin{aligned}\mathbf{x}' &= s_x * \mathbf{x} \\ \mathbf{y}' &= s_y * \mathbf{y}\end{aligned} \quad \begin{bmatrix} \mathbf{x}' \\ \mathbf{y}' \end{bmatrix} = \begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix} \begin{bmatrix} \mathbf{x} \\ \mathbf{y} \end{bmatrix}$$



# 2x2 Matrices

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What types of transformations can be represented with a 2x2 matrix?

2D Rotate around (0,0)?

$$\begin{aligned}x' &= \cos \Theta * x - \sin \Theta * y \\y' &= \sin \Theta * x + \cos \Theta * y\end{aligned}$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos \Theta & -\sin \Theta \\ \sin \Theta & \cos \Theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

2D Shear?

$$\begin{aligned}x' &= x + sh_x * y \\y' &= sh_y * x + y\end{aligned}$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & sh_x \\ sh_y & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

# 2x2 Matrices

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What types of transformations can be represented with a 2x2 matrix?

2D Mirror about Y axis?

$$\begin{aligned}x' &= -x \\ y' &= y\end{aligned} \quad \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

2D Mirror over (0,0)?

$$\begin{aligned}x' &= -x \\ y' &= -y\end{aligned} \quad \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

# 2x2 Matrices

---

What types of transformations can be represented with a 2x2 matrix?

2D Translation?

$$\begin{aligned}x' &= x + t_x \\ y' &= y + t_y\end{aligned} \quad \text{NO!}$$

Only linear 2D transformations  
can be represented with a 2x2 matrix

# All 2D Linear Transformations

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Linear transformations are combinations of ...

- Scale,
- Rotation,
- Shear, and
- Mirror

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Properties of linear transformations:

- Origin maps to origin
- Lines map to lines
- Parallel lines remain parallel
- Ratios are preserved
- Closed under composition

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} e & f \\ g & h \end{bmatrix} \begin{bmatrix} i & j \\ k & l \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

# Geometric Transformation

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- A **geometric transformation** refers to a combination of **translation**, **scaling**, and **rotation**, with a general form of

$$\begin{aligned}\mathbf{x} &= \mathbf{RS}(\mathbf{u} + \mathbf{t}) = \mathbf{A}\mathbf{u} + \mathbf{b}, \\ \mathbf{u} &= \mathbf{A}^{-1}(\mathbf{x} - \mathbf{b}) = \mathbf{A}^{-1}\mathbf{x} + \mathbf{c}, \\ \text{with } \mathbf{A} &= \mathbf{RS}, \mathbf{b} = \mathbf{RSt}, \mathbf{c} = -\mathbf{t}.\end{aligned}$$

- Note that interchanging the order of operations will lead to different results.

# Affine Mapping

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- All possible **geometric transformations** are special cases of the *Affine Mapping*:

$$\begin{cases} x = a_0 + a_1u + a_2v \\ y = b_0 + b_1u + b_2v \end{cases} \quad \text{or} \quad \mathbf{x} = \mathbf{A}\mathbf{u} + \mathbf{b}$$

$$\mathbf{A} = \begin{bmatrix} a_1 & a_2 \\ b_1 & b_2 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} a_0 \\ b_0 \end{bmatrix}$$

- When  $\mathbf{A}$  is a orthonormal matrix, it corresponds to a rotation matrix, and the corresponding affine mapping reduces to a geometric mapping.

# Matlab Functions

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- `T = MAKETFORM('affine',U,X)` builds a TFORM struct for a
- two-dimensional affine transformation that maps each row of U
- to the corresponding row of X. U and X are each 3-by-2 and
- define the corners of input and output triangles. The corners
- may not be collinear.
- Example
- -----
- Create an affine transformation that maps the triangle with vertices
- (0,0), (6,3), (-2,5) to the triangle with vertices (-1,-1), (0,-10),
- (4,4):
- 
- `u = [ 0 6 -2]';`
- `v = [ 0 3 5]';`
- `x = [-1 0 4]';`
- `y = [-1 -10 4]';`
- `tform = maketform('affine',[u v],[x y]);`

- 
- $G = \text{MAKETFORM}('affine', T)$  builds a TFORM struct  $G$  for an  $N$ -dimensional affine transformation.  $T$  defines a forward transformation such that  $\text{TFORMFWD}(U, T)$ , where  $U$  is a 1-by- $N$  vector, returns a 1-by- $N$  vector  $X$  such that  $X = U * T(1:N, 1:N) + T(N+1, 1:N)$ .  $T$  has both forward and inverse transformations.  $N=2$  for 2D image transformation

In MATLAB notation

$$T = \begin{bmatrix} a_1 & b_1 & 0 \\ a_2 & b_2 & 0 \\ a_0 & b_0 & 1 \end{bmatrix} = \begin{bmatrix} \mathbf{A}^T & 0 \\ \mathbf{b}^T & 1 \end{bmatrix}$$



- 
- $B = \text{IMTRANSFORM}(A, \text{TFORM}, \text{INTERP})$  transforms the image  $A$  according to the 2-D spatial transformation defined by  $\text{TFORM}$ ;  $\text{INTERP}$  specifies the interpolation filter
  - Example 1
  - -----
  - Apply a horizontal shear to an intensity image.
  - 
  - `I = imread('cameraman.tif');`
  - `tform = maketform('affine',[1 0 0; .5 1 0; 0 0 1]);`
  - `J = imtransform(I,tform);`
  - `figure, imshow(I), figure, imshow(J)`
  - Show in class

# Horizontal Shear Example

---



`tform = maketform('affine',[1 0 0; .5 1 0; 0 0 1]);`  
In MATLAB, 'affine' transform is defined by:  
`[a1,b1,0;a2,b2,0;a0,b0,1]`

With notation used in this lecture note

$$\mathbf{A} = \begin{bmatrix} 1 & 0.5 \\ 0 & 1 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Note in this example, first coordinate indicates horizontal position, second coordinate indicate vertic

# MATLAB function for image warping

---

- `B = IMTRANSFORM(A,TFORM, INTERP)` transforms the image A according to the 2-D spatial transformation defined by TFORM
- INTERP specifies the interpolation filter
- Example 1
  - -----
  - Apply a horizontal shear to an intensity image.
  - 
  - `I = imread('cameraman.tif');`
  - `tform = maketform('affine',[1 0 0; .5 1 0; 0 0 1]);`
  - `J = imtransform(I,tform);`
  - `figure, imshow(I), figure, imshow(J)`

# Horizontal Shear Example

---



`tform = maketform('affine',[1 0 0; .5 1 0; 0 0 1]);`  
In MATLAB, 'affine' transform is defined by:  
`[a1,b1,0;a2,b2,0;a0,b0,1]`

With notation used in this lecture note

$$\mathbf{A} = \begin{bmatrix} 1 & 0.5 \\ 0 & 1 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Note in this example,  $x, u$  indicates vertical position,  $y, v$  indicate horizontal position

# Example of Image Warping (1)

---

WAVE1



WAVE2



wave1: $x(u,v)=u+20\sin(2\pi v/128)$ ;  $y(u,v)=v$ ;  
wave2: $x(u,v)=u+20\sin(2\pi u/30)$ ;  $y(u,v)=v$ .

By Onur Guleyuz

# Example of Image Warping (2)

WARP



SWIRL



WARP  $x(u, v) = \text{sign}(u - x_0) * (u - x_0)^2 / x_0 + x_0; y(u, v) = v$

SWIRL  $x(u, v) = (u - x_0) \cos(\theta) + (v - y_0) \sin(\theta) + x_0;$   
 $y(u, v) = -(u - x_0) \sin(\theta) + (v - y_0) \cos(\theta) + y_0;$   
 $r = ((u - x_0)^2 + (v - y_0)^2)^{1/2}, \theta = \pi r / 512.$

By Onur Guleyuz

# Homogeneous Coordinates

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**Q: How can we represent translation as a 3x3 matrix?**

$$x' = x + t_x$$

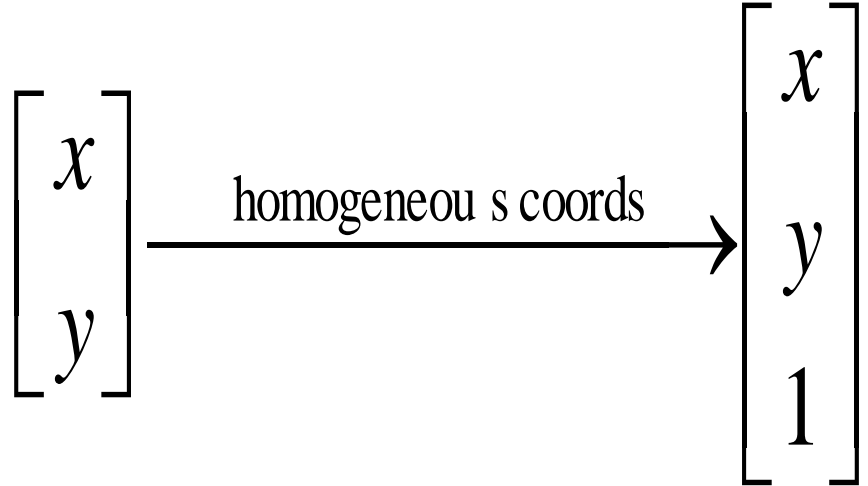
$$y' = y + t_y$$

# Homogeneous Coordinates

---

## *Homogeneous coordinates*

- represent coordinates in 2 dimensions with a 3-vector



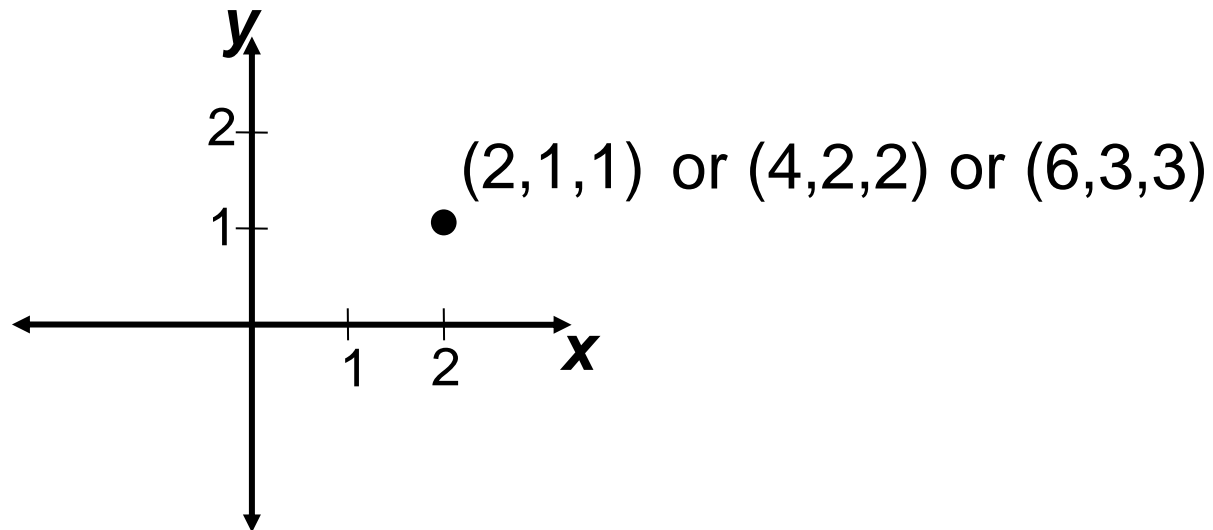


# Homogeneous Coordinates

---

Add a 3rd coordinate to every 2D point

- $(x, y, w)$  represents a point at location  $(x/w, y/w)$
- $(x, y, 0)$  represents a point at infinity
- $(0, 0, 0)$  is not allowed



Convenient  
coordinate system to  
represent many  
useful  
transformations

# Homogeneous Coordinates

---

**Q: How can we represent translation as a 3x3 matrix?**

$$x' = x + t_x$$

$$y' = y + t_y$$

**A: Using the rightmost column:**

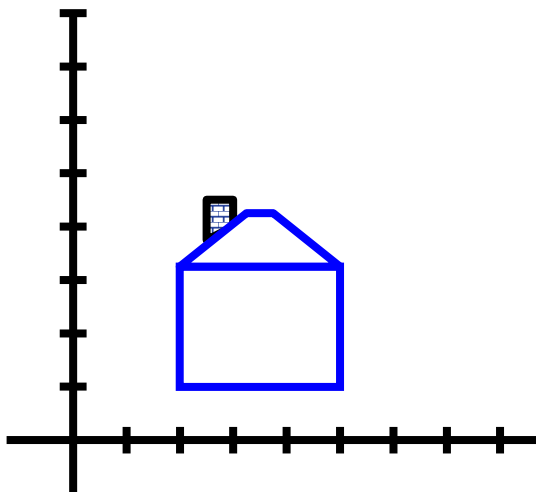
$$\mathbf{Translation} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix}$$

# Translation

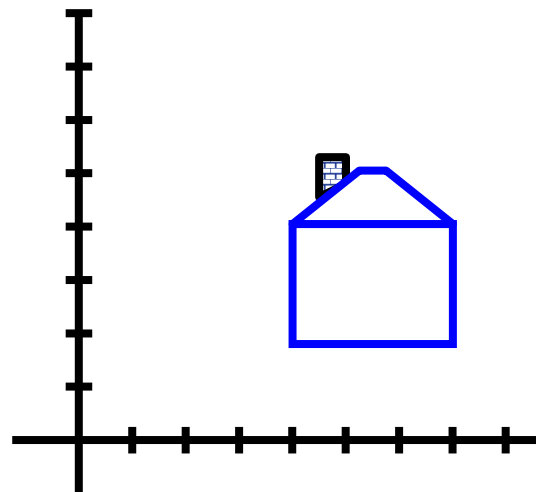
Example of translation

Homogeneous Coordinates

$$\begin{matrix} \downarrow & & \downarrow & & \downarrow \\ \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x+t_x \\ y+t_y \\ 1 \end{bmatrix} \end{matrix}$$



$$\begin{aligned} t_x &= 2 \\ t_y &= 1 \end{aligned}$$



# Basic 2D Transformations

---

## Basic 2D transformations as 3x3 matrices

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Translate

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Scale

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos \Theta & -\sin \Theta & 0 \\ \sin \Theta & \cos \Theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Rotate

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & sh_x & 0 \\ sh_y & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Shear

# Matrix Composition

---

Transformations can be combined by matrix multiplication

$$\begin{bmatrix} x' \\ y' \\ w' \end{bmatrix} = \left( \begin{bmatrix} 1 & 0 & tx \\ 0 & 1 & ty \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos \Theta & -\sin \Theta & 0 \\ \sin \Theta & \cos \Theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} sx & 0 & 0 \\ 0 & sy & 0 \\ 0 & 0 & 1 \end{bmatrix} \right) \begin{bmatrix} x \\ y \\ w \end{bmatrix}$$

$\mathbf{p}' = \mathbf{T}(t_x, t_y) \mathbf{R}(\Theta) \mathbf{S}(s_x, s_y) \mathbf{p}$

# Affine Transformations

---

Affine transformations are combinations of ...

- Linear transformations, and
- Translations

$$\begin{bmatrix} x' \\ y' \\ w \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ w \end{bmatrix}$$

Properties of affine transformations:

- Origin does not necessarily map to origin
- Lines map to lines
- Parallel lines remain parallel
- Ratios are preserved
- Closed under composition
- Models change of basis

Will the last coordinate  $w$  always be 1?

# Projective Transformations

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Projective transformations ...

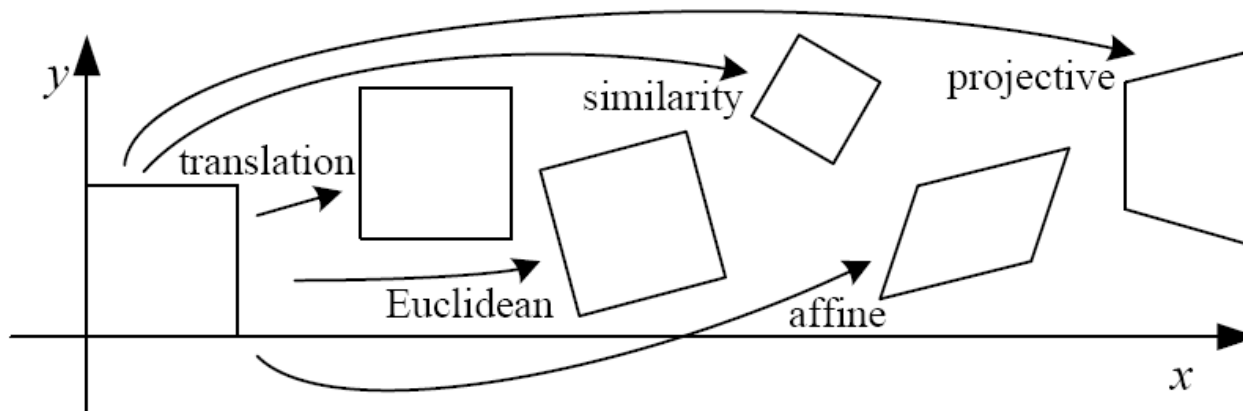
- Affine transformations, and
- Projective warps

$$\begin{bmatrix} x' \\ y' \\ w' \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} x \\ y \\ w \end{bmatrix}$$

Properties of projective transformations:

- Origin does not necessarily map to origin
- Lines map to lines
- Parallel lines do not necessarily remain parallel
- Ratios are not preserved
- Closed under composition
- Models change of basis

# 2D image transformations



Name	Matrix	# D.O.F.	Preserves:	Icon
translation	$\begin{bmatrix} \mathbf{I} & \mathbf{t} \end{bmatrix}_{2 \times 3}$			
rigid (Euclidean)	$\begin{bmatrix} \mathbf{R} & \mathbf{t} \end{bmatrix}_{2 \times 3}$			
similarity	$\begin{bmatrix} s\mathbf{R} & \mathbf{t} \end{bmatrix}_{2 \times 3}$			
affine	$\begin{bmatrix} \mathbf{A} \end{bmatrix}_{2 \times 3}$			
projective	$\begin{bmatrix} \tilde{\mathbf{H}} \end{bmatrix}_{3 \times 3}$			

These transformations are a nested set of groups

- Closed under composition and inverse is a member



# Image Morphing

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- Image morphing has been widely used in movies and commercials to create special visual effects. For example, changing a beauty gradually into a monster.
- The fundamental techniques behind image morphing is image warping.
- Let the original image be  $f(\mathbf{u})$  and the final image be  $g(\mathbf{x})$ . In image warping, we create  $g(\mathbf{x})$  from  $f(\mathbf{u})$  by changing its shape. In image morphing, we use a combination of both  $f(\mathbf{u})$  and  $g(\mathbf{x})$  to create a series of intermediate images.



$f(u)$



$g(x)$

# Examples of Image Morphing

Cross  
Dissolve

$$I(t) = (1-t)*S+t*T$$



Mesh  
based



*George Wolberg, "Recent Advances in Image Morphing",  
Computer Graphics Intl. '96, Pohang, Korea, June 1996.*

# Image Morphing Method

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- Suppose the mapping function between the two end images is given as  $\mathbf{x}=\mathbf{u}+\mathbf{d}(\mathbf{u})$ .  $\mathbf{d}(\mathbf{u})$  is the displacement between corresponding points in these two images.
- In image morphing, we create a series of images, starting with  $f(\mathbf{u})$  at  $k=0$ , and ending at  $g(\mathbf{x})$  at  $k=K$ . The intermediate images are a linear combination of the two end images:

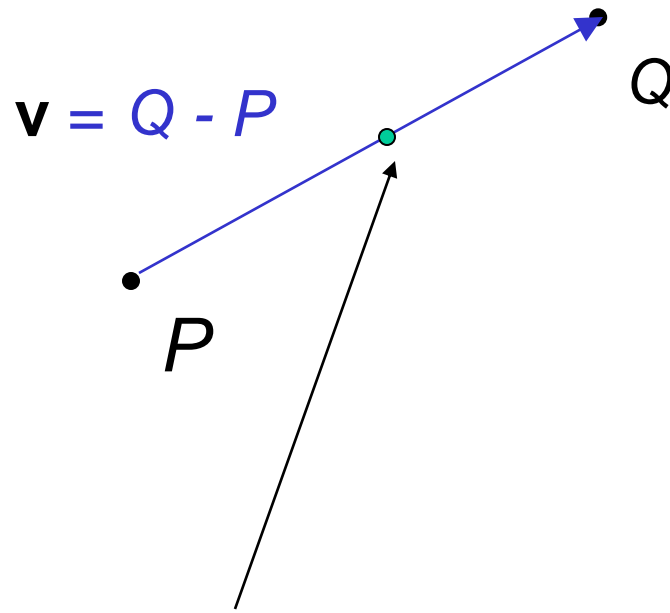
$$h_k(\mathbf{u} + s_k \mathbf{d}) = (1 - s_k) f(\mathbf{u}) + s_k g(\mathbf{u} + \mathbf{d}(\mathbf{u})), \quad k = 0, 1, \dots, K,$$

where  $s_k = k / K$ .

# Linear Interpolation

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How can we linearly transition between point  $P$  and point  $Q$ ?



$$P + t\mathbf{v} \\ = (1-t)P + tQ, \quad \text{e.g. } t = 0.5$$

$P$  and  $Q$  can be anything:

- points on a plane (2D) or in space (3D)
- Colors in RGB or HSV (3D)
- Whole images (m-by-n D)... etc.

# Idea #1: Cross-Dissolve

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Interpolate whole images:

$$\text{Image}_{\text{halfway}} = (1-t) \cdot \text{Image}_1 + t \cdot \text{Image}_2$$

This is called **cross-dissolve** in film industry

But what if the images are not aligned?

# Idea #2: Align, then cross-dissolve

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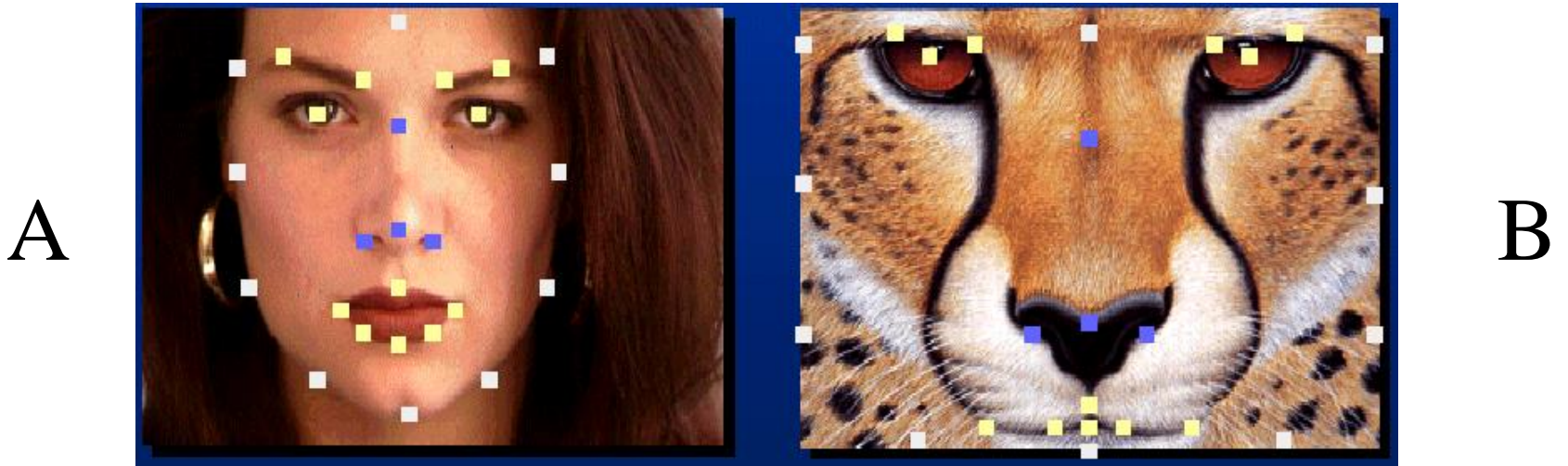


Align first, then cross-dissolve

- Alignment using global warp – picture still valid

# Full Morphing

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- What if there is no simple global function that aligns two images?
- User specifies corresponding feature points
- Construct warp animations  $A \rightarrow B$  and  $B \rightarrow A$
- Cross dissolve these



# Full Morphing

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# Full Morphing

Image A



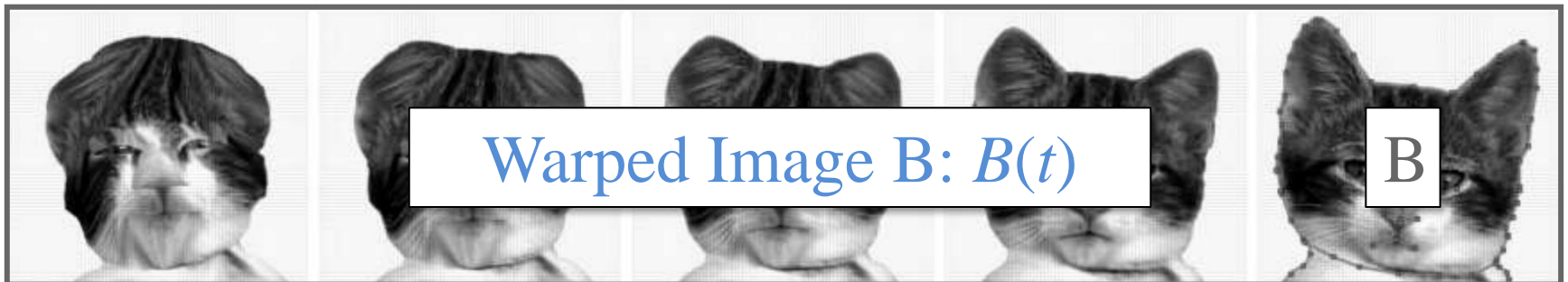
Image B



1. Find warping fields from user constraints (points or lines):
  - Warp field  $T_{AB}(x, y)$  that maps A pixel to B pixel
  - Warp field  $T_{BA}(x, y)$  that maps B pixel to A pixel
2. Make video  $A(t)$  that warps A over time to the shape of B
  - Start warp field at identity and linearly interpolate to  $T_{BA}$
  - Construct video  $B(t)$  that warps B over time to shape of A
3. Cross dissolve these two videos.

# Full Morphing

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# Catman!

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# Conclusion

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Illustrates general principle in graphics:

- First register, then blend

Avoids ghosting

[Michael Jackson - Black or White](#)