Eigenvalues and Timbre: Vibration Analysis of Acoustic Guitars

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***Abstract*—This study investigates the relationship between eigenvalues and timbral characteristics, specifically richness and brightness, in acoustic guitars through comprehensive vibration analysis. The research examines how the distribution and magnitude of eigenvalues in guitar strings and body systems correlate with perceived tonal qualities. The study reveals that the coupling between string modes and body resonances, characterized by their respective eigenvalues, plays a crucial role in determining timbral richness. This research provides valuable insights for luthiers and acoustic engineers, offering a mathematical framework for understanding and optimizing guitar design parameters to achieve desired tonal characteristics.**

***Keywords*— eigenvalues, guitar acoustics, timbre analysis, modal coupling, spectral content, vibration modes**

I. Introduction

The acoustic guitar is one of the most common musical instrument found in the world. The acoustic guitar phenomenon has been around for a long time and is recorded as history. This happened not without reason. The ease of use of the acoustic guitar is a significant factor in its popularity. Apart from that, the layers of tone color or timbre produced by each passage are unique to an acoustic guitar. The difference in timbre between one musical instrument and another is a very beautiful characteristic and is important to understand scientifically.

One interesting, but rarely explored, application of eigenvalue theory is its use in analyzing the sound characteristics of musical instruments, in this case, the acoustic guitar. Eigenvalue theory itself is a fundamental concept studied in Linear Algebra and Geometry courses. The vibrations of the strings on an acoustic guitar can be modeled using partial differential equations, which produces a matrix system with eigenvalues that correspond directly to the frequency of the note.

This research aims to explore the relationship between the eigenvalues of a matrix system that represents an acoustic guitar and the resulting tone color characteristics. The significance of this research lies in its contribution in building a bridge between abstract mathematical theories and measurable physical phenomena. A deep understanding of the relationship between the spectral structure of mathematical operators and acoustic characteristics can open new avenues in the design of musical instruments based on mathematical models.

II. Theoretical Basis

A. String Vibration

String vibration is a fundamental phenomenon in musical acoustics that can be explained by the principles of mathematical physics. In an acoustic guitar string, vibration occurs when the string is plucked or disturbed from its equilibrium position. Mathematically, string vibration can be modeled using a one-dimensional wave equation, which is expressed as

$$\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2},$$

where y is the string deflection, t is time, x is the position along the string, and c is the wave velocity on the string.

In the context of an acoustic guitar, the string is attached at both ends (bridge and nut), which provides the boundary conditions

$$y(0,t) = y(L,t) = 0,$$

in where L is the length of the string. This boundary condition determines the possible modes of vibration of the string. The solution of the wave equation with this boundary condition can be obtained using the method of separation of variables, which yields a function of the form

$$y(x,t) = X(x)T(t).$$

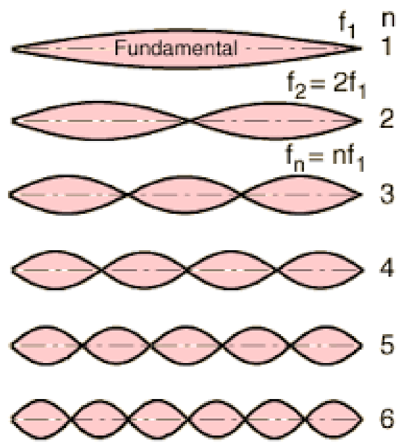

Figure 1. Standing waves on a string
Source: http://hyperphysics.phy-astr.gsu.edu/

The frequency of vibration of a string depends on several physical parameters, including the length of the string (L), the tension in the string (T), and the mass per unit length of the string (μ). The fundamental frequency of a string is given by the equation

$$f_1 = \frac{1}{2} L \sqrt{\frac{\mathrm{T}}{\mu}},$$

Whilst the n-th harmonic frequency of the string is given as $f_n = nf_1$.

These harmonics plays the part in forming characteristics of each instrument.

B. Eigenvalues

Eigenvalues and eigenvectors represent key mathematical properties that help understand how matrices and linear transformations behave. These concepts are essential for analyzing how vectors change within vector spaces during transformations like rotation, compression, and stretching.

Eigenvalues, which are scalar quantities, reveal the magnitude of scaling that occurs along specific directions during a linear transformation. They effectively measure how much a vector's length changes under transformation. The relationship between eigenvalues and eigenvectors is expressed through the fundamental equation

$$Av = \lambda v,$$

where A represents the matrix, v is the eigenvector, and λ denotes the eigenvalue. In this context, the eigenvalue acts as a scaling coefficient that determines how much stretching or compression occurs.

Eigenvectors are special non-zero vectors that maintain their original direction when subjected to the linear transformation described by the matrix. They identify the unique directions where the transformation results in pure scaling, without any directional change. When different eigenvalues have corresponding eigenvectors, these eigenvectors are linearly independent and form a fundamental basis for understanding the vector space.

C. Timbre

Timbre is the characteristic that distinguishes the sound of one instrument from another, even when playing the same note. The timbre is determined by the distribution and relative amplitude of the harmonics present in the sound. In the context of an acoustic guitar, timbre is influenced by a variety of factors including string material, body construction, and environmental conditions.

Timbre can be categorized into several characteristics, the most common of which are richness and brightness, used to describe the uniqueness of a musical instrument's sound.

Spectral analysis is a very important method in understanding the timbre characteristics of an acoustic guitar. The Fourier transform allows the decomposition of a complex signal into its constituent harmonic components. In practice, the Fast Fourier Transform (FFT) is used to efficiently analyze the frequency spectrum of a guitar sound.

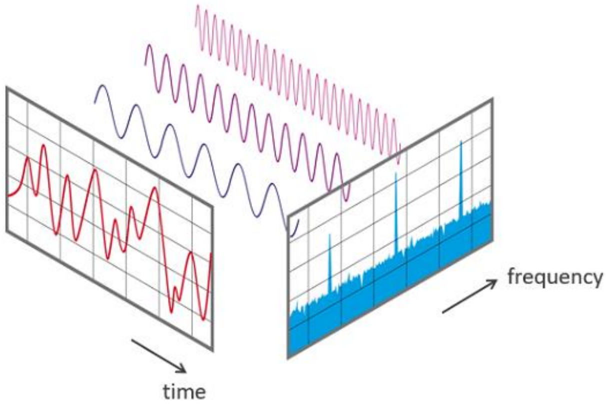

Figure 2. Fast Fourier Transform
Source: https://www.nti-audio.com/en/support/know-how/fast-fourier-transform-fft

D. Eigenvalues and Acoustic Characteristics

There is a correlation between the eigenvalue spectrum of a guitar system and its acoustic characteristics. The distribution of eigenvalues affects the frequency response of the system, which in turn determines the characteristics of the resulting tone color. The eigenvalues obtained from system analysis can be used to predict the resonance frequencies and vibration patterns that will occur.

The eigenvalues of the guitar have a direct relationship to the richness of the sound through the distribution of vibration modes that occur. When a guitar is played, the various eigenvalues produce resonances at different frequencies. The more eigenvalues that are excited in the frequency range, the higher the complexity of the sound produced. This creates richness because of the formation of a more complete overtone series.

The brightness of the guitar sound is correlated with the dominance of eigenvalues at high frequencies. Larger eigenvalues tend to produce vibration modes at higher frequencies. When acoustic energy is more concentrated in the high-frequency spectrum, a brighter sound character is produced.

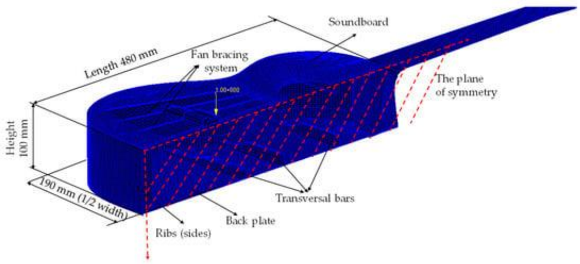

Figure 3. The structural symmetry of guitar body
Source: https://www.mdpi.com/2073-8994/12/5/795

The material and geometry of the guitar affect the distribution of eigenvalues. Changes in these physical parameters will change the eigenvalue spectrum, which is reflected in changes in the acoustic characteristics of the instrument.

III. Methodology

A. Research Design

This study uses a quantitative approach with an experimental method to analyze the relationship between eigenvalues and timbre characteristics of acoustic guitars. The research design involves systematic measurements and analysis of acoustic and mathematical parameters of several acoustic guitar samples. This approach was chosen to allow for an in-depth analysis of the relationship between the mathematical structure (eigenvalues) and physical characteristics (timbre) of the instrument.

B. Sampling

Two acoustic guitars with different specifications were used to obtain adequate data variation. The guitars were selected based on differences in string material (steel and nylon), and body size (1/2 and full). Each guitar was fitted with strings with identical brand specifications to eliminate uncontrolled variables.

The measuring equipment used included an iPhone 11 microphone with digital signal processing software. Recording was carried out in a 3x4 meter non-soundproof room.

C. Data Collection

The data collection process was carried out systematically with a standardized protocol. The microphone was placed at a distance of 10 cm from the guitar sound hole, at a 45-degree angle to the surface of the guitar. Each string was plucked with a standard plectrum at the same position (12 cm from the bridge) to maintain consistency of excitation.

For each string, recordings were made on all open strings (E, A, D, G, B, E) with a duration of 5 seconds per recording. Each measurement was repeated three times to ensure data reliability. The total data recorded was 12 sound samples (2 guitars × 2 strings × 3 repetitions). All recordings were done at room temperature (25°C) to maintain consistency of measurement conditions.

The sound recording results were stored in the form of wav files obtained by converting mp4a files from Voice Memo Software on Iphone 11 via website https://www.freeconvert.com/m4a-to-wav/ then processed with a Python programming language program.

D. Data Analysis

Data analysis is done in several stages using Python 3 software. The first stage is audio processing from wav files. The process_audio_file function reads the wav file and returns the normalized data and the sample rate of the data.

```
import numpy as np
from scipy.io import wavfile
def process_audio_file(filename):
    sample_rate, data = wavfile.read(filename)
    normalized_data = data / np.max(np.abs(data))
    window = np.hamming(len(data))
    windowed_data = normalized_data * window
return windowed_data, sample_rate
```

Fast Fourier Transform (FFT) is applied to each frame to obtain the frequency spectrum. In the program, FFT is performed using the built-in Scipy library. From this spectrum, information about the fundamental frequency, harmonic amplitudes, and spectral envelope is extracted. This spectral data is then used to construct a system matrix whose eigenvalues will be analyzed.

The get_harmonic_ratios() function is designed to analyze the harmonic composition of an audio signal using Fast Fourier Transform (FFT) data. The way this function works starts by finding the fundamental frequency index in the frequency array using np.argmin(np.abs(frequencies - fundamental_freq)), then calculating its fundamental amplitude. After that, the function iterates for each harmonic from the 2nd to the nth harmonic, where each harmonic has a frequency that is a multiple of the fundamental frequency. For each harmonic, the function calculates the ratio of its amplitude to the fundamental amplitude, which provides information about the relative strength of each harmonic in the signal.

Then, an analysis of the high-frequency content in the audio signal is carried out. The function accepts three parameters: the FFT data of the signal, the corresponding frequency array, and a cutoff frequency that defines the boundary between low and high frequencies (default 1000 Hz). The final result returned is the ratio of high frequency energy to total energy, which gives an indication of how much high frequencies contribute to the overall signal.

```
from scipy.fft import fft, fftfreq
import numpy as np

def spectral_matrix(fft_data, size=100):
    magnitude = np.abs(fft_data[:size])
    magnitude = magnitude / np.max(magnitude)

    matrix = np.zeros((size, size))
    for i in range(size):
        for j in range(size):
            matrix[i,j] = magnitude[abs(i-j)]
    matrix += np.eye(size) * 1e-10
    return matrix

def get_harmonic_ratios(fft_data, frequencies,
fundamental_freq, num_harmonics=8):
    harmonic_ratios = []
    fundamental_idx = np.argmin(np.abs(frequencies -
    fundamental_freq))
    fundamental_amp = np.abs(fft_data[fundamental_idx])
    for n in range(2, num_harmonics + 2):
        harmonic_freq = n * fundamental_freq
        harmonic_idx = np.argmin(np.abs(frequencies -
        harmonic_freq))
        harmonic_amp = np.abs(fft_data[harmonic_idx])
        ratio = harmonic_amp / fundamental_amp
        harmonic_ratios.append(ratio)
    return np.array(harmonic_ratios)

def get_high_frequency_content(fft_data, frequencies,
cutoff_freq=1000):
    magnitudes = np.abs(fft_data)
    high_freq_mask = frequencies > cutoff_freq
    total_energy = np.sum(magnitudes**2)
    high_freq_energy =
    np.sum(magnitudes[high_freq_mask]**2)

    high_freq_content = high_freq_energy / total_energy

    return high_freq_content
```

The program then analyzes the timbre of the pre-processed audio signal. Based on the parameters obtained from harmonic_ratios, high_frequency_content and spectral_matrix, the function calculates more interpretable timbre characteristics in the timbre_characteristics dictionary. These characteristics include 'brightness' calculated using the calculate_brightness function based on the spectral centroid and high-frequency content, and 'richness' calculated as the average of the harmonic ratios. Finally, the function returns a dictionary containing both sets of information: the raw parameters in 'parameters' and the interpreted timbre characteristics in 'characteristics'.

```
def calculate_brightness(spectral_centroid,
high_freq_content):
    normalized_centroid = np.clip(abs(spectral_centroid) /
      5000, 0, 1)
    normalized_high_freq = np.clip(abs(high_freq_content),
      0, 1)
    brightness = 0.6 * normalized_centroid + 0.4 *
      normalized_high_freq
    return np.clip(brightness, 0, 1)

def analyze_timbre(signal, sample_rate):
    fft_data = fft(signal)
    frequencies = fftfreq(len(signal), 1/sample_rate)

    magnitudes = np.abs(fft_data)
    fundamental_idx = np.argmax(magnitudes[1:]) + 1
    fundamental_freq = frequencies[fundamental_idx]

    params = {
        'harmonic_ratios': get_harmonic_ratios(fft_data,
         frequencies, fundamental_freq),
        'high_frequency_content':
         get_high_frequency_content(fft_data,
         frequencies),
        'spectral_centroid': np.sum(frequencies *
         magnitudes) / np.sum(magnitudes)
    }

    timbre_characteristics = {
        'brightness': calculate_brightness(
            params['spectral_centroid'],
            params['high_frequency_content']
        ),
        'richness': np.mean(params['harmonic_ratios'])
    }

    return {
        'parameters': params,
        'characteristics': timbre_characteristics
    }
```

E. Eigenvalue Calculation

The eigenvalues are calculated using the np.linalg.eig() function implemented in the NumPy library. This function implements the concept of linear algebra where for a matrix A, the eigenvalues λ and eigenvectors v satisfy the equation $Av = \lambda v$. In the context of sound analysis, these eigenvalues describe how much a linear transformation stretches or compresses the vector space of the audio signal.

The system matrix constructed from the spectral data is analyzed to obtain the complete eigenvalue spectrum. This process produces a set of eigenvalues that represent the dynamic characteristics of the guitar system.

After obtaining the eigenvalues, the program sorts the values from largest to smallest. This sorting is important because larger eigenvalues indicate more dominant components in the signal. Along with the sorting of the eigenvalues, the corresponding eigenvectors are also sorted to maintain the relationship between the two.

The program then extracts several important features from the sorted eigenvalues. The first feature is the five largest eigenvalues that represent the main components of the signal. Second, the program calculates the ratio between the largest eigenvalue and the total number of all eigenvalues, which shows how dominant the principal component is. Third, the program calculates the distribution of eigenvalues through the ratio between the standard deviation and the mean of the eigenvalues.

```
def analyze_audio_with_eigenvalues(filename):
   signal, sample_rate = process_audio_file(filename)
   fft_data = fft(signal)
   spec_matrix = spectral_matrix(fft_data)
   eigenvalues, eigenvectors = np.linalg.eig(spec_matrix)

   sorted_indices = np.argsort(eigenvalues)[::-1]
   eigenvalues = eigenvalues[sorted_indices]
   eigenvectors = eigenvectors[:, sorted_indices]

   timbre_results = analyze_timbre(signal, sample_rate)
   eigenvalue_features = {
       'dominant_eigenvalues': eigenvalues[:5],
       'eigenvalue_ratio': eigenvalues[0] /
       np.sum(eigenvalues),
       'eigenvalue_spread': np.std(eigenvalues) /
       np.mean(eigenvalues)
   }

       timbre_eigen_correlation =
       correlate_eigen_timbre(eigenvalue_features,
       timbre_results)
   results =  {
       'timbre_analysis': timbre_results,
       'eigenvalues': eigenvalue_features,
       'correlation': timbre_eigen_correlation
   }

   vis.visualize_analysis(results, filename)
   print(results)
```

F. Correlation and Interpretation

Next, the program correlates the eigenvalue features with the timbre analysis results and visualizes the results. There are three functions that work to analyze the correlation between the eigenvalue characteristics of a signal and its timbre characteristics. The first function calculates the correlation between the distribution of eigenvalues and the brightness level of the sound. This function normalizes the distribution of eigenvalues by dividing it by the maximum value and ensuring that the value is in the range of 0 to 1, then compares it with the brightness value that has also been normalized. The correlation is calculated as the complement of the absolute value of the difference between the two normalized values, so that closer values will produce a higher correlation.

The second function calculates the correlation between the ratio of eigenvalues and the richness level of the timbre. This function uses a slightly different approach by calculating the inverse of the ratio of eigenvalues (1 - ratio) before comparing it with the richness value. Like the previous function, the correlation is calculated as the complement of the absolute value of the difference between the two values. This approach allows a direct comparison between the structural characteristics represented by the eigenvalues and the characteristics of the perceived timbre.

The main function integrates the two previous functions to produce a comprehensive correlation analysis. This function accepts two parameters: eigenfeatures containing information about the distribution and ratio of eigenvalues, and timbre analysis results containing brightness and richness characteristics. This function returns a dictionary containing two correlation values: brightness correlation and timbre richness correlation. These results can be used to understand how strong the relationship is between the structural characteristics represented by the eigenvalues and the audible timbre characteristics, providing insight into how the mathematical properties of the vibratory system relate to auditory perception.

```
def calculate_brightness_correlation(eigenvalue_spread,
brightness):
    norm_spread = np.clip(eigenvalue_spread /
    np.max(eigenvalue_spread), 0, 1)
    norm_brightness = np.clip(brightness, 0, 1)
    correlation = 1 - abs(norm_spread - norm_brightness)
    return correlation

def calculate_richness_correlation(eigenvalue_ratio,
richness):
   inverse_ratio = 1 - eigenvalue_ratio
   correlation = 1 - abs(inverse_ratio - richness)
   return correlation

def correlate_eigen_timbre(eigen_features,timbre_results):
   correlations = {
       'brightness_correlation':
      calculate_brightness_correlation(
          eigen_features['eigenvalue_spread'],
          timbre_results['characteristics']['brightness']
      ),
       'richness_correlation':
      calculate_richness_correlation(
          eigen_features['eigenvalue_ratio'],
          timbre_results['characteristics']['richness']
      )
   }
   return correlations
```

G. Visualization

To accomodate reading the results of the data analysis, a Python program was created that uses the Matplotlib library to visualize the correlation between eigenvalues and guitar timbre characteristics.

```
import matplotlib.pyplot as plt
import numpy as np
def plot_complete_analysis(results):
    brightness_corr =
results['correlation']['brightness_correlation']
    richness_corr =
results['correlation']['richness_correlation']
    eigenvalues =
results['eigenvalues']['dominant_eigenvalues'][:5]
    brightness_char =
results['timbre_analysis']['characteristics']['brightnes
s']
    richness_char =
results['timbre_analysis']['characteristics']['richness'
]
    fig, (ax1, ax2, ax3) = plt.subplots(1, 3,
figsize=(20, 6))
    brightness_bar = ax1.bar(['Brightness'],
[brightness_corr],
                           color='g' if brightness_corr
>= 0 else 'r')
    ax1.set_title('Brightness Correlation', fontsize=14,
pad=20)
    ax1.grid(True, axis='y', linestyle='--', alpha=0.7)
    ax1.axhline(y=0, color='black', linestyle='-',
alpha=0.3)
    ax1.text(0, brightness_corr,
             f'Correlation:
{brightness_corr:.4f}\nCharacteristic:
{brightness_char:.4f}',
             ha='center',
             va='bottom' if brightness_corr >= 0 else
'top',
             bbox=dict(facecolor='white', alpha=0.8,
edgecolor='none'))
    brightness_margin = abs(brightness_corr) * 0.1
    ax1.set_ylim(min(0, brightness_corr -
brightness_margin),
                 max(0, brightness_corr +
brightness_margin))
    ax1.set_ylabel('Correlation Value')
    richness_bar = ax2.bar(['Richness'],
[richness_corr],
                          color='g' if richness_corr >=
0 else 'r')
    ax2.set_title('Richness Correlation', fontsize=14,
pad=20)
    ax2.grid(True, axis='y', linestyle='--', alpha=0.7)
    ax2.axhline(y=0, color='black', linestyle='-',
alpha=0.3)
    ax2.text(0, richness_corr,
             f'Correlation:
{richness_corr:.4f}\nCharacteristic:
{richness_char:.4f}',
             ha='center',
             va='bottom' if richness_corr >= 0 else
'top',
```

```
            bbox=dict(facecolor='white', alpha=0.8,
edgecolor='none'))
        richness_margin = abs(richness_corr) * 0.2
        ax2.set_ylim(min(0, richness_corr - richness_margin),
                     max(0, richness_corr + richness_margin))
        ax2.set_ylabel('Correlation Value')
        indices = range(1, 6)
        ax3.plot(indices, eigenvalues, 'b-o', linewidth=2,
markersize=8)
        ax3.set_title('Top 5 Eigenvalues', fontsize=14,
pad=20)
        ax3.grid(True, linestyle='--', alpha=0.7)
    ax3.set_xlabel('Eigenvalue Index')
    ax3.set_ylabel('Value')
    for idx, value in enumerate(eigenvalues):
        ax3.text(idx + 1, value, f'{value:.2f}',
                ha='center', va='bottom')
    ax3.set_xticks(indices)
    plt.setp(brightness_bar, visible=True)
    plt.setp(richness_bar, visible=True)
    plt.tight_layout()

    return fig

def visualize_analysis(results, filename):
    fig = plot_complete_analysis(results)
    output_filename = f"{filename.rsplit('.',
1)[0]}_analysis.png"
    fig.savefig(output_filename, dpi=300,
bbox_inches='tight')
    plt.close()
    print(f"Analysis visualization saved as:
{output_filename}")

    return output_filename
```

IV. RESULTS

The data used in this study was obtained from 4 types of samples according to the following table, where each sample has a different combination of string materials and guitar sizes.

Sample Number	String Material	Guitar Size
1	Nylon	½
2	Steel	½
3	Steel	1
4	Nylon	1

Data 1

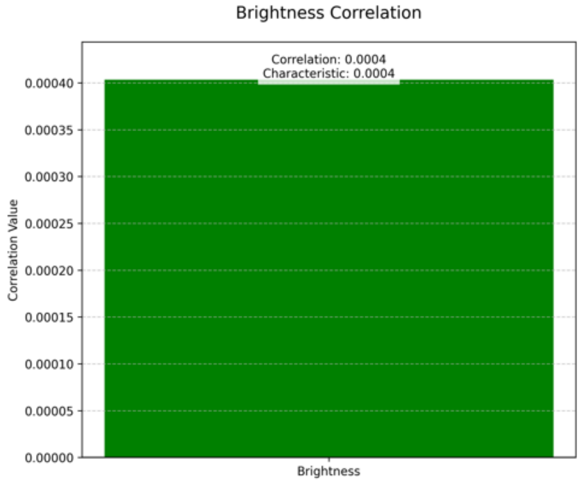

Figure 4. Data 1 brightness correlation

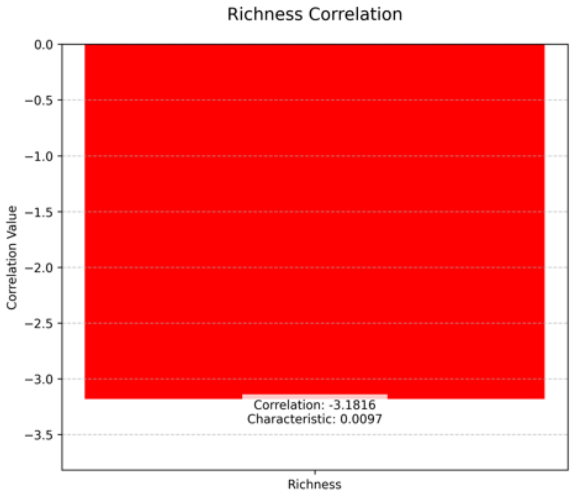

Figure 5. Data 1 richness correlation

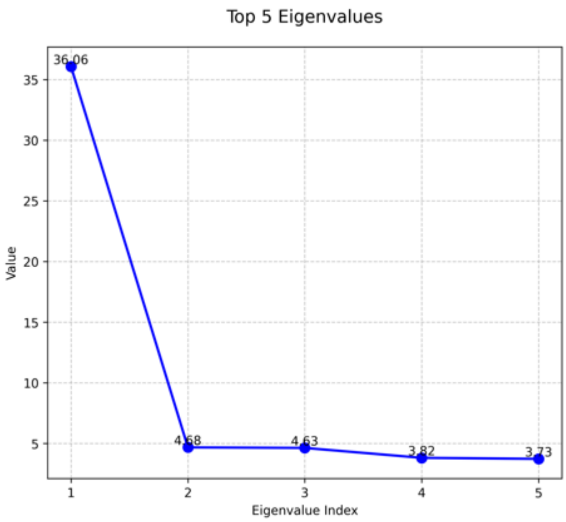

Figure 6. Data 1 eigenvalues

Data 2

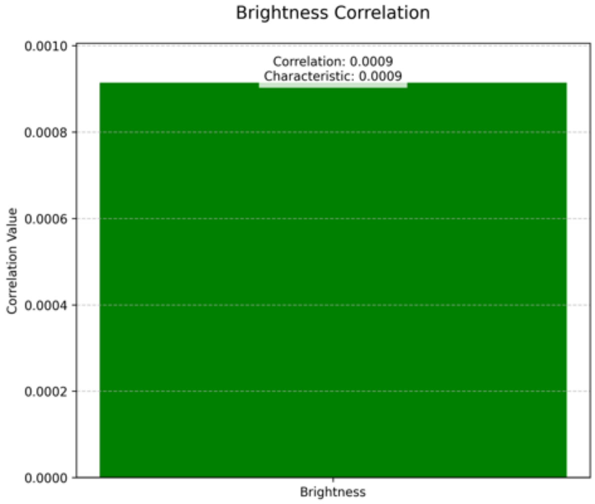

Figure 7. Data 2 brightness correlation

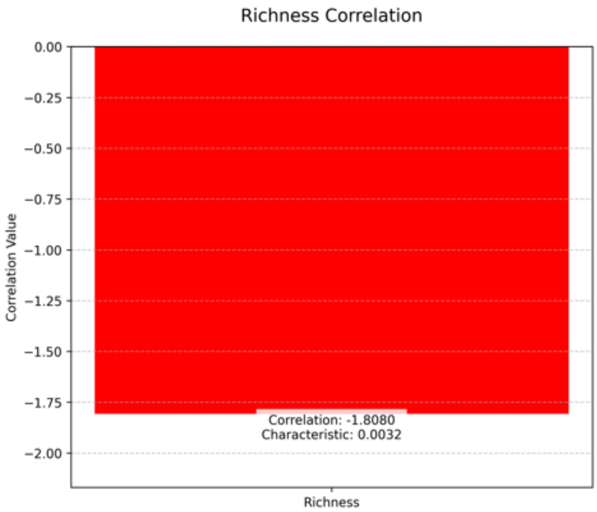

Figure 8. Data 2 richness correlation

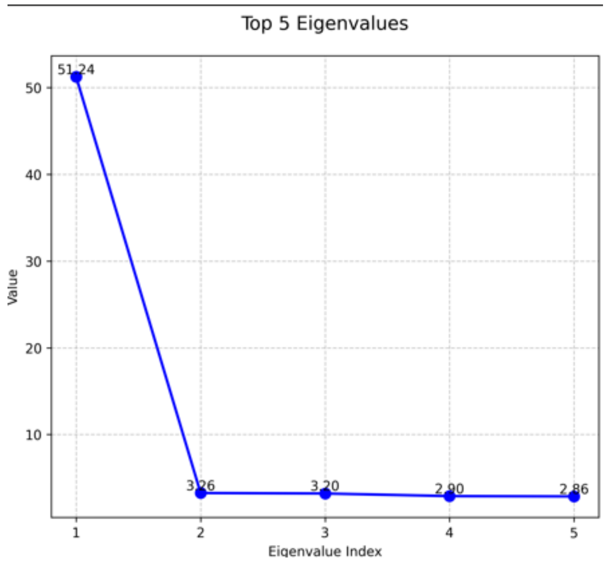

Figure 9. Data 2 eigenvalues

Data 3

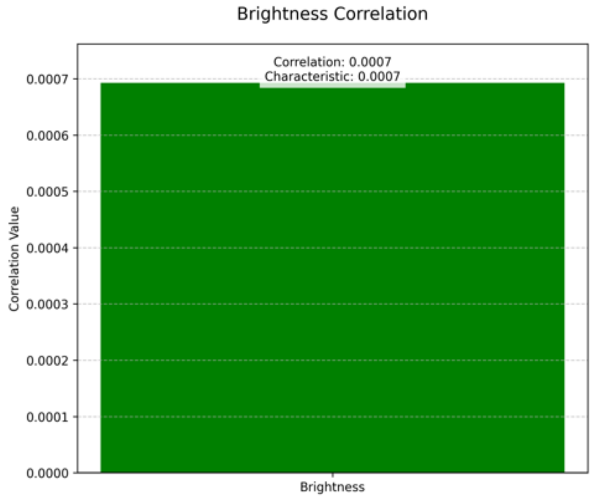

Figure 10. Data 3 brightness correlation

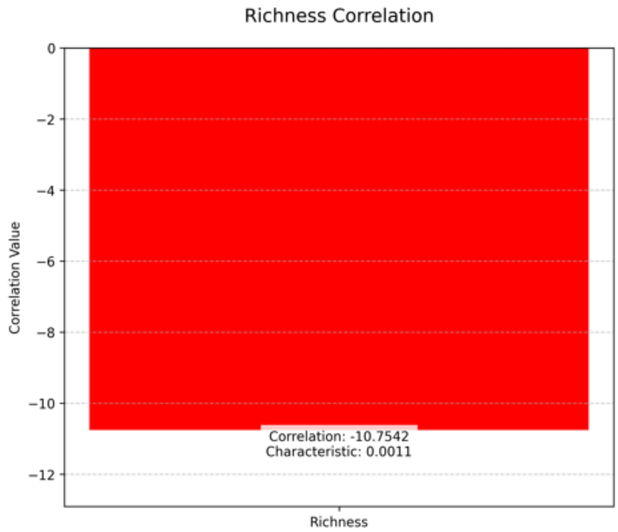

Figure 11. Data 3 richness correlation

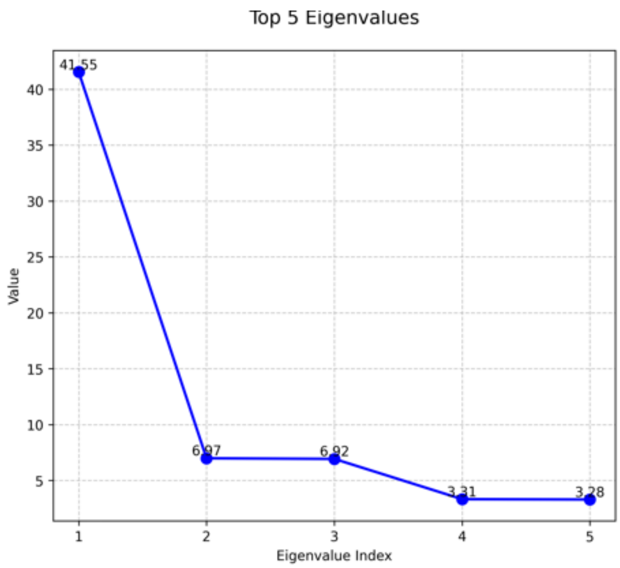

Figure 12. Data 3 eigenvalues

Data 4

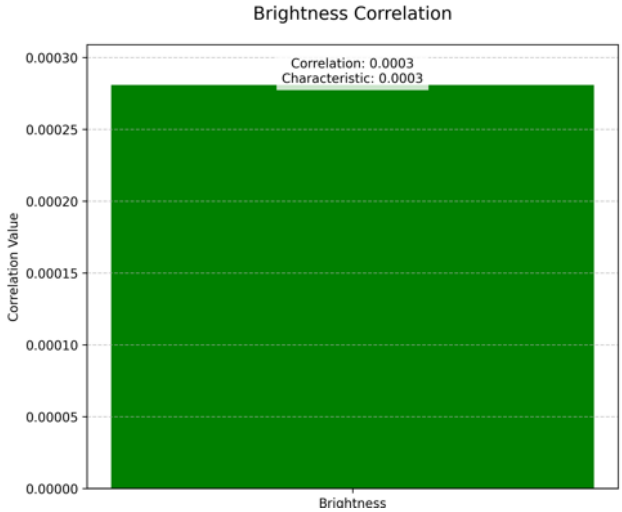

Figure 13. Data 4 brightness correlation

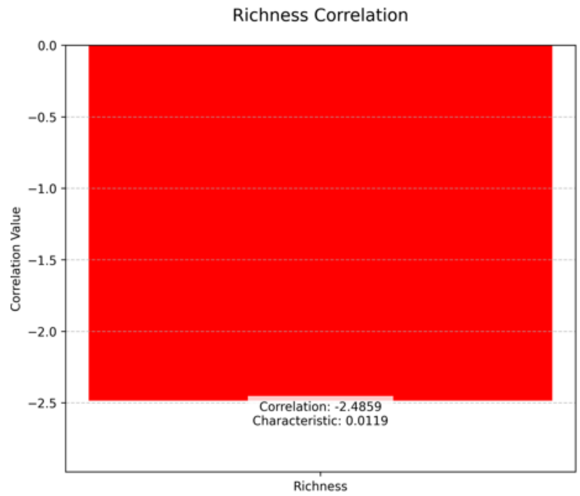

Figure 14. Data 4 richness correlation

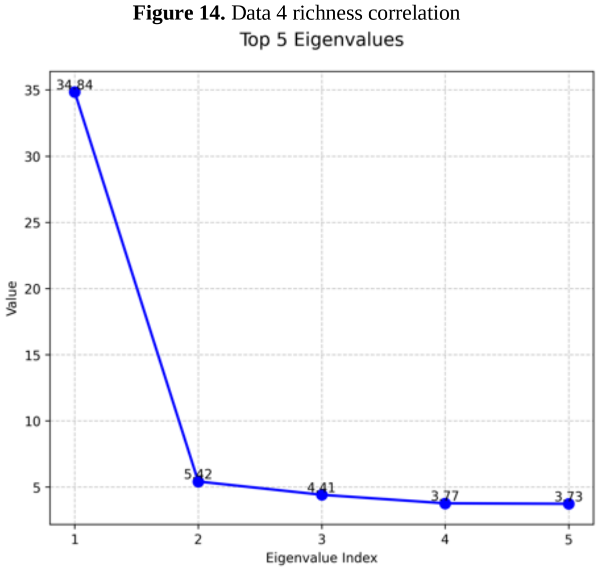

Figure 15. Data 4 eigenvalues

The correlation of eigenvalues with brightness is positive in all samples. In contrast, the correlation of eigenvalues with richness is negative in all data samples.

Sample 2 has the highest eigenvalue and brightness among the four, but the lowest richness. Sample 4 has the highest richness but the lowest brightness and eigenvalue.

The experimental results show a relationship between eigenvalue, brightness, and richness. The positive correlation between eigenvalue and brightness indicates that the higher the eigenvalue, the brighter the guitar sound color. This can be explained physically, where high eigenvalues reflect concentrated resonance energy, especially at high frequencies. For example, Sample 2, with the highest eigenvalue, produces the highest brightness level compared to other samples.

Conversely, the negative correlation between eigenvalue and richness indicates that high eigenvalues tend to reduce the complexity or richness of the sound spectrum. Richness is related to a more even distribution of energy across the harmonics of the sound. In sample 4, which has the lowest eigenvalue and brightness, the highest richness was found. This reflects that the energy is more distributed across the resonant modes, resulting in a richer and more balanced sound.

Comparison between samples provides further insight. Sample 2 shows the highest brightness but the lowest richness, possibly due to stiffer strings or a smaller body, which favor resonance at high frequencies. In contrast, Sample 4 has the highest richness despite its low brightness and eigenvalues, possibly due to a larger body or more flexible strings, which favor resonance across the frequency modes. Samples 1 and 3 fall somewhere in the middle, showing a moderate balance between brightness and richness.

V. Conclusion

The implications of these results are significant for guitar design. For brightness as the priority for producing crisp, bright melodies, a design like Sample 2 is more appropriate. However, for rich, full sounds, such as those required for chord or rhythm playing, a design like Sample 4 is more desirable. Body size and string material clearly have significant, complementary effects; a larger body increases richness, while stiffer or higher-tension strings increase brightness.

In conclusion, there is a clear trade-off between brightness and richness. The choice of guitar design, both in terms of body size and string material, should be tailored to musical needs. With additional data on string material type and body dimensions, this analysis could be expanded to provide more specific guidance for creating optimal guitar design.

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STATEMENT

I hereby declare that the paper I wrote is my own writing, not an adaptation or translation of someone else's paper, and is not plagiarized.

Bandung, 27 Desember 2024

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