Jawaban soal UAS – essay

1. Diberikan tiga buah vektor sebagai berikut: $\mathbf{a}$ = e1 + 2e2 -2e3; $\mathbf{b}$ = 2e1-e2+e3 ;   dan $\mathbf{c}$ = e1+3e2+e3, dan B = $\mathbf{b} \wedge \mathbf{c}$, hitunglah:
   a) $\mathbf{a} \wedge (\mathbf{b} + \mathbf{c})$ *(nilai 5)*
   b) $\mathbf{a} \cdot \mathbf{B}$ *(nilai 5)*

Jawaban: (diambil dari foto lembar jawaban I Putu Bakta Adi Sudewa, IF 2021)

a) $a \wedge (b+c)$

$b+c = 3e_1 + 2e_2 + 2e_3$

$a \wedge (b+c) = (2-6)e_{12} + (4+4)e_{23} + (-6-2)e_{31}$

$= -4e_{12} + 8e_{23} - 8e_{31}$ ✓

b) $a \cdot B = \frac{1}{2}(aB - Ba)$.

$aB = (e_1 + 2e_2 - 2e_3)(7e_{12} - 4e_{23} - e_{31})$

$= (7e_1 e_{12} - 4e_1 e_{23} - e_1 e_{31}$
$\quad + 14e_2 e_{12} - 8e_2 e_{23} - 2e_2 e_{31}$
$\quad - 14e_3 e_{12} + 8e_3 e_{23} + 2e_3 e_{31})$

$= (7e_2 - 4e_{123} + e_3 \qquad = -12e_1 - e_2 - 7e_3 - 20e_{123}$
$\quad -14e_1 - 8e_3 - 2e_{123}$
$\quad -14e_{123} - 8e_2 + 2e_1)$

$Ba = (7e_{12} - 4e_{23} - e_{31})(e_1 + 2e_2 - 2e_3)$

$= (7e_{12}e_1 + 14e_{12}e_2 - 14e_{12}e_3$
$\quad - 4e_{23}e_1 - 8e_{23}e_2 + 8e_{23}e_3$
$\quad - e_{31}e_1 - 2e_{31}e_2 + 2e_{31}e_3)$

$= (-7e_2 + 14e_1 - 14e_{123} \qquad = 12e_1 + e_2 + 7e_3 - 20e_{123}$
$\quad - 4e_{123} + 8e_3 + 8e_2$
$\quad - e_3 - 2e_{123} - 2e_1)$

$\therefore a \cdot B = \frac{1}{2}((-12e_1 - e_2 - 7e_3 - 20e_{123}) - (12e_1 + e_2 + 7e_3 - 20e_{123}))$

$= \frac{1}{2}(2(-12e_1 - e_2 - 7e_3)) = -12e_1 - e_2 - 7e_3$ ✓

2. Menggunakan vektor pada soal no 1) **a** = e1 + 2e2 -2e3; **b** = 2e1-e2+e3; **c** = e1+3e2+e3, hitunglah volume bangun ruang yang dibentuk oleh tiga vektor tersebut. (*nilai 5*)

Jawaban: (diambil dari foto lembar jawaban I Putu Bakta Adi Sudewa, IF 2021)

2). $a = e_1 + 2e_2 - 2e_3$
$b = 2e_1 - e_2 + e_3$
$c = e_1 + 3e_2 + e_3$

$B = b \wedge c$

Volume bangun ruang yang dibentuk adalah determinan matriks 3×3 yang terbentuk dari vektor a, b, c, yaitu.
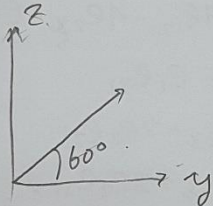
$$\begin{vmatrix} 1 & 2 & -2 \\ 2 & -1 & 1 \\ 1 & 3 & 1 \end{vmatrix} = 1(-1-3) - 2(2-1) - 2(6+1)$$

$$= -4 - 2 - 14 = -20$$

Jadi, volume bangun yang terbentuk adalah __20__

3. (Nilai: 10) Tentukanlah bayangan dari titik (1, −1, 2) setelah dirotasi dengan sudut 60◦ di sumbu bidang yz yang berinklinasi dengan sudut 60◦ terhadap sumbu y positif. Catatan : Kalau ketemu angka "akar kuadrat" maka tetap dalam bentuk akar kuadrat, tidak dihitung pakai kalkulator. Jika dihitung pakai kalkulator maka akan disalahkan.

Jawaban: (diambil dari foto lembar jawaban I Putu Bakta Adi Sudewa, IF 2021)

$P(1,-1,2)$
$\theta = 60°$
$P = \hat{i} - \hat{j} + 2\hat{k}$

$\hat{u} = \cos(60°)\hat{j} + \sin(60°)\hat{k}$
$= \frac{1}{2}\hat{j} + \frac{1}{2}\sqrt{3}\hat{k}$

$q = \cos\left(\frac{\theta}{2}\right) + \sin\left(\frac{\theta}{2}\right)\hat{u}$
$= \cos(30°) + \sin(30°)\left(\frac{1}{2}\hat{j} + \frac{1}{2}\sqrt{3}\hat{k}\right)$
$= \frac{1}{2}\sqrt{3} + \frac{1}{2}\cdot\frac{1}{2}\left(\hat{j} + \sqrt{3}\hat{k}\right)$
$= \frac{1}{4}\left(2\sqrt{3} + \hat{j} + \sqrt{3}\hat{k}\right)$

$q^{-1} = \cos\left(\frac{\theta}{2}\right) - \sin\left(\frac{\theta}{2}\right)\hat{u}$
$= \cos(30°) - \sin(30°)\left(\frac{1}{2}\hat{j} + \frac{1}{2}\sqrt{3}\hat{k}\right)$
$= \frac{1}{2}\sqrt{3} - \frac{1}{2}\cdot\frac{1}{2}\left(\hat{j} + \sqrt{3}\hat{k}\right)$
$= \frac{1}{4}\left(2\sqrt{3} - \hat{j} - \sqrt{3}\hat{k}\right)$

$$qP = \frac{1}{4}(2\sqrt{3} + \hat{j} + \sqrt{3}\,\hat{k})(\hat{i} - \hat{j} + 2\hat{k})$$

$$= \frac{1}{4}\left(\begin{array}{l} 2\sqrt{3}\,\hat{i} - 2\sqrt{3}\,\hat{j} + 4\sqrt{3}\,\hat{k} \\ -\hat{k} + 1 + 2\hat{i} \\ \sqrt{3}\,\hat{j} + \sqrt{3}\,\hat{i} - 2\sqrt{3} \end{array} \right. = \frac{1}{4}\left((1-2\sqrt{3}) + (2+3\sqrt{3})\hat{i} - \sqrt{3}\hat{j} + (4\sqrt{3}-1)\hat{k}\right)$$

$$qPq^{-1} = \frac{1}{4}\left((1-2\sqrt{3}) + (2+3\sqrt{3})\hat{i} - \sqrt{3}\hat{j} + (4\sqrt{3}-1)\hat{k}\right)\frac{1}{4}(2\sqrt{3} - \hat{j} - \sqrt{3}\,\hat{k})$$

$$= \frac{1}{16}\left(\begin{array}{l} (1-2\sqrt{3})2\sqrt{3} - (1-2\sqrt{3})\hat{j} - (1-2\sqrt{3})\sqrt{3}\,\hat{k} \\ +(2+3\sqrt{3})2\sqrt{3}\,\hat{i} - (2+3\sqrt{3})\hat{k} + (2+3\sqrt{3})\sqrt{3}\,\hat{j} \\ - \sqrt{3}(2\sqrt{3})\hat{j} - \sqrt{3} + \sqrt{3}(\sqrt{3})\hat{i} \\ +(4\sqrt{3}-1)2\sqrt{3}\,\hat{k} + (4\sqrt{3}-1)\hat{i} + (4\sqrt{3}-1)\sqrt{3} \end{array} \right)$$

$$= \frac{1}{16}\left(\begin{array}{l} (2\sqrt{3}-12) - (1-2\sqrt{3})\hat{j} - (\sqrt{3}-6)\hat{k} \\ (4\sqrt{3}+18)\hat{i} - (2+3\sqrt{3})\hat{k} + (2\sqrt{3}+9)\hat{j} \\ -6\hat{j} - \sqrt{3} + 3\hat{i} \\ +(24-2\sqrt{3})\hat{k} + (4\sqrt{3}-1)\hat{i} + (2-\sqrt{3}) \end{array} \right)$$

$$= \frac{1}{16}\left(0 + (20+8\sqrt{3})\hat{i} + (2+4\sqrt{3})\hat{j} + (28-6\sqrt{3})\hat{k}\right)$$

$$= \frac{1}{16}\left(2(10+4\sqrt{3})\hat{i} + 2(1+2\sqrt{3})\hat{j} + 2(14-3\sqrt{3})\hat{k}\right)$$

$$= \frac{1}{8}\left((10+4\sqrt{3})\hat{i} + (1+2\sqrt{3})\hat{j} + (14-3\sqrt{3})\hat{k}\right)$$

Maka bayangan dari titik P adalah $\dfrac{(10+4\sqrt{3})\hat{i}}{8} + \dfrac{(1+2\sqrt{3})\hat{j}}{8} + \dfrac{(14-3\sqrt{3})\hat{k}}{8}$

atau $\left(\dfrac{10+4\sqrt{3}}{8},\ \dfrac{1+2\sqrt{3}}{8},\ \dfrac{14-3\sqrt{3}}{8} \right)$ ✓ /10

4. (Nilai: 15) Diberikan tiga buah vektor sebagai berikut:

   $a = 2e1 + e2 - e3$

   $b = e1 - e2 - e3$

   $c = 2e1 + 2e2 - e3$

   Tentukan perpotongan bidang yang dibentuk oleh vektor a dan c dengan bidang $(e2 \wedge e3)$

Jawaban: (diambil dari foto lembar jawaban I Putu Bakta Adi Sudewa, IF 2021)

4). $a = 2e_1 + e_2 - e_3$ | $a \wedge c = (4-2)e_{12} + (-1+2)e_{23} + (-2+2)e_{31}$
$b = e_1 - e_2 - e_3$ |
$c = 2e_1 + 2e_2 - e_3$ | $= 2e_{12} + e_{23}$

$AV\, e_{23} = A^* \cdot e_{23}$

$\quad\quad = IA \cdot e_{23}$

$\quad\quad = e_{123}(a \wedge c) \cdot e_{23}$

$\quad\quad = e_{123}(2e_{12} + e_{23}) \cdot e_{23}$

$\quad\quad = (2e_{123}e_{12} + e_{123}e_{23}) \cdot e_{23}$

$\quad\quad = \underbrace{(-2e_3 - e_1)}_{a} \cdot \underbrace{e_{23}}_{B}$

$\circ\; aB = (-2e_3 - e_1)e_{23}$

$\quad\quad = -2e_3 e_{23} - e_1 e_{23}$

$\quad\quad = 2e_2 - e_{123}$

$\cdot\; Ba = e_{23}(-2e_3 - e_1)$

$\quad\quad = -2e_{23}e_3 - e_{23}e_1$

$\quad\quad = -2e_2 - e_{123}$

$\therefore\; a \cdot B = \frac{1}{2}(aB - Ba) = \frac{1}{2}((2e_2 - e_{123}) - (-2e_2 - e_{123}))$

$\quad\quad\quad\quad = \frac{1}{2}(4e_2) = 2e_2$

Jadi, perpotongan bidang yang dibentuk oleh vektor a dan c dengan bidang ($e_2 \wedge e_3$) adalah $2e_2$