

Jawaban Kuis 1 2019

1.

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Quiz

1. Diketahui Augmented matrix $A = \left[\begin{array}{cccc|c} 2 & -1 & -1 & 1 & -2 \\ -1 & 1 & 2 & 2 & -5 \\ 3 & 1 & -1 & -3 & 8 \\ 2 & 2 & -2 & -1 & 6 \end{array} \right]$

Find the solution as well as its determinant along the way of solution with OBE.

1. Swap $R_1 \leftrightarrow R_2$

$$\left[\begin{array}{cccc|c} -1 & 1 & 2 & 2 & -5 \\ 2 & -1 & -1 & 1 & -2 \\ 3 & 1 & -1 & -3 & 8 \\ 2 & 2 & -2 & -1 & 6 \end{array} \right]$$

2. $R_1 \leftarrow R_1 \times (-1)$

$$\left[\begin{array}{cccc|c} 1 & -1 & -2 & -2 & 5 \\ 2 & -1 & -1 & 1 & -2 \\ 3 & 1 & -1 & -3 & 8 \\ 2 & 2 & -2 & -1 & 6 \end{array} \right]$$

3. $R_2 \leftarrow R_2 - 2R_1$
 $R_3 \leftarrow R_3 - 3R_1$
 $R_4 \leftarrow R_4 - 2R_1$

$$\left[\begin{array}{cccc|c} 1 & -1 & -2 & -2 & 5 \\ 0 & 1 & 3 & 5 & -12 \\ 0 & 4 & 5 & 3 & -7 \\ 0 & 4 & 2 & 3 & -4 \end{array} \right]$$

4. $R_3 \leftarrow R_3 - 4R_2$
 $R_4 \leftarrow R_4 - 4R_2$

$$\left[\begin{array}{cccc|c} 1 & -1 & -2 & -2 & 5 \\ 0 & 1 & 3 & 5 & -12 \\ 0 & 0 & -7 & -17 & 41 \\ 0 & 0 & -10 & -17 & 44 \end{array} \right]$$

5. $R_4 \leftarrow R_4 - \frac{10R_3}{7}$

$$\left[\begin{array}{cccc|c} 1 & -1 & -2 & -2 & 5 \\ 0 & 1 & 3 & 5 & -12 \\ 0 & 0 & -7 & -17 & 41 \\ 0 & 0 & 0 & \frac{51}{7} & -\frac{102}{7} \end{array} \right]$$

a) Determinant $(A) = 1 \times 1 \times -7 \times \frac{51}{7} = -51$

catatan: jika jawaban betul, nilai 10.
 jika jawaban salah, proses serapa dgn. kesalahan simple, nilai 5.
 jika jawaban salah, proses tanpa OBE ke Echelon Form, nilai 0.
 jika jawaban betul, proses tanpa OBE, nilai 5.

Determinant

1. $\det(A) = \det(A) \times -1$
2. $\det(A) = \det(A)$
3. $\det(A) = \det(A)$
4. $\det(A) = \det(A)$
5. $\det(A) = \det(A)$

$\therefore \det(A) = -51$

$$\textcircled{6} \begin{aligned} R_3 &\leftarrow R_3 / -7 \\ R_4 &\leftarrow R_4 \times 7 \end{aligned}$$

$$\left[\begin{array}{cccc|c} 1 & -1 & -2 & -2 & 5 \\ 0 & 1 & 3 & 5 & -12 \\ 0 & 0 & 1 & \frac{17}{7} & -\frac{41}{7} \\ 0 & 0 & 0 & 51 & -102 \end{array} \right]$$

$$\textcircled{7} R_4 \leftarrow \frac{R_4}{51}$$

$$\left[\begin{array}{cccc|c} 1 & -1 & -2 & -2 & 5 \\ 0 & 1 & 3 & 5 & -12 \\ 0 & 0 & 1 & \frac{17}{7} & -\frac{41}{7} \\ 0 & 0 & 0 & 1 & -\frac{102}{51} \end{array} \right]$$

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$$\textcircled{8} R_1 \leftarrow R_1 + R_2$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 1 & 3 & -7 \\ 0 & 1 & 3 & 5 & -12 \\ 0 & 0 & 1 & \frac{17}{7} & -\frac{41}{7} \\ 0 & 0 & 0 & 1 & -\frac{102}{51} \end{array} \right]$$

$$\textcircled{9} R_1 \leftarrow R_1 - R_3 ; R_2 \leftarrow R_2 - 3R_3$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & \frac{4}{7} & 5 \\ 0 & 1 & 0 & -\frac{16}{7} & \frac{39}{7} \\ 0 & 0 & 1 & \frac{17}{7} & -\frac{41}{7} \\ 0 & 0 & 0 & 1 & -\frac{102}{51} \end{array} \right]$$

$$\textcircled{10} R_1 \leftarrow R_1 - \frac{4R_4}{7} ; R_2 \leftarrow R_2 + \frac{16R_4}{7} ; R_3 \leftarrow R_3 - \frac{17R_4}{7}$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & \frac{731}{119} \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & \frac{901}{189} \\ 0 & 0 & 0 & 1 & -\frac{102}{51} \end{array} \right]$$

$$\begin{aligned} \therefore x_1 &= \frac{731}{119} & x_3 &= \frac{901}{189} \\ x_2 &= 1 & x_4 &= -\frac{102}{51} \end{aligned}$$

∴ Jika ke-4 jawaban betul, nilai 20,,
tiap jawaban x_1, x_2, x_3, x_4 nilai
5,,

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 Mencari invers dengan OBE :

$$\left[\begin{array}{cccc|cccc} 2 & -1 & -1 & 1 & 1 & 0 & 0 & 0 \\ -1 & 1 & 2 & 2 & 0 & 1 & 0 & 0 \\ 3 & 1 & -1 & -3 & 0 & 0 & 1 & 0 \\ 2 & 2 & -2 & -1 & 0 & 0 & 0 & 1 \end{array} \right]$$

Dikarenakan dlm. proses yang lain, kita sudah membuat REF terhadap matriks asal, kita hanya mengikuti step yang sama terhadap matrik identitas (I).

1. Swap $R_1 \leftrightarrow R_2$

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

2. $R_1 \leftarrow R_1 \times (-1)$

$$\begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$R_2 \leftarrow R_2 - 2R_1$
 $R_3 \leftarrow R_3 - 3R_1$
 $R_4 \leftarrow R_4 - 2R_1$

$$\begin{bmatrix} 0 & -1 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ 0 & 3 & 1 & 0 \\ 0 & 2 & 0 & 1 \end{bmatrix}$$

4. $R_3 \leftarrow R_3 - 4R_2$
 $R_4 \leftarrow R_4 - 4R_2$

$$\begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 2 & 0 & 0 \\ -4 & -5 & 1 & 0 \\ -4 & -6 & 0 & 1 \end{bmatrix}$$

5. $R_4 \leftarrow R_4 - \frac{10R_3}{7}$

$$\begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 2 & 0 & 0 \\ -4 & -5 & 1 & 0 \\ \frac{12}{7} & \frac{8}{7} & -\frac{10}{7} & 1 \end{bmatrix}$$

6. $R_3 \leftarrow \frac{R_3}{-7}$; $R_4 \leftarrow R_4 \times 7$

$$\begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 2 & 0 & 0 \\ \frac{4}{7} & \frac{5}{7} & -\frac{1}{7} & 0 \\ 12 & 8 & -10 & 7 \end{bmatrix}$$

7. $R_4 \leftarrow \frac{R_4}{51}$

$$\begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 2 & 0 & 0 \\ \frac{4}{7} & \frac{5}{7} & -\frac{1}{7} & 0 \\ \frac{12}{51} & \frac{8}{51} & -\frac{10}{51} & \frac{7}{51} \end{bmatrix}$$

8. $R_1 \leftarrow R_1 + R_2$

$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 2 & 0 & 0 \\ \frac{4}{7} & \frac{5}{7} & -\frac{1}{7} & 0 \\ \frac{12}{51} & \frac{8}{51} & -\frac{10}{51} & \frac{7}{51} \end{bmatrix}$$

9. $R_1 \leftarrow R_1 - R_3$; $R_2 \leftarrow R_2 - 3R_3$

$$\begin{bmatrix} \frac{3}{7} & \frac{2}{7} & \frac{1}{7} & 0 \\ -\frac{5}{7} & -\frac{1}{7} & \frac{3}{7} & 0 \\ \frac{4}{7} & \frac{5}{7} & -\frac{1}{7} & 0 \\ \frac{12}{51} & \frac{8}{51} & -\frac{10}{51} & \frac{7}{51} \end{bmatrix}$$

10. $R_1 \leftarrow R_1 - \frac{4}{7}R_4$; $R_2 \leftarrow R_2 + \frac{16R_4}{7}$
 $R_3 \leftarrow R_3 - \frac{17R_4}{7}$

$$\begin{bmatrix} \frac{5}{17} & \frac{10}{51} & \frac{91}{357} & -\frac{4}{51} \\ -\frac{3}{17} & \frac{11}{51} & -\frac{1}{51} & \frac{16}{51} \\ 0 & \frac{119}{357} & \frac{119}{357} & -\frac{17}{51} \\ \frac{12}{51} & \frac{8}{51} & -\frac{10}{51} & \frac{7}{51} \end{bmatrix}$$

$$\begin{bmatrix} \frac{5}{17} & \frac{10}{51} & \frac{13}{51} & -\frac{4}{51} \\ -\frac{3}{17} & \frac{11}{51} & -\frac{1}{51} & \frac{16}{51} \\ 0 & \frac{1}{3} & \frac{1}{3} & -\frac{1}{3} \\ \frac{4}{17} & \frac{8}{51} & -\frac{10}{51} & \frac{7}{51} \end{bmatrix}$$

2.

cofactors	$\begin{pmatrix} 3 & 0 & 0 \\ 0 & -1 & 3 \\ 0 & -3 & -1 \end{pmatrix}$
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(a)

(b) $\det(A) = 30$

$$A^{-1} = 1/\det(A) \operatorname{adj}(A) = 1/30 \begin{pmatrix} 3 & 0 & 0 \\ 0 & -1 & -3 \\ 0 & 3 & -1 \end{pmatrix} = \begin{pmatrix} \frac{1}{10} & 0 & 0 \\ 0 & -\frac{1}{30} & -\frac{1}{10} \\ 0 & \frac{1}{10} & -\frac{1}{30} \end{pmatrix}$$

3. a) -56 b) $8/(-7)$ c) 7