Solusi Kuis 1 IF 2123 Aljabar Geometri, Semester 1 Tahun 2018/2019

1.

I a).
$$\left[\begin{array}{cccccc|c} 1 & 3 & -2 & 0 & 2 & 0 & 0 \\ 2 & 6 & -5 & -2 & 4 & -3 & -1 \\ 0 & 0 & 5 & 10 & 0 & 15 & 5 \\ 2 & 6 & 0 & 8 & 4 & 18 & 6 \end{array}\right]$$

nilai 5

b).
$$\begin{bmatrix} 1 & 3 & -2 & 0 & 2 & 0 & 0 \\ 0 & 0 & -1 & -2 & 0 & -3 & -1 \\ 0 & 0 & 5 & 10 & 0 & 15 & 5 \\ 0 & 0 & 4 & 8 & 0 & 18 & 6 \end{bmatrix} \overset{OBE}{\sim} \begin{bmatrix} 1 & 3 & -2 & 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 2 & 0 & 3 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 6 & 2 \end{bmatrix} \overset{OBE}{\sim}$$

$$\begin{bmatrix} 1 & 3 & -2 & 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 2 & 0 & 3 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1/3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(nilai 1 matriks 5
betul 3 total $5 \times 3 = 15$

c). Solusi

$$x_1 = -3x_2 - 4x_4 - 2x_5$$
$$x_3 = -2x_4$$
$$x_6 = \frac{1}{3}$$

nilai 10

Variable bebas x_2, x_4, x_5 dinyatakan dlm r, s, t menjadi

$$x_1 = -3r - 4s - 2t$$
$$x_2 = r$$
$$x_3 = -2s$$
$$x_4 = s$$
$$x_5 = t$$
$$x_6 = 1/3.$$

nilai 5

total $10 + 5 = 15$

Jawab 1b : mungkin hasil OBE nya berbeda.
tidak apa², tapi matriks terakhir
harus sama.

2.

Q2. Diketahui matriks:

$$B = \begin{bmatrix} 1 & 3 & 1 & 1 \\ 2 & 5 & 2 & 2 \\ 1 & 3 & 8 & 9 \\ 1 & 3 & 2 & 9 \end{bmatrix}$$

Answer:

a) Hitung determinant B dengan metoda row-reduction (OBE)

nilai:

step 1 : swap $R2 \leftrightarrow R1$.

$$B_1 = \begin{bmatrix} 2 & 5 & 2 & 2 \\ 1 & 3 & 1 & 1 \\ 1 & 3 & 8 & 9 \\ 1 & 3 & 2 & 9 \end{bmatrix}$$ [3]

step 2 : $R_2 \leftarrow R_2 - \frac{1}{2}R_1$
$R_3 \leftarrow R_3 - \frac{1}{2}R_1$
$R_4 \leftarrow R_4 - \frac{1}{2}R_1$

$$B_2 = \begin{bmatrix} 2 & 5 & 2 & 2 \\ 0 & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 7 & 8 \\ 0 & \frac{1}{2} & 1 & 1 \end{bmatrix}.$$ [3]

step 3 : $R_3 \leftarrow R_3 - 1 R_2$
$R_4 \leftarrow R_4 - 1 \cdot R_2$

$$B_3 = \begin{bmatrix} 2 & 5 & 2 & 2 \\ 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 7 & 8 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$ [3].

step 4 : $R_4 \leftarrow R_4 - \frac{1}{7} R_3$

$$B_4 = \begin{bmatrix} 2 & 5 & 2 & 2 \\ 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 7 & 8 \\ 0 & 0 & 0 & -\frac{1}{7} \end{bmatrix}.$$ [3]

Dari B_4 dapat kita hitung determinant matriks B :

$$\det|B| = \left(2 \cdot \tfrac{1}{2} \cdot 7 \cdot -\tfrac{1}{7}\right)\underbrace{(-1)}_{\text{efek dari step-1}}$$ [3]

$$\det|B| = 1.$$

nilai total a = [15]

Jika ketemu B_4, maka nilai otomatis [12]

Jika nilai determinant dan B_4 betul [15].

6. Hitung matriks balikan dari B^{-1}: dgn eliminasi Gauss-Jordan

① Buat Augmented Matrik $[B \mid I]$.

$$= \left[\begin{array}{cccc|cccc} 1 & 3 & 1 & 1 & 1 & 0 & 0 & 0 \\ 2 & 5 & 2 & 2 & 0 & 1 & 0 & 0 \\ 1 & 3 & 8 & 9 & 0 & 0 & 1 & 0 \\ 1 & 3 & 2 & 9 & 0 & 0 & 0 & 1 \end{array}\right]$$

nilai:
Jika B^{-1} benar:
nilai = [20]
Jika B^{-1} salah:
nilai = step betul (MAX 10)

②. Swap $R_2 \leftrightarrow R_1$.

$$= \left[\begin{array}{cccc|cccc} 2 & 5 & 2 & 2 & 0 & 1 & 0 & 0 \\ 1 & 3 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 3 & 8 & 9 & 0 & 0 & 1 & 0 \\ 1 & 3 & 2 & 9 & 0 & 0 & 0 & 1 \end{array}\right]$$

③. ⊖ $R_2 \leftarrow R_2 - \frac{1}{2}R_1$
⊖ $R_3 \leftarrow R_3 - \frac{1}{2}R_1$
⊖ $R_4 \leftarrow R_4 - \frac{1}{2}R_1$

$$\longrightarrow = \left[\begin{array}{cccc|cccc} 2 & 5 & 2 & 2 & 0 & 1 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & 1 & -\frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 7 & 8 & 0 & -\frac{1}{2} & 1 & 0 \\ 0 & \frac{1}{2} & 1 & 8 & 0 & -\frac{1}{2} & 0 & 1 \end{array}\right]$$

④. ⊖ $R_3 \leftarrow R_3 - R_2$.
⊖ $R_4 \leftarrow R_4 - R_2$

$$\longrightarrow = \left[\begin{array}{cccc|cccc} 2 & 5 & 2 & 2 & 0 & 1 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & 1 & -\frac{1}{2} & 0 & 0 \\ 0 & 0 & 7 & 8 & -1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 8 & -1 & 0 & 0 & 1 \end{array}\right]$$

⑤ ⊖ $R_4 \leftarrow R_4 - \frac{1}{7}R_3$

$$\longrightarrow = \left[\begin{array}{cccc|cccc} 2 & 5 & 2 & 2 & 0 & 1 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & 1 & -\frac{1}{2} & 0 & 0 \\ 0 & 0 & 7 & 8 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & \frac{48}{7} & -\frac{6}{7} & 0 & -\frac{1}{7} & 1 \end{array}\right]$$

⑥ ⊖ $R_4 \leftarrow \frac{7}{48} \cdot R_4$

$$\longrightarrow = \left[\begin{array}{cccc|cccc} 2 & 5 & 2 & 2 & 0 & 1 & 0 & 0 \\ 0 & 1/2 & 0 & 0 & 1 & -1/2 & 0 & 0 \\ 0 & 0 & 7 & 8 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & -\frac{1}{8} & 0 & -1/48 & 7/48 \end{array}\right]$$

⑦ ⊖ $R_3 \leftarrow R_3 - 8R_4$
⊖ $R_1 \leftarrow R_1 - 2R_4$

$$\longrightarrow = \left[\begin{array}{cccc|cccc} 2 & 5 & 2 & 0 & 1/4 & 1 & 1/24 & -7/24 \\ 0 & 1/2 & 0 & 0 & 1 & -1/2 & 0 & 0 \\ 0 & 0 & 7 & 0 & 0 & 0 & 7/6 & -7/6 \\ 0 & 0 & 0 & 1 & -1/8 & 0 & -1/48 & 7/48 \end{array}\right]$$

⑧. ⊖ $R_3 \leftarrow \frac{1}{7}R_3$

$$\left[\begin{array}{cccc|cccc} 2 & 5 & 2 & 0 & 1/4 & 1 & 1/24 & -7/24 \\ 0 & 1/2 & 0 & 0 & 1 & -1/2 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1/6 & -1/6 \\ 0 & 0 & 0 & 1 & -1/8 & 0 & -1/48 & 7/48 \end{array}\right]$$

⑨. a. $R_1 \leftarrow R_1 - 2R_3$
b. $R_2 \leftarrow 2R_2$
c. $R_1 \leftarrow R_1 - 5R_2$
d. $R_1 \leftarrow \frac{1}{2}R_1$

step

$$\longrightarrow = \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & -39/8 & 3 & -7/48 & 1/48 \\ 0 & 1 & 0 & 0 & 2 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1/6 & -1/6 \\ 0 & 0 & 0 & 1 & -1/8 & 0 & -1/48 & 7/48 \end{array}\right]$$

$$\therefore B^{-1} = \begin{bmatrix} -39/8 & 3 & -7/48 & 1/48 \\ 2 & -1 & 0 & 0 \\ 0 & 0 & 1/6 & -1/6 \\ -1/8 & 0 & -1/48 & 7/48 \end{bmatrix}$$

3. (a)

$$A = \begin{bmatrix} 4 & 4 & 2 & 4 \\ 1 & 1 & 0 & -1 \\ 3 & 0 & -3 & 1 \\ 6 & 14 & 3 & 6 \end{bmatrix}$$

$$\begin{bmatrix} 4 & 4 & 2 & 4 \\ 1 & 1 & 0 & -1 \\ 3 & 0 & -3 & 1 \\ 6 & 14 & 3 & 6 \end{bmatrix} \begin{matrix} R1-4R2 \\ R3-3R2 \\ R4-6R2 \end{matrix} \begin{bmatrix} 0 & 0 & 2 & 8 \\ 1 & 1 & 0 & -1 \\ 0 & -3 & -3 & 4 \\ 0 & 8 & 3 & 12 \end{bmatrix}$$ (pakai OBE dulu) (nilai 10)

Det(A) = -(1) $\begin{vmatrix} 0 & 2 & 8 \\ -3 & -3 & 4 \\ 8 & 3 & 12 \end{vmatrix}$ = (-1){-(-3) $\begin{vmatrix} 2 & 8 \\ 3 & 12 \end{vmatrix} + 8 \begin{vmatrix} 2 & 8 \\ -3 & 4 \end{vmatrix}$}

= (-1){3(24 – 24) + 8(8 + 24)}

= (-1)(0 + 256)

= -256 (nilai 10)

(b) Sudah diketahui dari soal matriks kofaktor dari A adalah

$$\begin{bmatrix} -63 & 48 & -68 & -15 \\ -112 & 0 & -64 & -144 \\ -16 & 0 & 64 & -16 \\ 26 & -32 & 24 & -6 \end{bmatrix}$$

maka, adjoin dari A adalah transpose dari matriks kofaktor, yaitu:

adj(A) = $\begin{bmatrix} -63 & -112 & -16 & 26 \\ 48 & 0 & 0 & -32 \\ -68 & -64 & 64 & 24 \\ -15 & -144 & -16 & -6 \end{bmatrix}$ (nilai = 4)

Maka, balikan (invers) dari A adalah

$$A^{-1} = \frac{1}{\det(A)} adj(A) = \frac{1}{-256} \begin{bmatrix} -63 & -112 & -16 & 26 \\ 48 & 0 & 0 & -32 \\ -68 & -64 & 64 & 24 \\ -15 & -144 & -16 & -6 \end{bmatrix} = \begin{bmatrix} 63/256 & 112/256 & 16/256 & -26/256 \\ -48/256 & 0 & 0 & 32/256 \\ 68/256 & 64/256 & -64/256 & -24/256 \\ 15/256 & 144/256 & 16/256 & 6/256 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{63}{245} & \frac{7}{16} & \frac{1}{16} & -\frac{13}{128} \\ -\frac{3}{16} & 0 & 0 & \frac{1}{8} \\ \frac{17}{64} & \frac{1}{4} & -\frac{1}{4} & -\frac{3}{32} \\ \frac{15}{26} & \frac{9}{16} & \frac{1}{16} & \frac{3}{128} \end{bmatrix}$$

(nilai = 6)