Program Studi Teknik Informatika
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Kuis ke-1 IF2123 Aljabar Geometri (3 SKS) – Matriks dan Sistem Persamaan Lanjar
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1. Selesaikan SPL berikut dengan metode eliminasi Gauss-Jordan:

$$3x_1 + 2x_2 - x_3 = -15$$
$$5x_1 + 3x_2 + 2x_3 = 0$$
$$3x_1 + x_2 + 3x_3 = 11$$
$$11x_1 + 7x_2 = -30$$

Penyelesaian:

$$\begin{bmatrix} 3 & 2 & -1 & -15 \\ 5 & 3 & 2 & 0 \\ 3 & 1 & 3 & 11 \\ 11 & 7 & 0 & -30 \end{bmatrix} \sim .. \sim \begin{bmatrix} 1 & 0 & 0 & -4 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 7 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Diperoleh: $x_1 = -4$, $x_2 = 2$, $x_3 = 7$

2. Apakah SPL berikut dapat dipecahkan? Jelaskan alasannya dengan menggunakan properti matriks (ada perhitungannya)

$$3w + x + 7y + 9z = 4$$
$$w + x + 4y + 4z = 7$$
$$-w - 2y - 3z = 0$$
$$-2w - x - 4y - 6z = 6$$

Penyelesaian:

$$A = \begin{bmatrix} 3 & 1 & 7 & 9 \\ 1 & 1 & 4 & 4 \\ -1 & 0 & -2 & -3 \\ -2 & -1 & -1 & -6 \end{bmatrix} \sim .. \sim \begin{bmatrix} 1 & 1 & 4 & 4 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Hitung determinan matriks A dengan metode apapun, diperoleh det(A) ≠ 0, sehingga SPL dapat dipecahkan

3. Tentukan determinan dan balikan dari matriks berikut dengan mengunakan konsep-konsep kofaktor, adjoin, kaidah Cramer, dan bantuan operasi baris elementer:

$$\begin{bmatrix} 4 & 4 & 0 & 4 \\ 1 & 1 & 0 & -1 \\ 3 & 0 & -3 & 1 \\ 6 & 14 & 3 & 6 \end{bmatrix}$$

Penyelesaian:

- Mencari determinan matriks

3) $Adj(A) = C^T$

C: matriks kofaktor

$C_{ij} = (-1)^{i+j} \cdot M_{ij} = adj(A)_{ji}$

M: matriks minor

$C_{11} = (-1)^2 \begin{pmatrix} 1 & 0 & -1 \\ 0 & -3 & 1 \\ 14 & 3 & 6 \end{pmatrix} = 1\,(-18+0+0-42-3-0)$

$C_{11} = -63$

$C_{12} = (-1)^3 \begin{pmatrix} 1 & 0 & -1 \\ 3 & -3 & 1 \\ 6 & 3 & 6 \end{pmatrix} = -1\,(-18+0-9-18-3-0)$

$= 48$

$C_{13} = (-1)^4 \begin{pmatrix} 1 & 1 & -1 \\ 3 & 0 & 1 \\ 6 & 14 & 6 \end{pmatrix} = 1\,(0+6-42-0-14-18)$

$= -68$

$C_{14} = (-1)^5 \begin{pmatrix} 1 & 1 & 0 \\ 3 & 0 & -3 \\ 6 & 14 & 3 \end{pmatrix} = -1\,(0-18+0-0+42-9)$

$= -15$

$C_{21} = (-1)^3 \begin{pmatrix} 4 & 0 & 4 \\ 0 & -3 & 1 \\ 14 & 3 & 6 \end{pmatrix} = (-1)\,(-72+0+0+168-12-0)$

$= -84$

$C_{22} = (-1)^4 \begin{pmatrix} 4 & 0 & 4 \\ 3 & -3 & 1 \\ 6 & 3 & 6 \end{pmatrix} = 1\,(-72+0+36+72-12-0)$

$= 24$

$C_{23} = (-1)^5 \begin{pmatrix} 4 & 4 & 4 \\ 3 & 0 & 1 \\ 6 & 14 & 6 \end{pmatrix} = -1\,(0+24+168-0-56-72)$

$= -64$

$C_{24} = (-1)^6 \begin{pmatrix} 4 & 4 & 0 \\ 3 & 0 & -3 \\ 6 & 14 & 3 \end{pmatrix} = 1\,(0-72+0-0+168-36)$

$= 60$

$C_{31} = (-1)^4 \begin{pmatrix} 4 & 0 & 4 \\ 1 & 0 & -1 \\ 14 & 3 & 6 \end{pmatrix} = 1\,(0+0+12-0+12-0)$

$= 24$

$C_{32} = (-1)^5 \begin{pmatrix} 4 & 0 & 4 \\ 1 & 0 & -1 \\ 6 & 3 & 6 \end{pmatrix} = -1\,(0+0+12-0+12-0)$

$= -24$

$C_{33} = (-1)^6 \begin{pmatrix} 4 & 4 & 4 \\ 1 & 1 & -1 \\ 6 & 14 & 6 \end{pmatrix} = 1\,(24-24+56-24+56-24)$

$= 64$

$C_{34} = (-1)^7 \begin{pmatrix} 4 & 4 & 0 \\ 1 & 1 & 0 \\ 6 & 14 & 3 \end{pmatrix} = -1\,(12+0+0-0-0-12)$

$= 0$

$C_{41} = (-1)^5 \begin{pmatrix} 4 & 0 & 4 \\ 1 & 0 & -1 \\ 0 & -3 & 1 \end{pmatrix} = -1\,(0+0-12-0-12-0)$

$= 24$

$C_{42} = (-1)^6 \begin{pmatrix} 4 & 0 & 4 \\ 1 & 0 & -1 \\ 3 & -3 & 1 \end{pmatrix} = 1\,(0+0-12-0-12-0)$

$= -24$

$C_{43} = (-1)^7 \begin{pmatrix} 4 & 4 & 4 \\ 1 & 1 & -1 \\ 3 & 0 & 1 \end{pmatrix} = -1\,(4-12+0-12-0-4)$

$= 24$

$C_{44} = (-1)^8 \begin{pmatrix} 4 & 4 & 0 \\ 1 & 1 & 0 \\ 3 & 0 & -3 \end{pmatrix} = 1\,(-12+0+0-0-0+12)$

$= 0$

$$\det(A) = a_{11}(C_{11}) + a_{12}(C_{12}) + a_{13}(C_{13}) + a_{14}(C_{14})$$
$$\det(A) = 4(-63) + 4(48) + 0(-68) + 4(-15)$$
$$\det(A) = 4(-63+48-15) = -120$$

determinan matriks A juga bisa didapat melalui operasi baris elementer (OBE)

Determinan matriks tersebut adalah -120

- Mencari balikan matriks

Invers matriks $A \longrightarrow A^{-1} = \frac{1}{\det(A)} \cdot adj(A)$

$$adj(A) = \begin{pmatrix} C_{11} & C_{21} & C_{31} & C_{41} \\ C_{12} & C_{22} & C_{32} & C_{42} \\ C_{13} & C_{23} & C_{33} & C_{43} \\ C_{14} & C_{24} & C_{34} & C_{44} \end{pmatrix}$$

$$adj(A) = \begin{pmatrix} -63 & -84 & 24 & 24 \\ 48 & 24 & -24 & -24 \\ -68 & -64 & 64 & 24 \\ -15 & 60 & 0 & 0 \end{pmatrix}$$

$$A^{-1} = \frac{1}{\det(A)} \cdot adj(A)$$

$$A^{-1} = \frac{-1}{120} \begin{pmatrix} -63 & -84 & 24 & 24 \\ 48 & 24 & -24 & -24 \\ -68 & -64 & 64 & 24 \\ -15 & 60 & 0 & 0 \end{pmatrix}$$

$$A^{-1} = \begin{pmatrix} 21/40 & 7/10 & -1/5 & -1/5 \\ -2/5 & -1/5 & 1/5 & 1/5 \\ 17/30 & 8/15 & -8/15 & -1/5 \\ 1/8 & -1/2 & 0 & 0 \end{pmatrix}$$

Balikan matriks dari soal diatas adalah

$$\begin{bmatrix} 21/40 & 7/10 & -1/5 & -1/5 \\ -2/5 & -1/5 & 1/5 & 1/5 \\ 17/30 & 8/15 & -8/15 & -1/5 \\ 1/8 & -1/2 & 0 & 0 \end{bmatrix}$$