Applications of Linear Algebra in Economics

Nathan James Runtuwene 13514083
Program Studi Informatika
Sekolah Teknik Elektro dan Informatika
Institut Teknologi Bandung, Jl. Ganesha 10 Bandung 40132, Indonesia
13514083@stei.itb.ac.id

***Abstract*—The input-output method is a well-known method in economics which uses linear algebra. It uses property of matrices to find a way to calculate the effects of changes in the market to price, demand, or supply.**

***Keywords*—economics, input-output matrices, linear algebra, supply and demand**

I. Introduction

Economics is always closely related to linear algebra. Even from around 200 BC people used linear algebra for economy. Linear algebra can be said to originate from peoples need to count their earnings. In this paper we will talk about one of the methods of analyzing economy using a method called the input-output model.

In 1973, an American economist named Wassily Wassilyevich Leontief won a Nobel price in economic sciences for his work on the input-output analysis. The input-output analysis creates an environment to make it easier to calculate and predict the supply and demand of a system. The input-output analysis can work for small businesses or even major companies, and also for both closed systems and systems that has commodities flowing in and out of it without much difference in the method.

Because the input-output model is based on linear algebra, computation on it can be done very rapidly. Its linear nature also makes it very flexible to compute changes in demand. Input-output models from different businesses or regions can also be analysed to investigate the trade relationship between those businesses or regions.

The input-output model can even show the relation between things that are relatively not related to economics such as pollution and population with prices.

II. Input-Output Matrices

The input-output analysis starts from creating the input-output matrix. The input-output matrix represents the amount of resources an industry produces which are used by other industries, including itself. The rows of the matrix represents the producing industry, while the columns represent he consuming industry. An example of the matrix is shown on Figure 1.

$$\begin{array}{r} \\ Agriculture(A) \\ Manufacture(M) \\ Labor(L) \end{array} \begin{array}{c} \begin{array}{ccc} A & M & L \end{array} \\ \begin{bmatrix} 10 & 20 & 30 \\ 5 & 30 & 12 \\ 12 & 8 & 8 \end{bmatrix} \end{array}$$

Figure 1. Input-output matrix example

On the example matrix shown on figure 1, cell (1,1) represents the amount of product that the agriculture sector uses on its own, cell (1,2) and cell (1,3) represents the amount of product that the agriculture sector produces that the manufacturing sector and the labor sector uses, and so on. The input-output matrix is very scalable and is not restricted to just three sectors.

The total internal demand of a certain product is simply the sum of values on that row. For example, again using the matrix on figure 1, the total internal demand for the products that the manufacturing sector produces will be 5+30+12=47 units of product. All of the internal demand can be represented as a total internal demand vector. An example of an internal demand vector for the matrix on figure 1:

$$\begin{array}{r} \\ Agriculture(A) \\ Manufacture(M) \\ Labor(L) \end{array} \begin{array}{c} \text{Internal Demand} \\ \begin{bmatrix} 120 \\ 235 \\ 168 \end{bmatrix} \end{array}$$

Figure 2. Internal Demand Vector

The above matrices show the flow of goods between the industries, however we can not know the value of those goods that are traded and thus we can not know the relative size of an industry related to currency. To convert the input-output matrix to a valued input-output matrix (an input-output matrix based on currency) we simply need to multiply the input-output matrix with the price/value matrix. The price matrix is a matrix where the diagonals of the matrix is equal to the price of a unit produced by the related industry, that is for cell (i,i) in the matrix, its value will be the price of one unit of the product produced by industry i. The example of a price matrix:

$$\begin{array}{r} \\ Agriculture(A) \\ Manufacture(M) \\ Labor(L) \end{array} \begin{array}{c} \text{Price} \\ \begin{bmatrix} 2 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 6 \end{bmatrix} \end{array}$$

Figure 3. Price Matrix

After we have this matrix, we multiply it with the input-output matrix to get the valued input-output matrix.

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 6 \end{bmatrix} \begin{bmatrix} 10 & 20 & 30 \\ 5 & 30 & 12 \\ 12 & 8 & 8 \end{bmatrix} = \begin{bmatrix} 2*10 & 2*20 & 2*30 \\ 5*5 & 5*30 & 5*12 \\ 6*12 & 6*8 & 6*8 \end{bmatrix}$$

$$\text{Valued Input-Output Matrix} = \begin{bmatrix} 20 & 40 & 60 \\ 25 & 150 & 60 \\ 72 & 48 & 48 \end{bmatrix}$$

Figure 4. Valued Input-Output Matrix

The valued demand vector can also be done simply by adding the values on the same row.

$$\begin{array}{r} \\ Agriculture(A) \\ Manufacture(M) \\ Labor(L) \end{array} \begin{array}{c} \text{Internal Demand} \\ \begin{bmatrix} 120 \\ 235 \\ 168 \end{bmatrix} \end{array}$$

Figure 5. Valued Internal Demand Vector

In addition to the internal demand, the input-output model also supports demands from either unproductive sectors (sectors that only consumes), sectors from outside, or both. These external demands can be represented by a vector called the final demand vector. Also, define a vector which will have the total products produced by a sector which will fulfill both the internal demand and the final demand and call it the amount produced vector.

$$\begin{bmatrix} \text{Amount} \\ \text{Produced} \end{bmatrix} = \begin{bmatrix} \text{Internal} \\ \text{Demand} \end{bmatrix} + \begin{bmatrix} \text{Final} \\ \text{Demand} \end{bmatrix} \quad (1)$$

III. Technical Coefficients and Structural Matrices

Technical coefficients are the amount of goods per unit of good produced that is consumed. A general way of calculating a technical coefficient is to divide a cell in fig. 1, expressed with $a_{i,j}$, with the corresponding value in the amount produced vector expressed as x_j. That is :

$$c_{i,j} = \frac{a_{i,j}}{x_j} \quad (2)$$

Let's say the amount produced vector is equal to:

$$\text{Amount Produced} = \begin{bmatrix} 135 \\ 350 \\ 200 \end{bmatrix}$$

Figure 6. Amount Produced Vector

So the structural matrix of the matrix on figure 1 is

$$\begin{array}{r} Agriculture(A) \\ Manufacture(M) \\ Labor(L) \end{array} \begin{array}{c} \begin{array}{ccc} A & M & L \end{array} \\ \begin{bmatrix} \frac{10}{135} & \frac{20}{350} & \frac{30}{200} \\ \frac{5}{135} & \frac{30}{350} & \frac{12}{200} \\ \frac{12}{135} & \frac{8}{350} & \frac{8}{200} \end{bmatrix} \end{array}$$

Figure 7. Structural Matrix

If the structural matrix is composed of technical coefficients that are using the ratio of currency to amount produced, the matrix is usually called the consumption matrix.

The sum of the columns of a consumption matrix can indicate some important traits of a business. If the sum is less than 1, then it means that the cost to produce the products is less than the value they sell for, which means the company is currently profiting. If the sum is exactly one, then it means that the cost used in production is equal to the value that the products are sold for. If the sum is more than one, then that means the cost to produce is more than the value the products sell for and the company is suffering a loss.

IV. Unit Consumption Vectors

As previously mentioned, the column of an input-output matrix is equal to the amount of products that a sector consumes. The unit consumption vectors are the columns of a structural matrix. This unit consumption vector represents the amount of consumption needed to produce a single product.

If a sector wants to produce an amount of goods, lets call it x_i, then the amount of goods they need to consume to produce x_i amount of goods is equal to x_i times its unit consumption vector.

The total of demand for all the sectors is also the internal demand, which can now be written as:

$$\text{Internal Demand} = x_1 c_1 + x_2 c_2 + \ldots + x_n c_n = xC \quad (3)$$

With C is a consumption matrix. And x is the amount produced vector.

V. THE LEONTIFF INPUT-OUTPUT MODEL

With the formulas we have written above we can now get

$$x = Cx + f \tag{4}$$

x is the amount produced vector, C is the consumption matrix and f is the final demand vector.

Applying the identity rule on the left hand side we get

$$Ix = Cx + f \tag{5}$$

$$(I - C)x = f \tag{6}$$

A consumption matrix is considered economically feasible if the sum of any column of C is less then 1. Then the next theorem follows:

If C and f have non-negative entries and C is economically feasible then matrix (I-C) has an inverse and thus:

$$x = (I - C)^{-1} f \tag{7}$$

With (7), we can now calculate the expected amount produced and internal demand by simply knowing the final demand and the consumption matrix.

VI. CONCLUSION

In conclusion, the input-output model is very useful in economics because with this model we can calculate the expected effects of changes in the final demand or the consumption matrix. For example, the price of a certain item is changed, so we simply need to change the consumption matrix to adjust and we will have our expected production amount. This method can also be used as a tool to help companies set up prices so that they can have the maximum profit. We can also try to change our output as the producer and see what effects it has on the consumption matrix and the final demand vector.

VII. ACKNOWLEDGMENT

I would like to thank Mr. Rinaldi and Mr. Judhi for guiding me through this class. I would also like to thank my family and friends who helped me finish this paper.

REFERENCES

[1] http://www.math.unt.edu/~tushar/S10Linear2700%20%20Project_files/Davidson%20Paper.pdf , last opened on December 14, 2015

[2] https://math.dartmouth.edu/archive/m22f06/public_html/leontief_slides.pdf , last opened on December 15, 2015

[3] Thijs Ten Raa, *Input–output economics: theory and applications: featuring Asian economies*, World Scientific, 2009

PERNYATAAN

Dengan ini saya menyatakan bahwa makalah yang saya tulis ini adalah tulisan saya sendiri, bukan saduran, atau terjemahan dari makalah orang lain, dan bukan plagiasi.

Bandung, 16 Desember 2015

Nathan James Runtuwene 13514083