Orbital Trajectory Simulation of Satellite around Space Object by Fractal Animation Model

based on Shifting Centroid from a Fixed Point

Tedjo Darmanto
Informatics Department
STMIK AMIK Bandung
Bandung, Indonesia
tedjodarmanto@gmail.com

Iping Supriana Suwardi
Program Studi Teknik Informatika
Institut Teknologi Bandung
Bandung, Indonesia
iping@informatika.org

Rinaldi Munir
Program Studi Teknik Informatika
Institut Teknologi Bandung
Bandung, Indonesia
rinaldi@informatika.org

***Abstract*—Animation of two celestial objects as a pair moving around a fixed point and one of them is as a satellite of another as a planet is very interesting to be simulated, so the orbital trajectory of the satellite rotating around the planet which is rotating around a fixed point can be observed, predicted and explained for educational purpose. Compared to the conventional animation, the fractal animation model is easier to be handled just by calling the rotation procedure for the satellite object around the planet object and the same revolution procedure around a fixed point for both objects with a certain parameter to determine the speed of rotation and revolution.**

Keywords—fractal; animation model; orbital trajectory; shifting centroid; fixed point

I. INTRODUCTION

In the educational point of view especially in Mechanics, the basic of the Newtonian Physics, the movement animation of two objects dynamically relative from a fixed point in a pair, one object is as a planet and the other is as a satellite is very interesting to be observed, so it can be predicted and explained better as a new model than in static model. The Fractal animation model has advantages over the non-fractal model. One of the advantages is the animation algorithm in fractal animation model is more simple than in traditional way, especially to handle the rotation of the satellite around planet, which is revolved around a fixed point. To most advantageous of fractal model is the ability to control the rotation and revolution a pair of fractal object in synchronous mode, so the combination of different speed between the rotation and revolution can be simulated easily. In this paper, to propose a new method to simulate the orbital trajectory of space objects, the use of two fractal objects are enough for simplifying the simulation and animation instead of two multi-objects, because a space object can be represented by as a simple fractal object as a star-like object of IFS fractal. In the case of more complicated object, multi-object of fractal can be used instead of a simple object as described in the last reference [14].

II. FRACTAL MODELS

A. Fractal

The term fractal itself is coined for the first time by Mandelbrot [1] and as a honor to his contribution, his name cannot be separated from and became the name of the famous fractal, the Mandelbrot set besides the Julia set in the early time of the emerging fractals as a new field of science especially in mathematics and computer science. In general there are at least two major fractal models are emerged as the applicative models, i.e. the IFS fractal model and the L-system fractal model, as described in the next sub-sections.

B. L-System Fractal

The L-system is the fractal model which is introduced by Lindenmayer, so the letter-'L' can be interrelated as the initial name of him. The L-system model is suitable to model and reconstruct plant-like objects governed by rules like turtle movements iteratively in such a way so the appearance of plant-like objects so natural [5].

C. IFS Fractal

The IFS code model is the fractal model which is introduced for the first time by Barnsley and Demko [3] based on Hutchinson's idea, selif-similarity [2] and is became popular when the famous fern-like fractal is modeled by Barnsley in his book entitled: "Fractals Everywhere" [6]. Basically the 2D IFS fractal model is based on the contractive affine transformation function or is just called as CAT function which has six coefficients (**a, b, c, d, e,** and **f**) and can be expressed as a matrix and a vector in mathematical equation (1) below.

$$w\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}a & b\\c & d\end{bmatrix} * \begin{bmatrix}x\\y\end{bmatrix} + \begin{bmatrix}e\\f\end{bmatrix} \qquad (1)$$

The coefficients in IFS code set as expressed in equation (1) above are the affine transformation coefficients that can be classified into two kinds of transformation, the linear and the rotational transformations as described in the next section. Both transformations are needed in animation of orbital trajectory simulation of a space object around another space object and are explained later in the next section. Actually the linear transformation consists of collection of several primitive functions, such as move in X and Y-directions (2D), scaled up and down (zoom-in and zoom-out) and mirror to the X, Y, or both X and Y-axis (2D) [3, 4], but in accordance with the context of this paper, only the move functions are explained in the next sub-sections of the next section.

III. AFFINE TRANSFORMATION AND COLLAGE THEOREM

A. Linear Transformation

By moving fractal object in **X**-direction by a unit distance dX, the new coefficient-**e’** and **f’** of the 2D IFS code are changed based on the new coefficient-**a’** and **c’** which depend on the previous coefficient-**a** and **c** [4] as described in equation (2.a) to (2.d) below in iterative way:

$$a' = 1.0 - a * dX \quad (2.a)$$

$$c' = dX * c \quad (2.b)$$

$$e' = e + a' \quad (2.c)$$

$$f' = f - c' \quad (2.d)$$

By moving fractal object in **Y-**direction by a unit distance **dY**, the new coefficient-**e’** and **f’** of the 2D IFS code are changed based on the new coefficient-**b’** and **d’** which depend on the previous coefficient-**b** and **d** [4] as described in equation (3.a) to (3.d) below in iterative way:

$$b' = dY * b \quad (3.a)$$

$$d' = 1.0 - d * dY \quad (3.b)$$

$$e' = e - b' \quad (3.c)$$

$$f' = f + d' \quad (3.d)$$

Both move functions will be used in the rotation of satellite around the planet explained in the corresponding section below.

B. Rotational Transformation

To accomplish the rotational animation in X-Y plane around Z-axis or a fixed point (0, 0) as centroid, the next six 2D IFS code coefficients are changed based on the current six 2D IFS code coefficients iteratively by a small deviation angle (dA) [4] as described in the six equations (4.a) to (4.f) below:

$$a'=a*\cos(dA)*\cos(dA)-(b+c)*\cos(dA)*\sin(dA)+ d*\sin(dA)*\sin(dA) \quad (4.a)$$

$$b'=(a-d)*\cos(dA)*\sin(dA)+b*\cos(dA)*\cos(dA)- c*\sin(dA)*\sin(dA) \quad (4.b)$$

$$c'=(a-d)*\cos(dA)*\sin(dA)-b*\sin(dA)*\sin(dA)+ c*\cos(dA)*\cos(dA) \quad (4.c)$$

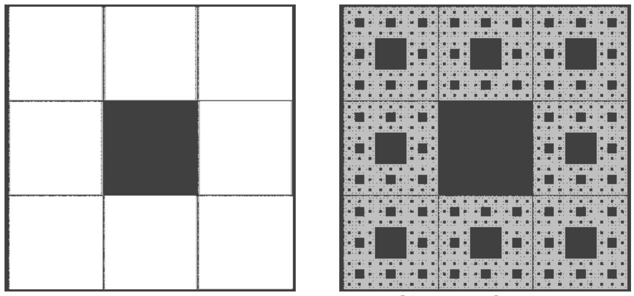
$$d'=a*\sin(dA)*\sin(dA)+(b+c)*\cos(dA)*\sin(dA)+ d*\cos(dA)*\cos(dA) \quad (4.d)$$

$$e'=e*\cos(dA)-f*\sin(dA) \quad (4.e)$$

$$f'=e*\sin(dA)+f*\cos(dA) \quad (4.f)$$

C. Collage Theorem

The collage theorem is explained concisely by Barnsley and with the self-similarity property of fractal are became the foundation of the IFS model [6]. The best way to explained what is collage theorem is by giving an example such as illustrated in figure 1 (a) and 1 (b). Each region in collage layout is as part of and resembles the form of the whole the correlated IFS code is displayed in Table 1 below. All coefficients in each column is the same for all rows, except for the last two columns, column with coefficient-**e** is representing the position of south-west corner of each region in collage layout relatively to the axis-**Y** and column with coefficient-**f** is representing the position of south-west corner of each region in collage layout relatively to the axis-**X**. So the Sierpinsky gasket object as an example object has the centroid at the south-west corner of the object.



(a) 8 regions of collage layout (b) Sierpinsky carpet

Fig 1. Implementation example of collage theorem

TABLE I. IFS CODE OF SIERPINSKY CARPET AS AN EXAMPLE

a	b	c	d	e	f
0.33	0.00	0.00	0.33	0.00	0.00
0.33	0.00	0.00	0.33	0.33	0.00
0.33	0.00	0.00	0.33	0.66	0.00
0.33	0.00	0.00	0.33	0.00	0.33
0.33	0.00	0.00	0.33	0.66	0.33
0.33	0.00	0.00	0.33	0.00	0.66
0.33	0.00	0.00	0.33	0.33	0.66
0.33	0.00	0.00	0.33	0.66	0.66

IV. RELATED WORKS

There are not many related works in accordance with this paper topic except the generic theory in IFS fractal field such as collage theorem, IFS inverse problem and decoding algorithm, and the multi-object of fractal model. There are at least two papers representing the research in collage theorem, as are proposed by Olien et.al and Honda et.al [7, 8]. The proposed theorem as the extension of collage theorem can reconstruct fractal image with less errors and fewer iteration than the previous theorem. In the IFS inverse problem algorithm, there are also at least two papers, the Algorithm proposed by Wardstromer is used to automate the Barnsley algorithm [10] and the algorithm proposed by Sarafopoulos et.al is based on evolutionary algorithm which is more general solution than the previous one [11]. As the representation papers in the IFS decoding or construction algorithm, there are also two papers, in both proposed papers the fast algorithm are presented. The first paper is proposed by Chu et.al which is faster on large scale of computation [9]. The second paper is proposed by Zhao et.al based on twice pre-searching method which is causing 80% coding time can be reduced and the quality of reconstructed image can be increased [12]. To generate multi-object of fractal, the partitioned-random iteration algorithm can be used, so the dense of pixels population in individual object as the part of multi-object can be controlled [13, 14]. The hybrid animation model based on the metamorphic interpolation model and partitioned-random iteration algorithm model recently has been proposed to simulate the complicated multi-object of fractal [14].

V. METHOD

Two things need to be discussed in this section, as already mentioned in the previous section, the combination of rotational and linear transformations are needed to create a new methods the rotation, revolution and shifting centroid as described in the sub-sections below.

A. Rotation and Revolution

The rotation and revolution animation can be handled by rotation operation as described in equation (4.a) to (4.f) at the Rotational transformation sub-section above. The only different between rotation and revolution procedures is the position of centroid relatively to the origin point (0,0), so in this paper the method of shifting centroid is proposed as described in the next sub-section.

B. Shifting Centroid

From a fixed point of view at (0, 0), any objects in 2D plane have two kinds of centroid, the general centroid which is the same as absolute centroid and the local centroid which are existed at any points other than (0, 0). To generate an animation of one object (A) which has local centroid orbiting around another object (B) which has another local centroid or centroid not at (0, 0) with a certain distance (D), first the centroid of A is shifted to the absolute centroid then operate the rotation procedure by a small angle (dA) once and the centroid is shifted back to the new position which has the distance (D). The new X and Y values calculated based on (dA) as illustrated in figure-2 below. Those steps are repeated as many as the rotation animation steps are needed until are stopped.

VI. ANIMATION

For the preparation of animation, two star-like objects which have a point as a center are prepared. Both object have the same form but is differ in size and have different centroid. The first object as a planet has bigger size and the center of object has offset farther than the second object as a satellite as illustrated in figure-3 below. The IFS code of both objects are displayed in table-2 and table-3 below. The difference of each IFS code can be observed at column-**e** and **f** (in bold type). The first eight rows in table-2 and table-3 are representing eight lines of object with the same probability-**p** values. The last row in table-2 and table-3 is representing the center of object as a dot with smallest probability-**p** value.

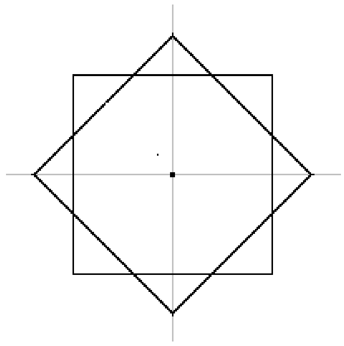


Fig 2. Star-like fractal with center point

TABLE II. IFS CODE OF STAR-LIKE FRACTAL (1) WITH PROBABILITY FACTOR P

a	b	c	d	e	f	p
0.712	0.178	0.000	0.000	1.245	**-.267**	0.12
0.000	0.000	-.712	-.178	1.767	**-.255**	0.12
-.712	-.178	0.000	0.000	1.755	**0.267**	0.12
0.000	0.000	0.712	0.178	1.233	**0.255**	0.12
0.267	0.445	0.267	0.445	0.652	**-.487**	0.12
0.267	0.445	-.267	-.445	1.987	**-.848**	0.12
-.267	-.445	-.267	-.445	2.348	**0.487**	0.12
-.267	-.445	0.267	0.445	1.013	**0.848**	0.12
0.020	0.000	0.000	0.020	1.470	**0.000**	0.04

TABLE III. IFS CODE OF STAR-LIKE FRACTAL (1) WITH PROBABILITY FACTOR P

a	b	c	d	e	f	p
0.712	0.178	0.000	0.000	**0.202**	**-.074**	0.12
0.000	0.000	-.712	-.178	**0.487**	**-.211**	0.12
-.712	-.178	0.000	0.000	**0.624**	**0.074**	0.12
0.000	0.000	0.712	0.178	**0.340**	**0.211**	0.12
0.267	0.445	0.267	0.445	**0.080**	**-.035**	0.12
0.267	0.445	-.267	-.445	**0.448**	**-.333**	0.12
-.267	-.445	-.267	-.445	**0.746**	**0.035**	0.12
-.267	-.445	0.267	0.445	**0.379**	**0.333**	0.12
0.020	0.000	0.000	0.020	**0.405**	**0.000**	0.04

VII. SIMULATION

The simulation is conducted and observed every 3 unit of time as illustrated in figure-4 below. At T0 object-1 as a planet at X-axis with offset 1.47 from origin and object-2 as a satellie at X-axis with offset 0.405 from origin. At T3 object-2 orbiting object-1 in counter-clockwise fashion, while object-1 revolving the origin in clockwise fashion, etc. The object-2 is always facing object-1, because the centroid when the rotation operation is occurred at the center of object-1 represented by dot in the middle of object-1. The object-1 as also facing the origin as a fixed point as the centroid when the rotation operation is occurred.

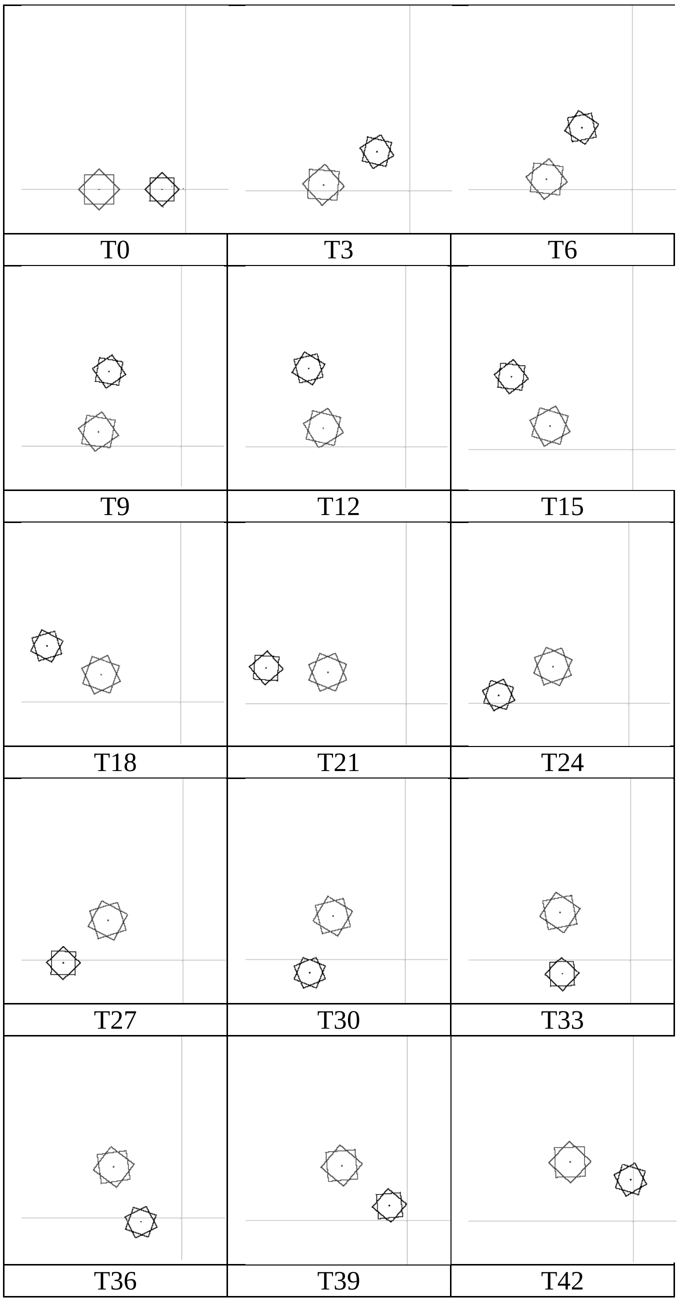


Fig 3. Orbital trajectory simulation

VIII. CONCLUSION

By applying shifting centroid method to the centroid of satellite from the original position to the origin point as a fixed point back and forth step by step, the animation of a pair of celestial objects can be accomplished by fractal model

The orbital trajectory of satellite rotating planet can be observed, predicted and calculated any time by recording the IFS coefficients for every movement and the satellite is always facing the planet while rotating it

The rotation simulation of the satellite which is always rotating and facing the planet and the revolution simulation of the planet which is always rotating and facing the origin point can be accomplished by fractal method

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