Optimizing Flight Cost in Indonesia Using Undirected Multi-Graph Theory and Dijkstra’s Algorithm

Vidia Anindhita - 13512034
Program Studi Teknik Informatika
Sekolah Teknik Elektro dan Informatika
Institut Teknologi Bandung, Jl. Ganesha 10 Bandung 40132, Indonesia
vidianindhita@students.itb.ac.id

Abstract—Air traffic management is the most important thing which encompassing all systems that assist airplanes to depart and arrive, controlling airplanes flight in sky, and also preserving safety flight in order to prevent airplanes crash. The air traffic management is needed so that it can establish the air transportation, especially in Indonesia. As a developing country, air transportation in Indonesia has not balanced yet. This paper will be further discuss about how to optimize the flight cost by understanding more about graph theory especially undirected multi-graph and using Dijkstra’s algorithm, so the air transportation in Indonesia then will be more efficient.

Index Terms—air traffic, flight network, graph theorem, undirected multi-graph, Dijkstra’s algorithm.

I. INTRODUCTION

As an archipelago and a maritime country, Indonesia needs more air transportations to connect each of the islands. Although more than half of the citizens are in Java Island, but for supporting economic growth Indonesia really needs to increase the air transportation system. Indonesian citizens still think that air transportation is not cheap or safe. The price and safety are actually depends on the air traffic and airports disposition. The more airplanes that transit to some areas, the much more gasoline that is needed. It will specify the flight cost.

Indonesian air traffic is getting busy each year. It is clearly shown that the user of air transportation is increasing from year to year. In 2012, air transportation users are recorded about 72.2 millions. This enhancement then will have impact to the air traffic in Indonesia. Nowadays, about 5000 airplanes pass through Indonesia’s sky each day. This problem requires the provision of sufficient air space capacity where of course the dimension of Indonesia’s air territory cannot be expanded. So that the air traffic management is needed and should be developed in order to connect more cities in the most efficient way.

Along with the air navigation technology development with the support of navigation tools in land and the development of space satellites, will take effects to the flight safety and air space capacity enhancement in Indonesia. This enhancement then will of course help the air traffic management so that the flight efficiency will be fulfilled.

This paper will discuss a way of making air traffic management more efficient by concerning about how to connect countless cities in the most efficient way that will lead to moving the most passengers with the fewest possible trips, so that the flight cost can be pressed. When finding the most efficient flight cost, we can use undirected multi-graph and Dijkstra’s algorithm. Thus, the basic understanding of graph theory must be needed because this theory will bring us to know about flight network that can establish the efficiency of air traffic management.

II. FUNDAMENTAL THEORIES

2.1 Definition of Graph

Graphs are discrete structures consisting of vertices and edges that connect to these vertices. Mathematically, graph is defined: [1]

Definition 2.1. Graph G is defined as set (V,E) where:
- \( V \) = a nonempty set of vertices
  = \{v_1, v_2, v_3, ..., v_n\}
- \( E \) = a set of edges that connect a pair of vertices. Each edge has either one or two vertices associated with it, called endpoints. An edge is said to connect its endpoints.
  \( E = \{e_1, e_2, e_3, ..., e_n\} \)

Graph can be grouped into several categories regarding whether edges have direction, same pair of vertices, or loops are allowed. Here is the example of graph:

![Figure 2.1 A Graph with 5 Vertices and 5 Edges](image-url)
The graph above is an undirected graph with 5 vertices and 5 edges. The vertices are 1, 2, 3, 4, 5.

2.2 Directed Graph
A directed graph $G = (V, E)$ consists of $V$, a nonempty set of vertices (or nodes), and $E$, a set of directed edges or arcs. Each edge is an ordered pair of vertices. The directed edge $(u,v)$ is said to start at $u$ and end at $v$.

**Definition 2.2** Let $(u,v)$ be an edge in $G$. Then $u$ is the initial vertex of this edge and is adjacent to $v$ and $v$ is the terminal (or end) vertex of this edge and is adjacent from $u$. The initial and terminal vertices of a loop are the same.\(^2\)

Here is an example of a directed graph:

![Directed Graph Example](image)

Figure 2.2 A Directed Graph

The directed graph above has 5 vertices with different oriented directions. The edges or arcs determine the direction of graph $G$. The arc $e_1$ shows that the direction is from vertex 1 to vertex 2, or can be written $(1, 2)$, which is not the same as $(2, 1)$.

2.3 Multi-graph
A multi-graph $G = (V,E)$ consists of a set of vertices $V$ and a set of edges $E$ such that set $E$ may contain multiple edges and self loops.\(^3\) For example:

![Multi-Graph Example](image)

Figure 2.3 A Multi-Graph

From the figure 2.3, it is shown that $e_1$, $e_2$, $e_4$, and $e_5$ are multiple edges.

2.4 Directed Multi-graph
A directed multi-graph is an interflow of directed graph and multi-graph. Like directed graphs, but there may be more than one edge from a vertex to another. A directed multi-graph $G = (V,E,f)$ consists of a set $V$ of vertices, a set $E$ of edges, and a function $f : E \rightarrow V \times V$. Here is an example of a directed multi-graph:

![Directed Multi-Graph Example](image)

Figure 2.4 A Directed Multi-graph

The directed multi-graph above shown that edge $e_1$ is different from edge $e_2$ because edge $e_1$ is an edge which connect vertex 2 to vertex 1 whether $e_2$ is an edge which connect vertex 1 to vertex 2. And so that for edge $e_3$ is different from edge $e_5$.

2.5 Weighted Graph
A weighted graph is a graph $G$ in which each edge $e$ has been assigned a real number called weight of $e$.\(^1\) This weight of each edge represents the distance between two vertices. Below is the example of weighted graph.

![Weighted Graph Example](image)

Figure 2.5 Weighted Graph

The weighted graph above means that every edge between to vertices has weight that represents the distance between them.

2.6 Dijkstra’s Algorithm

The Dijkstra’s algorithm is an algorithm which determines the shortest path in graph $G = (V,E)$ to the remaining vertices in the graph. When the shortest path to a particular vertex is desired, the algorithm is terminated when the shortest path to that vertex has been found. In what follows, we denote the source vertex by A and the particular destination vertex we are interested in by Z.

**Definition 2.6** Let $d(i)$ denote the distance of vertex $i$ ($i \in V$) from source vertex $A$; it is the sum of arcs in a possible path from vertex $A$ to vertex $i$. Let $P(i)$ denote the predecessor of vertex $i$ on the same path. Note that $d(A) = 0$. The following steps result in the determination of the shortest path from $A$ to $Z$:

**Step 1.** Start with $d(A) = 0$, $d(i) = l(Ai)$, if $i \in \Gamma_A$, $\infty$, otherwise ($\infty$ is a large number defined below); \(\Gamma\) = set of neighbor vertices of vertex i, $l(ij)$ = length of arc from vertex $i$ to vertex $j$.

Assigns $S = V - \{A\}$, where $V$ is the set of vertices in the given graph.
Assigns \( P(i) = A \) if \( i \in S \).

Step 2. a) Find \( j \in S \) such that \( d(j) = \min d(i), i \in S \).
b) Set \( S = S \setminus \{ j \} \).
c) If \( j = Z \) (the destination vertex), END; otherwise, go step 3.

Step 3. \( \forall i \in \Gamma \) and \( i \in S \), if \( d(j) + l(ji) < d(i) \), set \( d(i) = d(j) + l(ji), P(i) = j \).

Go to step 2.\[4\]

In Step 1, the vertices of the graph are assigned an initial value of distance of \( \infty \) except for vertex A. The distance assignments (temporary labels) of the vertices then will be updated in Step 3 via scanning neighbors of a vertex selected (permanently labeled) in Step 2a (scanning is limited to only those neighbors that belong to the set \( S \)). When the label of a vertex is updated, so is its predecessor vertex in Step 3.

As a vertex is selected (permanently labeled) in Step 2a, the shortest path to that vertex from vertex A is considered found. This algorithm finds the shortest path to the nearest vertex from the source vertex A, and then the second nearest vertex, and so on. Examining of the predecessor array \( P(i) \) for the above example shows that \( P(Z) = D, P(D) = C, P(C) = A \), which implies the shortest path is ACDZ.

Or, we can take a look at the pseudo-code of Dijkstra’s algorithm below.\[5\]

\[
\begin{align*}
\text{dist}[s] &\leftarrow 0 \\
\text{for all } v \in V \setminus \{s\} &\text{ do } \text{dist}[v] \leftarrow \infty \\
S &\leftarrow \emptyset \\
Q &\leftarrow V \\
\text{While } Q \neq \emptyset &\text{ do } u \leftarrow \text{mindistance}(Q, \text{dist}) \\
S &\leftarrow S \cup \{ u \} \\
\text{for all } v \in \text{neighbors}[u] &\text{ do if } \text{dist}[v] > \text{dist}[u] + w(u,v) \text{ then } \text{dist}[v] \leftarrow \text{dist}[u] + w(u,v) \\
\text{return } \text{dist}
\end{align*}
\]

2.7 Flight Network

First we should know what network is. Formally, networks are defined as

\[
\text{Network} = \text{objects} + \text{connections}
\]

A network \( N \) can easily be understood as a set \( V \) of vertices and a set \( E \) of edges. The maximum number of edges in any given network is the Cartesian product of \( V \times V \), because edges can only be constructed between nodes. And set \( E \) of edges in a network is at least a subset of \( V \times V \).

Definition 2.7 Network \( N \) consists of set \( V \) of vertices and set \( E \) of edges, where \( E \in V \times V \).

In flight network, it is also known a circle, which is a sequence of edges in which the first and the last nodes are the same. Also in flight network, the nodes represent airports and the edges indicate flights. The use of directed graph and weighted graph are also known in flight network.

### III. ANALYZING AIR TRAFFIC MANAGEMENT IN INDONESIA

#### 3.1 Domestic Flight Route Map

The domestic flight in Indonesia generally centered in the big cities. Take a look at Indonesian domestic flight route map below.
In Figure 3.1.1, we notice that the flight route is the implementation of multi-graph. And basically the flights are in two-way. Here, even the flights are in two-way and because actually flight from airport A to airport B (A,B) is different from flight from airport B to airport A (B,A), as a directed graph, but we can make it as undirected graph because the paths that are used in flights are the same path. Here is the description why we can make it as undirected graph for airline flights.

Figure 3.1.2 Airline Flight Illustration [8]

For enhancing airspace capacity, the air traffic uses the same path for two-way flight with the 2000 feet safe distance separation. Now we can make the undirected multi-graph for the route map.

Figure 3.1.3 Undirected Multi-graph from Indonesian Domestic Flight Route

According to Figure 3.1.3, the vertices represent the airports while the edges represent the flights.

3.2 Analyzing Flight Cost in Indonesia

The flight cost is affected by several aspects, which are from the fuel cost, airplane maintenance cost, airport tax, until the cost for the airlines. But here we assume the flight cost depends only from the fuel cost, because it is the biggest factor in determining the flight cost. In Indonesia, the aviation fuel is still imported from other countries, so that the cost is depended on US Dollar. Nowadays the US Dollar is reaching about IDR 12,000. That means the aviation fuel cost is also increasing. By December 2013, a liter of aviation fuel is about IDR 9,845.

For the commercial airplanes that usually use Boeing 737, the fuel needs is about 3200 liter for an hour of running time. So then we can calculate the flight cost considering the time travel and the fuel cost. For example the time travel from Soekarno-Hatta International Airport (CGK) in Jakarta to Achmad Yani International Airport (SRG) in Semarang takes about 1.05 hour. Then we can calculate for one-way trip, it cost $1.05 \times 3,200 \times IDR 9,845 = IDR 33,079,200 or 33.08 million rupiahs.

According to Indonesia’s geographical location, not all cities and its destinations can be reached through a one-way trip. There has to be a transit in some places. And of course this will affect the flight cost. To reduce the flight cost, we can determine which flight path should be taken by choosing the right path or the shortest path to the destination. Therefore, the writer would consider using a Dijkstra’s algorithm within the undirected multi-graph to choose the shortest path so that the flight cost will be reduced.

IV. IMPLEMENTATION OF DIJKSTRA’S ALGORITHM IN OPTIMIZING FLIGHT COST IN INDONESIA

The Dijkstra’s algorithm will be implemented in optimizing flight cost in Indonesia by finding the shortest path that cost less fuel. Here we assume the airports as the vertices, the flight paths as edges, and the cost of fuel per one-way trip as the weights. Let us define the Figure 4.1 below as an undirected multi-
Here is the table of domestic flight cost calculation in national range:

<table>
<thead>
<tr>
<th>Airport</th>
<th>Time Travel (hour)</th>
<th>Fuel Cost (million)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CGK</td>
<td>1.05</td>
<td>33.0792</td>
</tr>
<tr>
<td>CGK</td>
<td>1</td>
<td>31.504</td>
</tr>
<tr>
<td>CGK</td>
<td>1.5</td>
<td>47.256</td>
</tr>
<tr>
<td>CGK</td>
<td>1.8</td>
<td>56.7072</td>
</tr>
<tr>
<td>CGK</td>
<td>2</td>
<td>63.008</td>
</tr>
<tr>
<td>CGK</td>
<td>2.2</td>
<td>69.3088</td>
</tr>
<tr>
<td>CGK</td>
<td>1.45</td>
<td>45.6808</td>
</tr>
<tr>
<td>CGK</td>
<td>4.5</td>
<td>141.768</td>
</tr>
<tr>
<td>CGK</td>
<td>1.4</td>
<td>44.1056</td>
</tr>
<tr>
<td>CGK</td>
<td>3.1</td>
<td>99.2376</td>
</tr>
<tr>
<td>CGK</td>
<td>1.05</td>
<td>33.0792</td>
</tr>
<tr>
<td>CGK</td>
<td>1.35</td>
<td>42.5304</td>
</tr>
<tr>
<td>CGK</td>
<td>1.45</td>
<td>45.6808</td>
</tr>
<tr>
<td>CGK</td>
<td>1.1</td>
<td>34.6544</td>
</tr>
<tr>
<td>CGK</td>
<td>1.2</td>
<td>37.8048</td>
</tr>
<tr>
<td>CGK</td>
<td>1.25</td>
<td>39.38</td>
</tr>
<tr>
<td>CGK</td>
<td>1.05</td>
<td>33.0792</td>
</tr>
<tr>
<td>CGK</td>
<td>1.45</td>
<td>45.6808</td>
</tr>
<tr>
<td>CGK</td>
<td>1.4</td>
<td>44.1056</td>
</tr>
<tr>
<td>CGK</td>
<td>2.1</td>
<td>66.1584</td>
</tr>
<tr>
<td>CGK</td>
<td>3.15</td>
<td>99.2376</td>
</tr>
<tr>
<td>CGK</td>
<td>1.05</td>
<td>33.0792</td>
</tr>
<tr>
<td>CGK</td>
<td>1.3</td>
<td>42.5304</td>
</tr>
<tr>
<td>CGK</td>
<td>1.2</td>
<td>37.8048</td>
</tr>
<tr>
<td>CGK</td>
<td>1.25</td>
<td>39.38</td>
</tr>
<tr>
<td>CGK</td>
<td>1.05</td>
<td>33.0792</td>
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<tr>
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<td>1.45</td>
<td>45.6808</td>
</tr>
<tr>
<td>CGK</td>
<td>1.2</td>
<td>37.8048</td>
</tr>
<tr>
<td>CGK</td>
<td>3.1</td>
<td>99.2376</td>
</tr>
<tr>
<td>CGK</td>
<td>1.05</td>
<td>33.0792</td>
</tr>
<tr>
<td>CGK</td>
<td>1.35</td>
<td>42.5304</td>
</tr>
<tr>
<td>CGK</td>
<td>1.2</td>
<td>37.8048</td>
</tr>
<tr>
<td>CGK</td>
<td>1.5</td>
<td>47.256</td>
</tr>
<tr>
<td>CGK</td>
<td>1.3</td>
<td>42.5304</td>
</tr>
<tr>
<td>CGK</td>
<td>1.1</td>
<td>34.6544</td>
</tr>
<tr>
<td>DPS</td>
<td>0.75</td>
<td>23.628</td>
</tr>
<tr>
<td>DPS</td>
<td>1.5</td>
<td>47.256</td>
</tr>
<tr>
<td>LOP</td>
<td>3.25</td>
<td>102.388</td>
</tr>
<tr>
<td>LOP</td>
<td>1.2</td>
<td>37.8048</td>
</tr>
<tr>
<td>LOP</td>
<td>1</td>
<td>31.504</td>
</tr>
</tbody>
</table>

The Figure 4.1 represents the undirected multi-graph of domestic flight with the weight of time travels in an hour. While the Table I shows the time travel between two airports and the fuel cost in million rupiahs. The fuel cost is obtained by multiplying the time travel, the fuel needs for an hour, and the aviation fuel price per liter.

Now that we have calculated the fuel cost for each flight, then we can determine which flight cost less than another. This will lead us to know which flight path that is right in order to reduce flight cost. To simplify the graph reading, let the graph divides into several parts, so that we can choose the minimum flight cost by using Dijkstra’s algorithm. Note that we only compute the flight cost that has more than one path from the origin airport to the destination airport. Take a look at the flight path from Soekarno-Hatta Int. Airport (CGK) as the origin airport (A) to Sentani Airport (DJJ) in Jayapura as the destination airport (Z).
From the Figure 4.2, which is an undirected multi-graph with weights of the fuel cost in each one-way trip, we can determine the shortest path or in this case the lowest price for the fuel. So that when we get the right flight path, we can also optimize the flight cost in Indonesia. Here we assume the CGK Airport as vertex A, and the destination airport DJJ Airport as vertex Z. Now we have to choose which path is right to reduce the cost.

Note that there is more than one circle, which the first and the last nodes are the same, so we cannot just ignore them. Look at the Figure 4.3 below.

The lowest cost from the shortest path from CGK to DJJ is 179.56 million rupiahs by transiting at DPS, UPG, and BIK Airports. So the right path is CGK-DPS-UPG-BIK-DJJ. Note that we do not choose SRG for the transit airport even though the distance is shorter than DPS because when the weights of path CGK-SRG-SUB are added in the temporary labels, it will be more than the CGK-DPS path. Like the SRG case, we also do not choose DPS-TIM path because when we add the weights and put it in the temporary label it cost more than the DPS-UPG-BIK paths. And here is the path with the lowest cost.

Figure 4.4 Flight Path with the Lowest Cost from Jakarta to Jayapura

The red lines in the Figure 4.4 show the flight path with the lowest cost. Hereafter finding the lowest flight cost to the east part of Indonesia let us take a look at the west part.

Figure 4.5 Weighted and Undirected Multi-graph to Determine the Lowest Cost from Jakarta to Medan

Now we will find the lowest flight cost from Jakarta (CGK) to Medan (MES). We would not find the lowest cost from Jakarta (CGK) to Banda Aceh (BTJ), because it already has the one-way trip from Jakarta to Banda Aceh without transiting to other airport, it means that the airline is capable to flight directly to Banda Aceh from Jakarta, so the flight cost has been minimized. Here is the using of Dijkstra’s algorithm.

Step 1.  
\[ d(CGK) = 0, d(PGK) = 33.08, d(BTH) = 45.69, d(PDG) = 47.26, d(BTJ) = 78.76, d(PKU) = d(MES) = INFINITY(>373.74). \]
\[ S = \{ PGK, BTH, PDG, BTJ \}. \]
\[ P(PGK) = P(BTH) = P(PDG) = P(BTJ) = CGK. \]

Step 2.  
\[ a) j = PGK, d(PGK) = 94.5. \]
\[ b) S = \{ SRG, SUB, DOP, LOP, BDI, BPN, UPG, TIM \}. \]

Step 3.  
\[ \Gamma_{DPS} = \{ CGK, LOP, UPG, TIM \}. \]  
\[ \Gamma_{DPS} \Lambda S = \{ UPG, TIM \}. \]  
\[ P(LOP) = P(UPG) = P(TIM) = DPS. \]

Step 2.  
\[ a) j = UPG, d(UPG) = 144.91. \]
\[ b) S = \{ SRG, SUB, LOP, BDI, BPN, TIM \}. \]

Step 3.  
\[ \Gamma_{UPG} = \{ SUB, DPS, LOP, BPN, DJJ, BIK \}. \]  
\[ \Gamma_{UPG} \Lambda S = \{ SUB, LOP, BPN, DJJ, BIK \}. \]  
\[ d(BIK) = 233.12, d(DJJ) = 179.56. \]  
\[ P(BIK) = P(DJJ) = UPG. \]

Step 2.  
\[ a) j = DJJ, d(DJJ) = 179.56. \]
\[ b) S = \{ BIK, UPG, TIM \}. \]  
\[ c) END. \]
Step 2.  
a) \( j = \text{BTH} \), \( d(\text{BTH}) = 45.68 \).
b) \( S = \{\text{PGK}, \text{PDG}, \text{PKU}, \text{MES}, \text{BTJ}\} \).

Step 3.  
\[ \Gamma_{\text{BTH}} = \{\text{CGK}, \text{PKU}, \text{MES}\} \]
\[ \Gamma_{\text{BTH}} \Lambda S = \{\text{PKU}, \text{MES}\} \]
\[ d(\text{PKU}) = 70.88, \ d(\text{MES}) = 86.64. \]
\[ P(\text{PKU}) = P(\text{MES}) = \text{BTH}. \]

Step 2.  
a) \( j = \text{MES} \), \( d(\text{MES}) = 86.64 \).
b) \( S = \{\text{BTH}, \text{PKU}, \text{PDG}, \text{BTJ}\} \).
c) END.

According to the Dijkstra’s algorithm, the lowest flight cost from Jakarta to Medan is through Batam (BTH) with the cost of 86.64 million rupiahs each one flight. Below is the route pass by the flight.

![Figure 4.6 The Flight Path with the Lowest Cost from Jakarta to Medan](image)

The Figure 4.6 above describes the route that is passed by the airplane in order to get the minimum flight cost from Jakarta to Medan. The red lines show the flight paths that transit to Batam first. The most efficient flight path is CGK-BTH-MES by the red lines.

Due to the implementation of Dijkstra’s algorithm, the most efficient flight path can be found and the flight cost then can be optimized. It cannot be denied that to optimize the flight cost is not that simple because we have to consider another aspects like the operational costs, airplane maintenance, airport taxes, and personnel costs. But at least we can calculate most of the flight cost by knowing the fuel costs in a one-way trip.

V. CONCLUSION

The application of graph theories can be used in representing the Indonesian domestic flight route map. The vertices represent the airports meanwhile the edges represent the flight path. To determine the most efficient flight cost, the graph that is used is undirected multi-graph and weighted graph as the weights are the fuel costs in million rupiahs.

The implementation of Dijkstra’s algorithm is very useful in determining the efficiency of fuel needs as it effects the optimization of flight costs. The Dijkstra’s algorithm is able to find the shortest path in graph. In this case the paths represent the flight costs. By finding the shortest path then the most efficient flight cost can also be determined. As a maritime country, Indonesia needs to optimize the air transportation due to economic needs. By knowing which is the most efficient flight route then the fuel needs can be reduced. This then will influence the air traffic management, if the flight costs can be pressed then the fee can be allocated into some other aspect in order to enhance air transportation quality in Indonesia.

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REFERENCES


PERNYATAAN

Dengan ini saya menyatakan bahwa makalah yang saya tulis ini adalah tulisan saya sendiri, bukan saduran, atau terjemahan dari makalah orang lain, dan bukan plagiasi.

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Vidia Anindhita 13512034