

Aplikasi Transformasi Linier di dalam *Computer Graphics*

Oleh: Rinaldi Munir

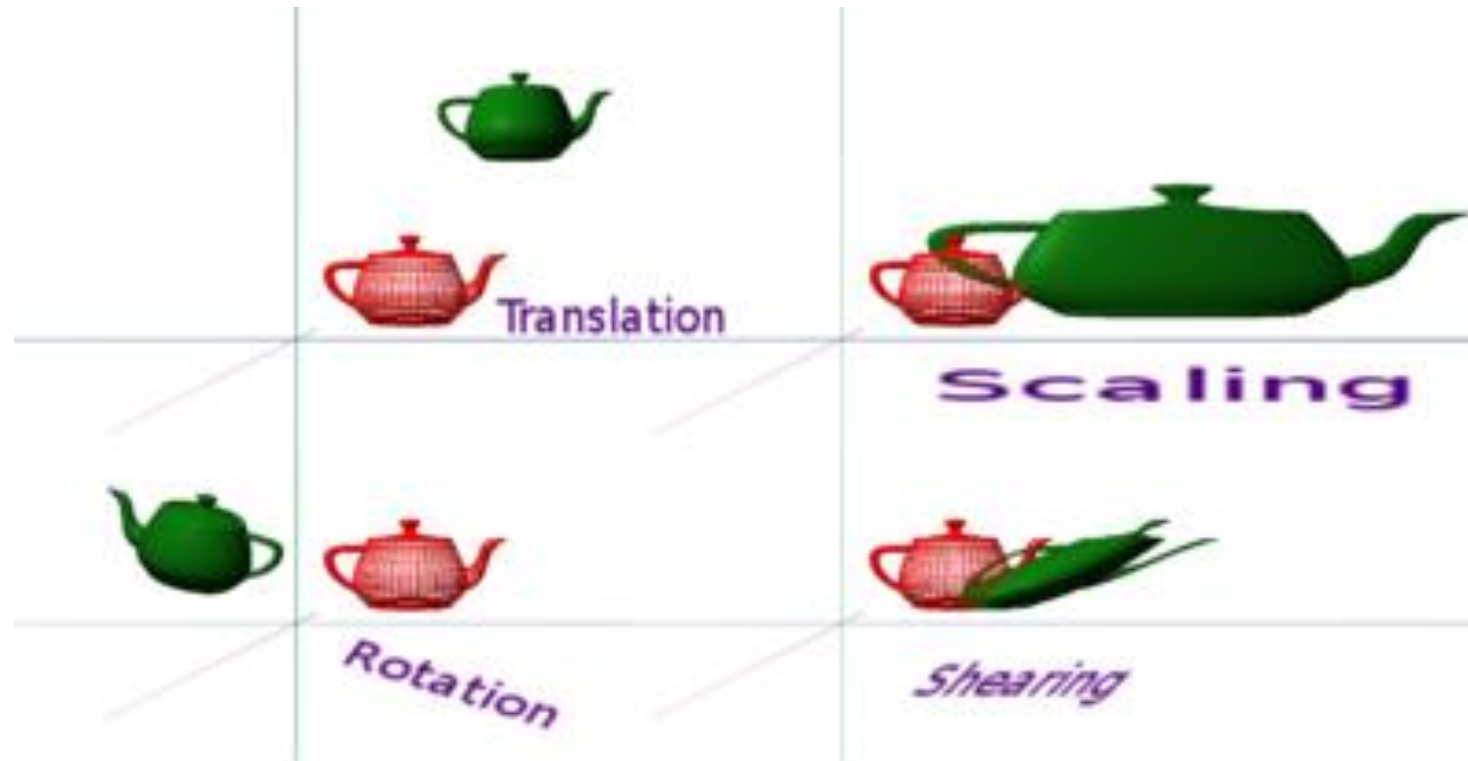
Program Studi Informatika
Sekolah Teknik Elektro dan Informatika
ITB

Aplikasi Transformasi Linier di dalam *Computer Graphics*

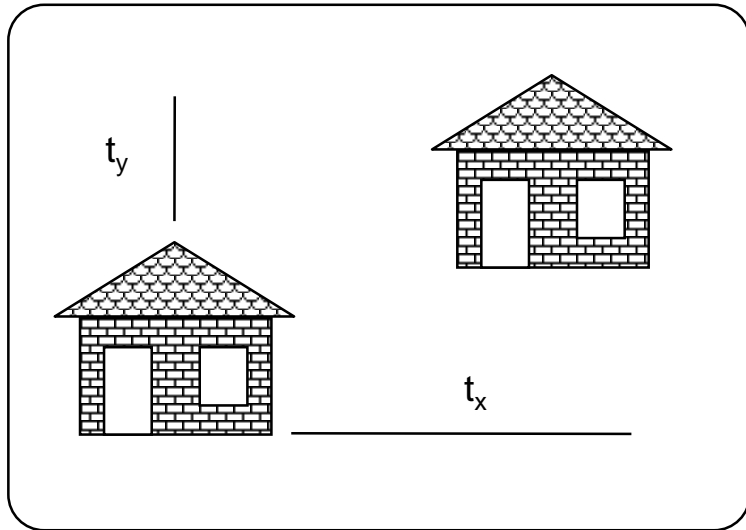
- **Definisi:** Jika $T : V \rightarrow W$ adalah sebuah fungsi dari ruang vektor V ke ruang vektor W , maka T dinamakan transformasi linier jika
 - (i) $T(\mathbf{u} + \mathbf{v}) = T(\mathbf{u}) + T(\mathbf{v})$ untuk semua vektor \mathbf{u} dan \mathbf{v} di dalam V
 - (ii) $T(k\mathbf{u}) = kT(\mathbf{u})$ untuk semua vektor \mathbf{u} di dalam V
- Transformasi linier $T : R^n \rightarrow R^m$ dapat dinyatakan sebagai sebuah perkalian matriks

$$T(\mathbf{x}) = A\mathbf{x}$$

Jenis-Jenis Tranformasi Linier 2D ($T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$)



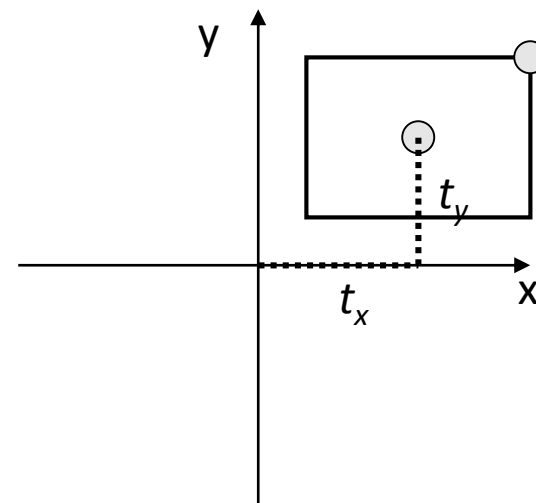
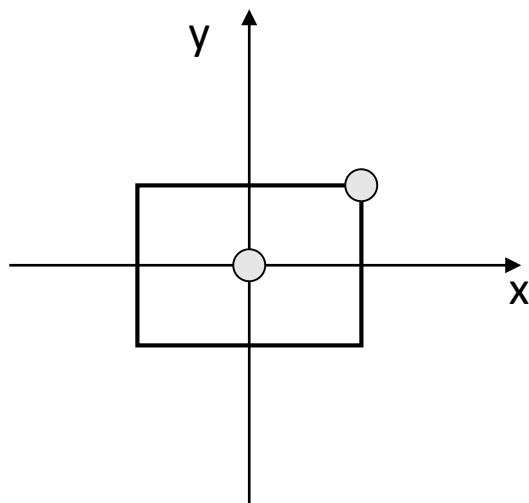
1. Translasi



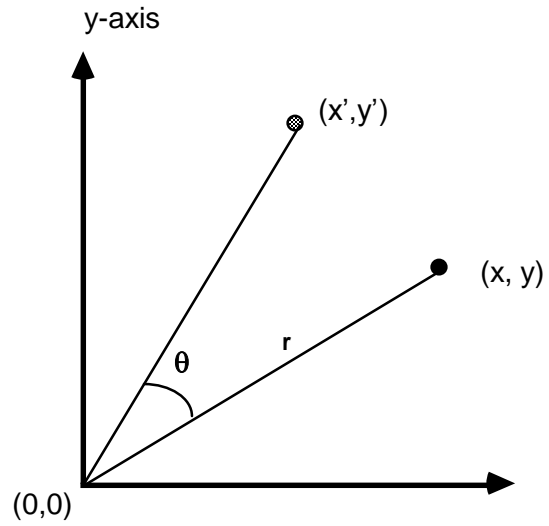
$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \end{bmatrix}$$

atau

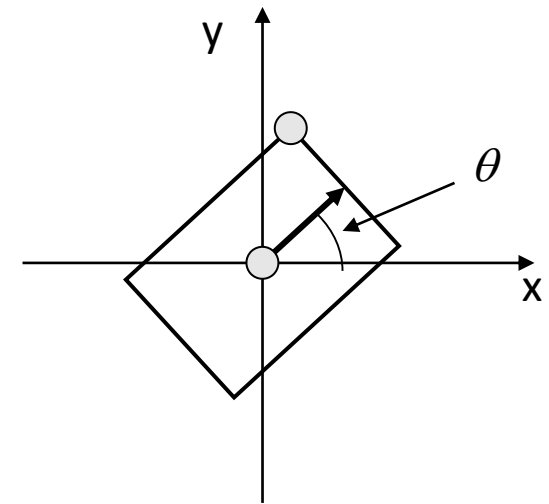
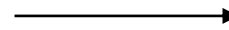
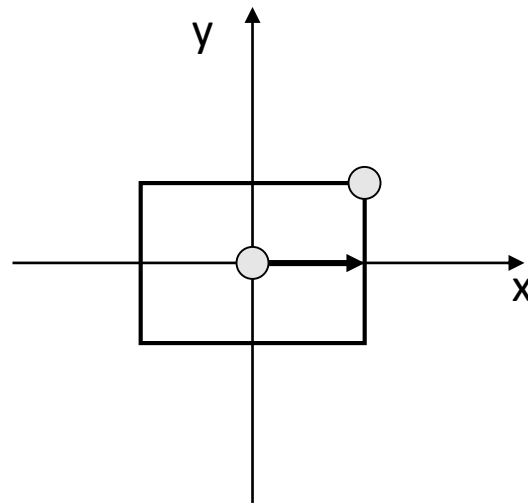
$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \end{bmatrix}$$



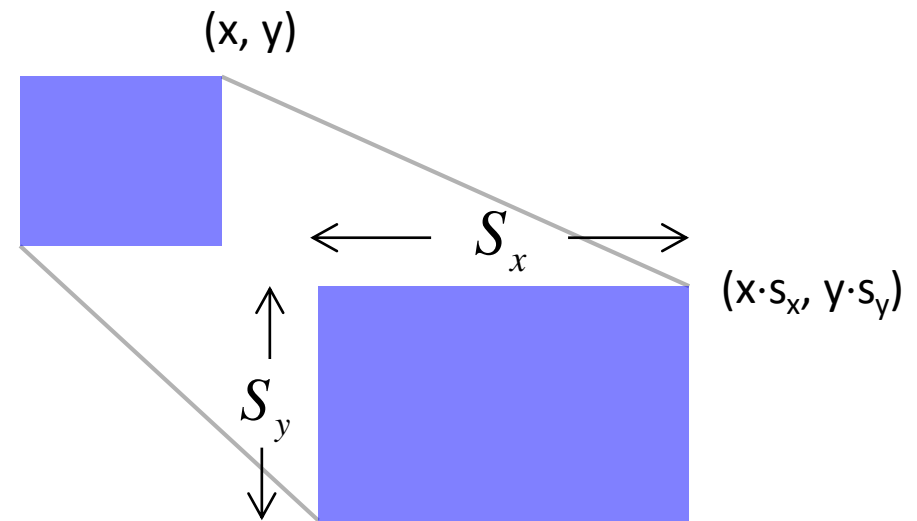
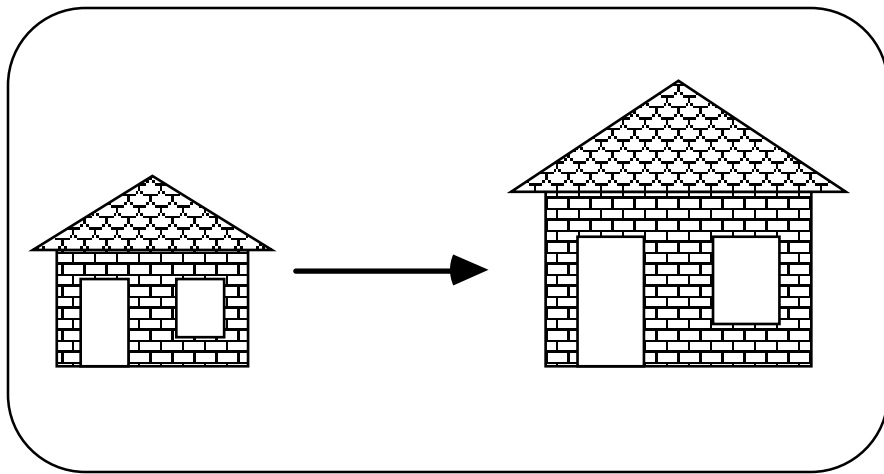
2. Rotasi



$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$



3. Penskalaan (*scaling*)



$$x' = x \cdot s_x, \quad y' = y \cdot s_y$$

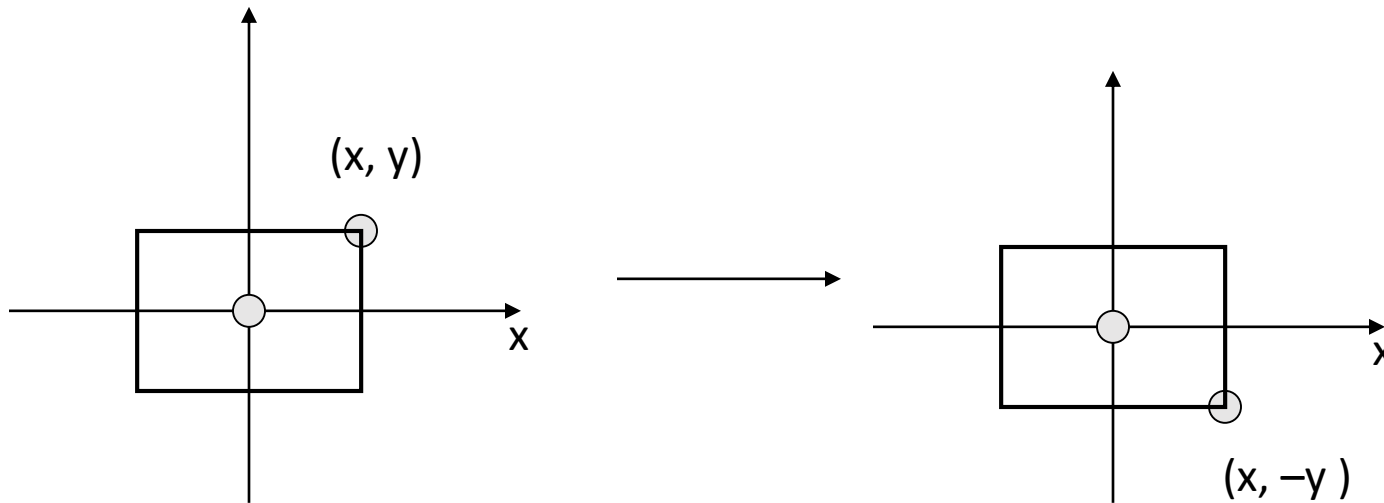
$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\mathbf{P}' = \mathbf{S} \cdot \mathbf{P}$$

4. Pencerminkan (*reflection*)

Pencerminkan pada sumbu-X:

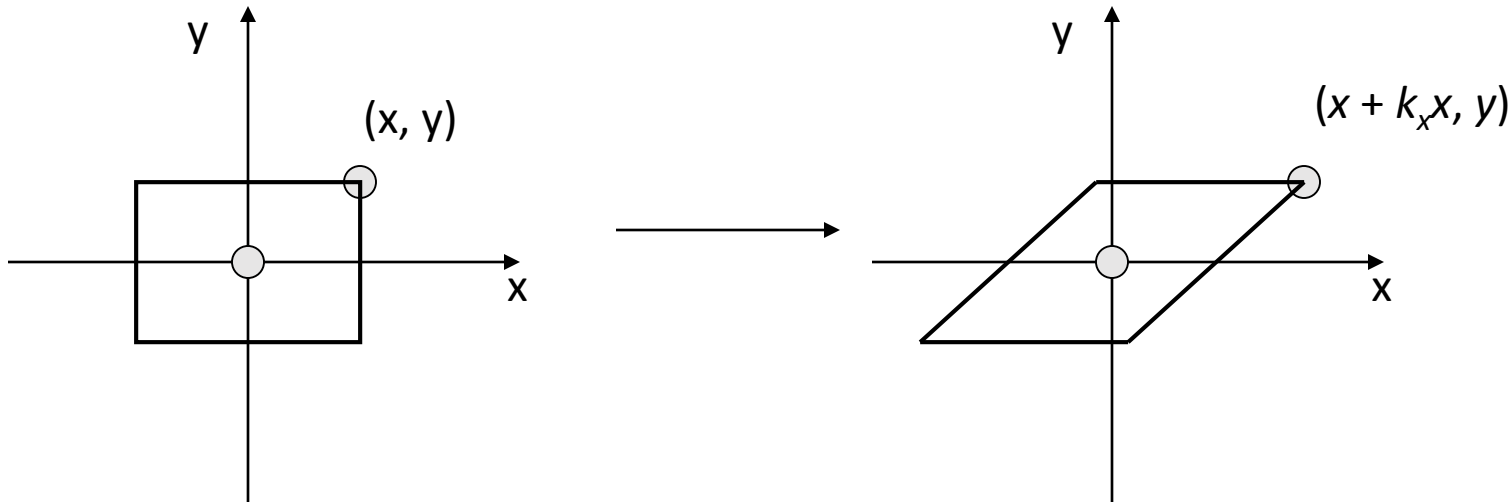
$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$



5. Peregangan (*shear*)

Peregangan sepanjang sumbu-X:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & k_x \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$



Koordinat Homogen

- Di dalam grafika computer, sebuah gambar dapat dibangun dari dari sekumpulan bentuk terdefinisi (kotak, lingkaran, segitiga, dll).
- Tiap bentuk mungkin diskalakan, dirotasi, atau ditranslasi ke posisi gambar yang sebenarnya.
- Agar perhitungan koordinat akhir dapat langsung dihitung dari koordinat awal dengan efisien, maka diperlukan sebuah sistem koordinat yang homogen
- Pada koordinat homogen, setiap titik direpresentasikan dengan tiga angka:

$$(x, y) \rightarrow (x \cdot w, y \cdot w, w) \quad \text{dengan syarat } w \neq 0$$

Translasi 2D

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}, \quad \mathbf{P}' = \mathbf{T}(t_x, t_y) \cdot \mathbf{P}$$

Rotasi 2D

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}, \quad \mathbf{P}' = \mathbf{R}(\theta) \cdot \mathbf{P}$$

Penskalaan 2D

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}, \quad \mathbf{P}' = \mathbf{S}(S_x, S_y) \cdot \mathbf{P}$$

Transformasi *inverse*:

$$\mathbf{T}^{-1} = \begin{bmatrix} 1 & 0 & -t_x \\ 0 & 1 & -t_y \\ 0 & 0 & 1 \end{bmatrix}, \quad \mathbf{R}^{-1} = \begin{bmatrix} \cos \theta & \sin \theta & 0 \\ -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad \mathbf{S}^{-1} = \begin{bmatrix} 1/S_x & 0 & 0 \\ 0 & 1/S_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Komposisi tranformasi: $\mathbf{P}' = \mathbf{M}_2 (\mathbf{M}_1 \cdot \mathbf{P}) = (\mathbf{M}_2 \cdot \mathbf{M}_1) \cdot \mathbf{P} = \mathbf{M} \cdot \mathbf{P}$

$$\mathbf{P}' = \mathbf{T}(t_{2x}, t_{2y}) \{ \mathbf{T}(t_{1x}, t_{1y}) \cdot \mathbf{P} \} = \{ \mathbf{T}(t_{2x}, t_{2y}) \cdot \mathbf{T}(t_{1x}, t_{1y}) \} \cdot \mathbf{P}$$

Komposisi translasi:

$$\begin{bmatrix} 1 & 0 & t_{2x} \\ 0 & 1 & t_{2y} \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & t_{1x} \\ 0 & 1 & t_{1y} \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_{1x} + t_{2x} \\ 0 & 1 & t_{1y} + t_{2y} \\ 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{T}(t_{2x}, t_{2y}) \cdot \mathbf{T}(t_{1x}, t_{1y}) = \mathbf{T}(t_{1x} + t_{2x}, t_{1y} + t_{2y})$$

$$\mathbf{P}' = \mathbf{R}(\theta_2) \{ \mathbf{R}(\theta_1) \cdot \mathbf{P} \} = \{ \mathbf{R}(\theta_2) \cdot \mathbf{R}(\theta_1) \} \cdot \mathbf{P}$$

Komposisi rotasi:

$$\mathbf{R}(\theta_2) \cdot \mathbf{R}(\theta_1) = \mathbf{R}(\theta_1 + \theta_2)$$

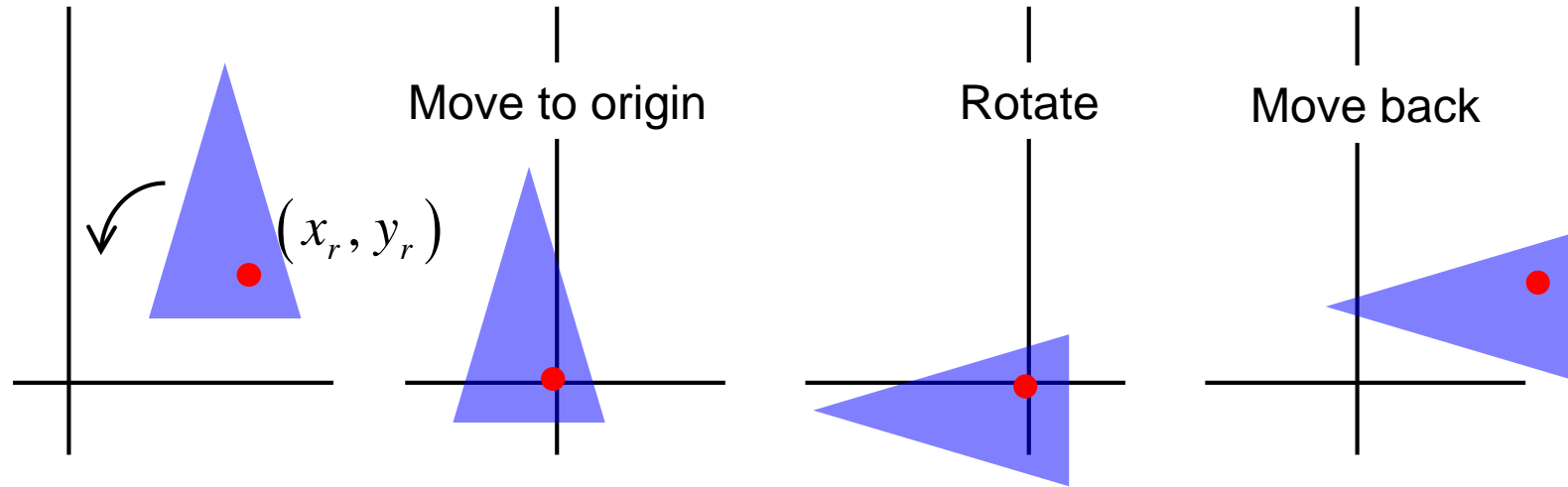
$$\mathbf{P}' = \mathbf{R}(\theta_1 + \theta_2) \cdot \mathbf{P}$$

$$\begin{bmatrix} S_{2x} & 0 & 0 \\ 0 & S_{2y} & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} S_{1x} & 0 & 0 \\ 0 & S_{1y} & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} S_{1x} \cdot S_{2x} & 0 & 0 \\ 0 & S_{1y} \cdot S_{2y} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Komposisi
penskalaan:

$$\mathbf{S}(S_{2x}, S_{2y}) \cdot \mathbf{S}(S_{1x}, S_{1y}) = \mathbf{S}(S_{1x} \cdot S_{2x}, S_{1y} \cdot S_{2y})$$

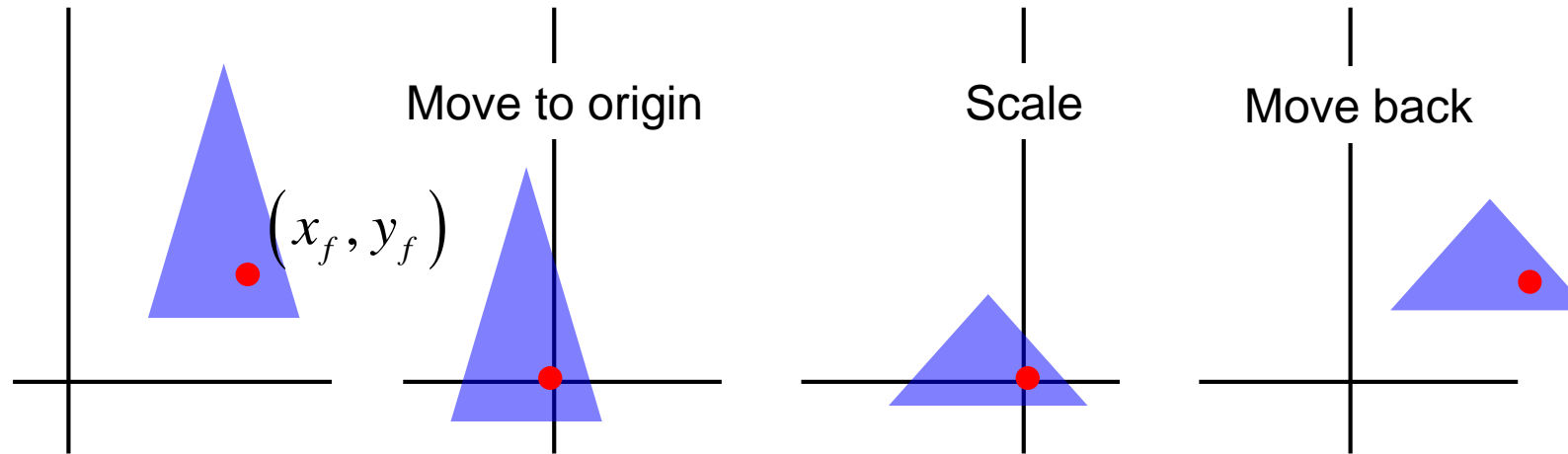
General 2D Rotation



$$\begin{bmatrix} 1 & 0 & x_r \\ 0 & 1 & y_r \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & -x_r \\ 0 & 1 & -y_r \\ 0 & 0 & 1 \end{bmatrix} =$$

$$\begin{bmatrix} \cos \theta & -\sin \theta & x_r (1 - \cos \theta) + y_r \sin \theta \\ \sin \theta & \cos \theta & y_r (1 - \cos \theta) - x_r \sin \theta \\ 0 & 0 & 1 \end{bmatrix}$$

General 2D Scaling

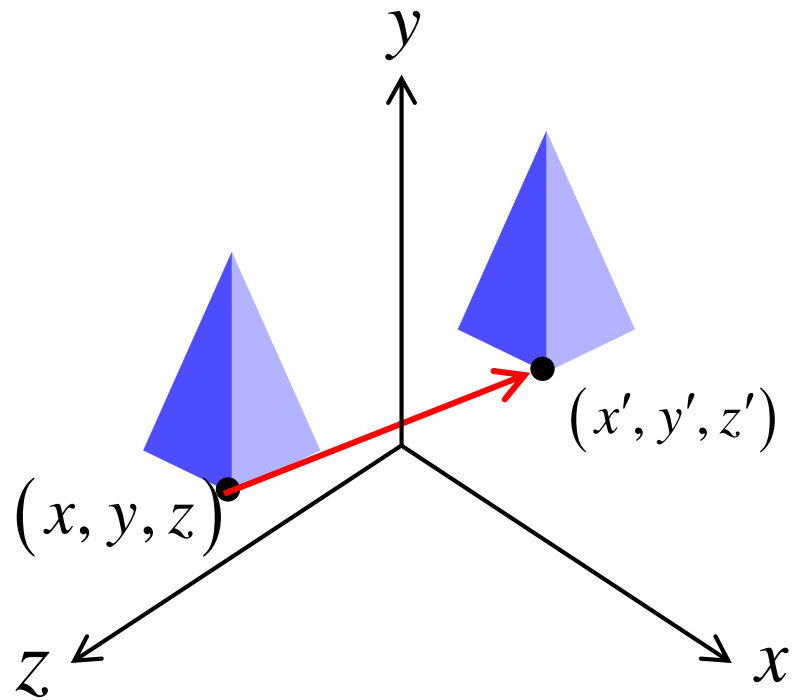


$$\begin{bmatrix} 1 & 0 & x_f \\ 0 & 1 & y_f \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & -x_f \\ 0 & 1 & -y_f \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} S_x & 0 & x_f(1-S_x) \\ 0 & S_y & y_f(1-S_y) \\ 0 & 0 & 1 \end{bmatrix}$$

Transformation 3D

Mirip dengan transformasi 2D. Menggunakan matriks 4x4

Translation



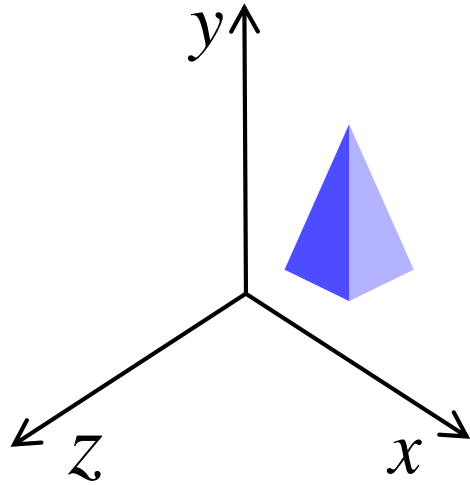
$$x' = x + t_x$$

$$y' = y + t_y$$

$$z' = z + t_z$$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 1 & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

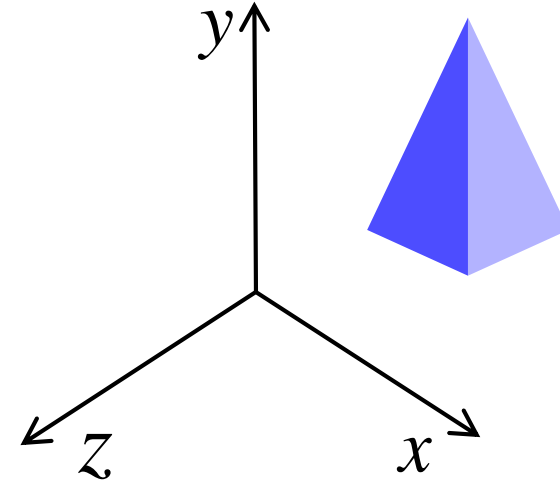
Penskalaan 3D



$$x' = x \cdot S_x$$

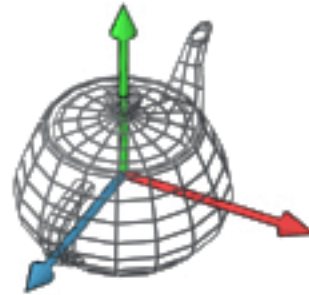
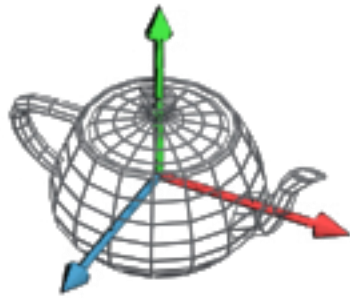
$$y' = y \cdot S_y$$

$$z' = z \cdot S_z$$

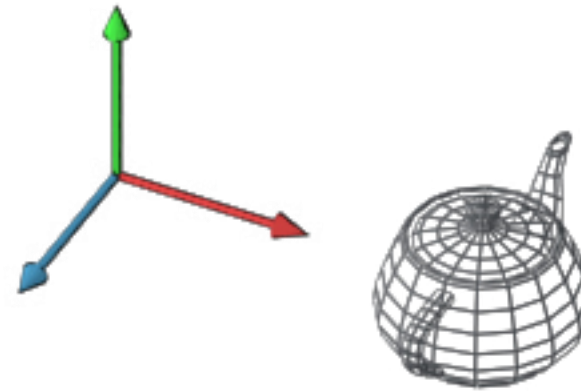


$$\mathbf{P}' = \begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} S_x & 0 & 0 & 0 \\ 0 & S_y & 0 & 0 \\ 0 & 0 & S_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \mathbf{S} \cdot \mathbf{P}$$

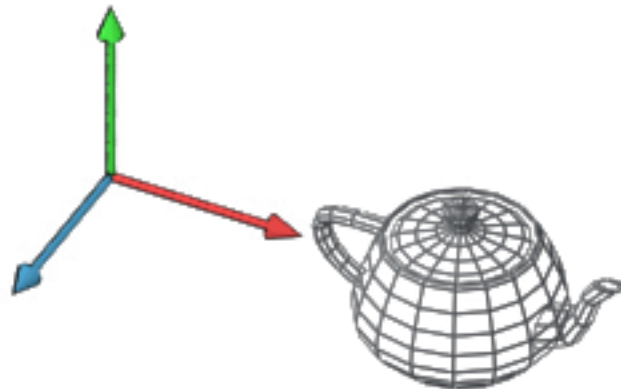
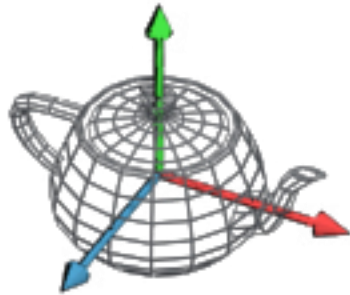
Rotation 90° around Y



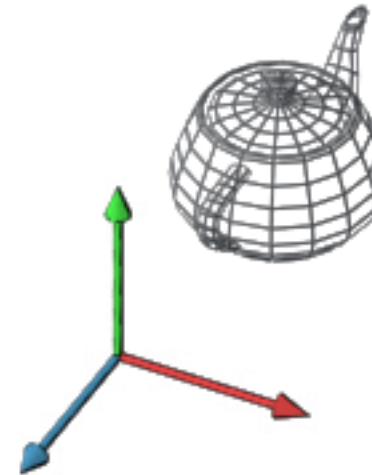
Translate along X

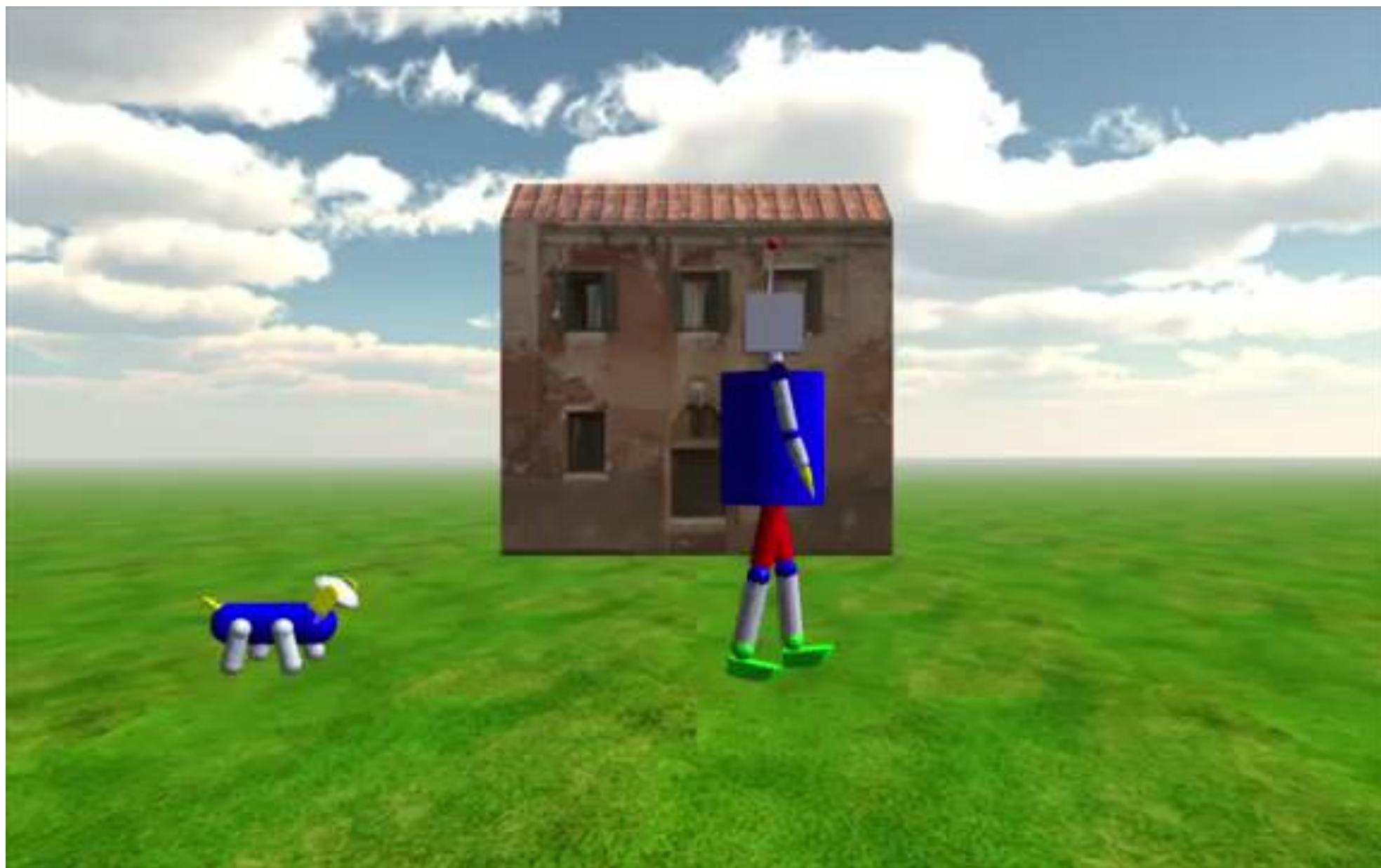


Translate along X



Rotation 90° around Y





Referensi

1. Shmuel Wimer, *Geometric Transformations for Computer Graphics*, Bar Ilan Univ., School of Engineering
2. Larry F. Hodges, *2D Transformation*