Penentuan Rute (*Route/Path Planning*)

Bagian 2: Algoritma A*

Bahan Kuliah IF2211 Strategi Algoritma

Oleh: Nur Ulfa Maulidevi & Rinaldi Munir

Program Studi Teknik Informatika
Sekolah Teknik Elektro dan Informatika ITB
2025

Referensi

1. Materi kuliah IF3170 Inteligensi Buatan Teknik Informatika ITB, Course Website:

   http://kuliah.itb.ac.id → STEI → Teknik Informatika → IF3170

2. Stuart J Russell & Peter Norvig, *Artificial Intelligence: A Modern Approach, 3rd Edition*, Prentice-Hall International, Inc, 2010, Textbook

   Site: http://aima.cs.berkeley.edu/ (2nd edition)

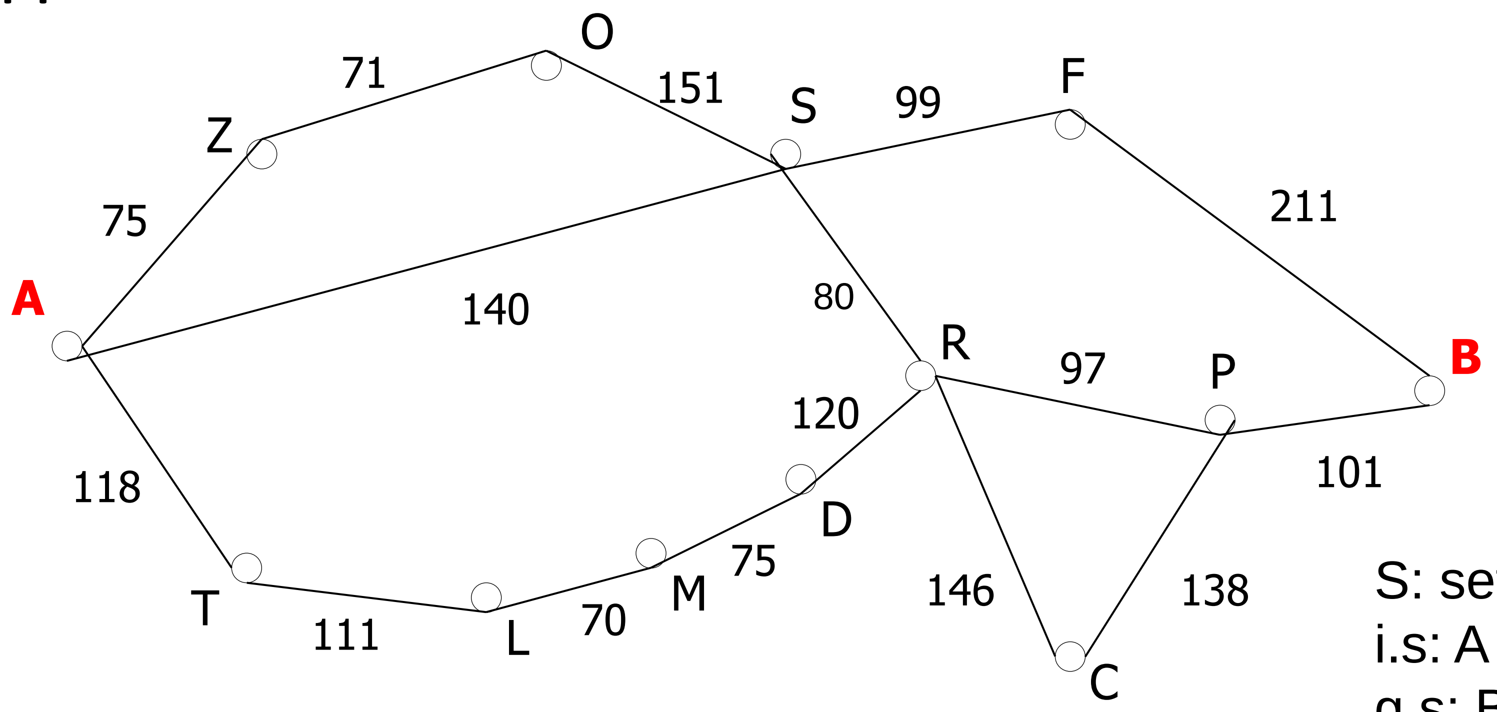
3. Free online course materials | MIT OpenCourseWare Website: Site: http://ocw.mit.edu/courses/electrical-engineering-and-computer-science/

4. Lecture Notes in Informed Heuristic Search, ICS 271 Fall 2008, http://www.ics.uci.edu/~dechter/courses/ics-271/fall-08/lecture-notes/4.InformedHeuristicSearch.ppt

Route Planning

Source: Russell's book

Search



S: set of cities
i.s: A (Arad)
g.s: B (Bucharest)
Goal test: s = B ?
Path cost: time ~ distance

Persoalan: Carilah lintasan **terpendek** dari Arad (A) ke Bucharest (B)!

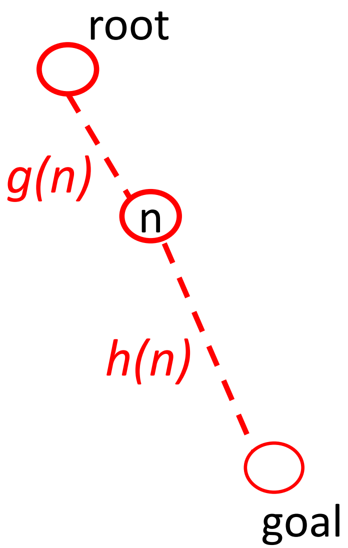
Heuristic Search

- Heuristic estimates value of a node
    - promise of a node
    - difficulty of solving the subproblem
    - quality of solution represented by node
    - the amount of information gained

- f(n)- heuristic evaluation function.
    - depends on n, goal, search so far, domain

A* Search

- Idea: avoid expanding paths that are already expensive



- Evaluation function $f(n) = g(n) + h(n)$
  - $g(n)$ = cost so far to reach n
  - $h(n)$ = estimated cost from n to goal
  - $f(n)$ = estimated total cost of path through n to goal

- if $f(n) = g(n)$ → Uniform Cost Search (UCS)

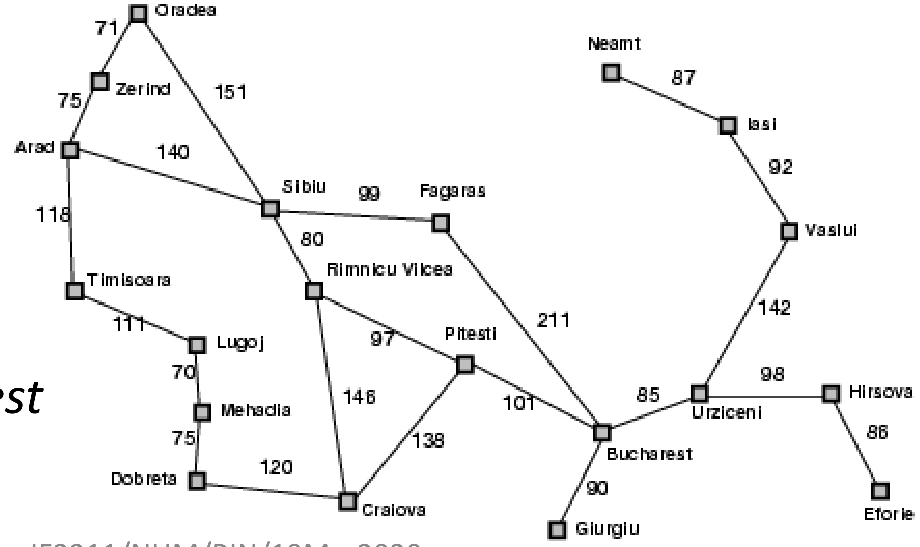
  if $f(n) = h(n)$ → Greedy Best First Search

  if $f(n) = g(n) + h(n)$ → A*

A* search example

$f(n) = g(n) + h(n)$

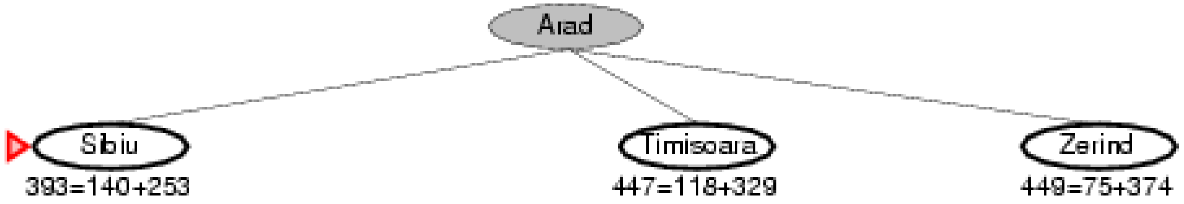
g(n) = distance from Arad to n
h(n) = straight-line distance from n to Bucharest



Straight–line distance
to Bucharest

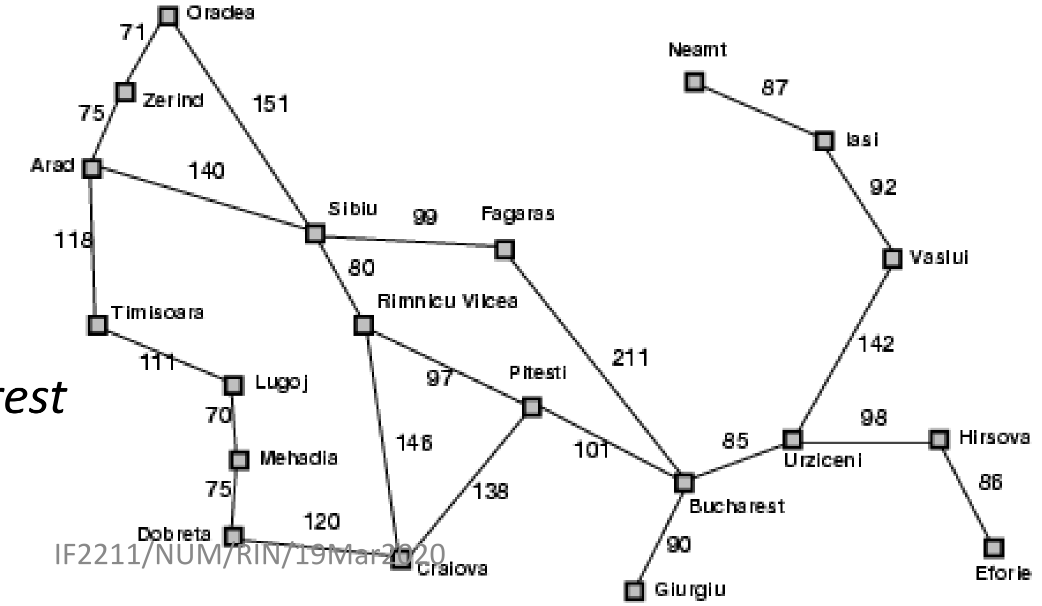
Arad	366
Bucharest	0
Craiova	160
Dobreta	242
Eforie	161
Fagaras	176
Giurgiu	77
Hirsova	151
Iasi	226
Lugoj	244
Mehadia	241
Neamt	234
Oradea	380
Pitesti	10
Rimnicu Vilcea	193
Sibiu	253
Timisoara	329
Urziceni	80
Vaslui	199
Zerind	374

A* search example



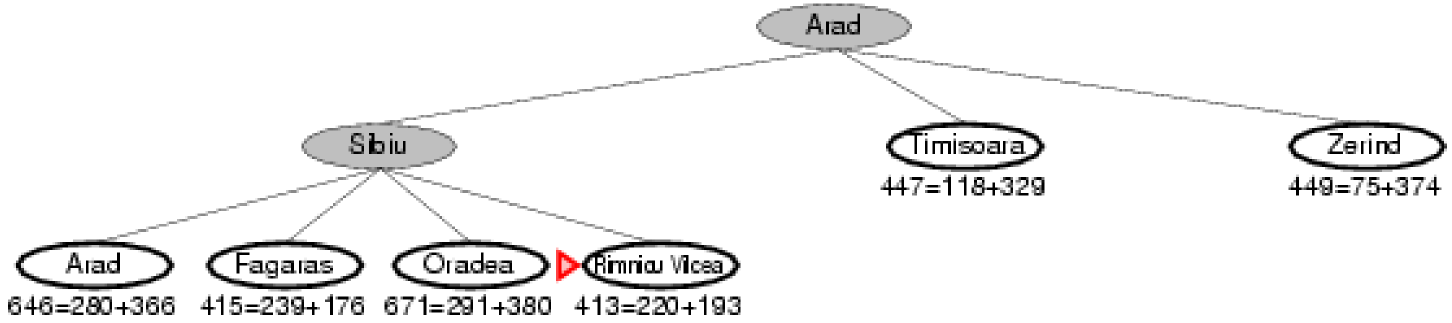
$f(n) = g(n) + h(n)$

$g(n)$ = *distance from Arad to n*
$h(n)$ = *straight-line distance from n to Bucharest*



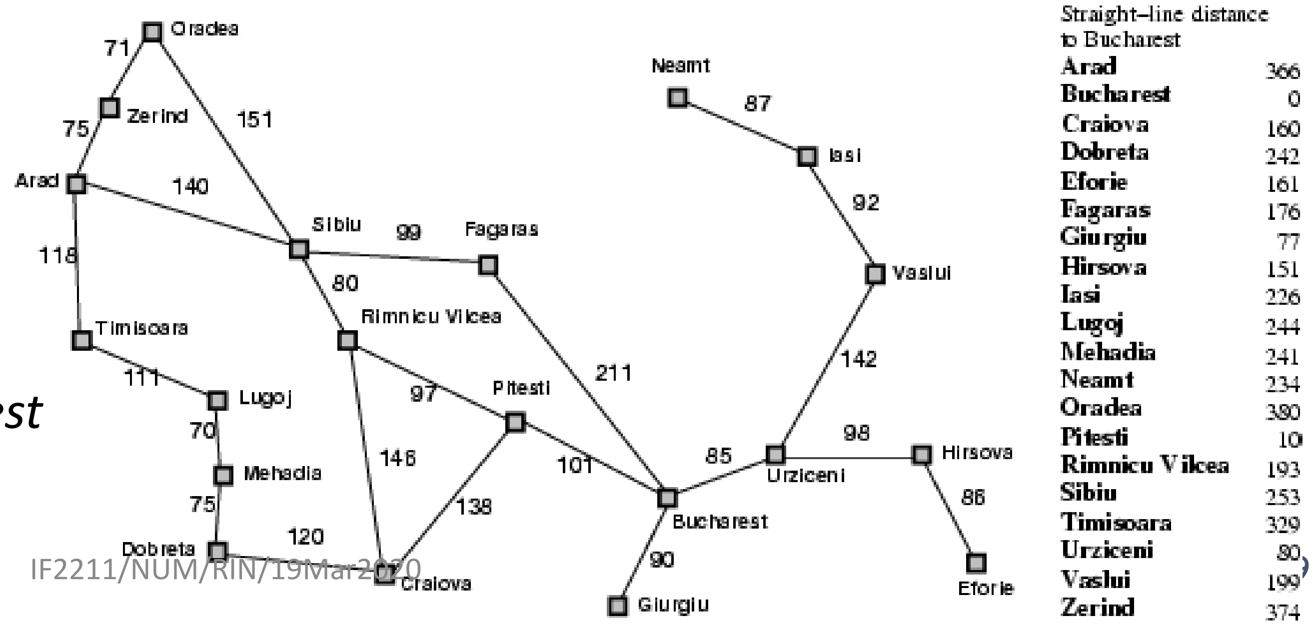
Straight−line distance to Bucharest	
Arad	366
Bucharest	0
Craiova	160
Dobreta	242
Eforie	161
Fagaras	176
Giurgiu	77
Hirsova	151
Iasi	226
Lugoj	244
Mehadia	241
Neamt	234
Oradea	380
Pitesti	10
Rimnicu Vilcea	193
Sibiu	253
Timisoara	329
Urziceni	80
Vaslui	199
Zerind	374

A* search example

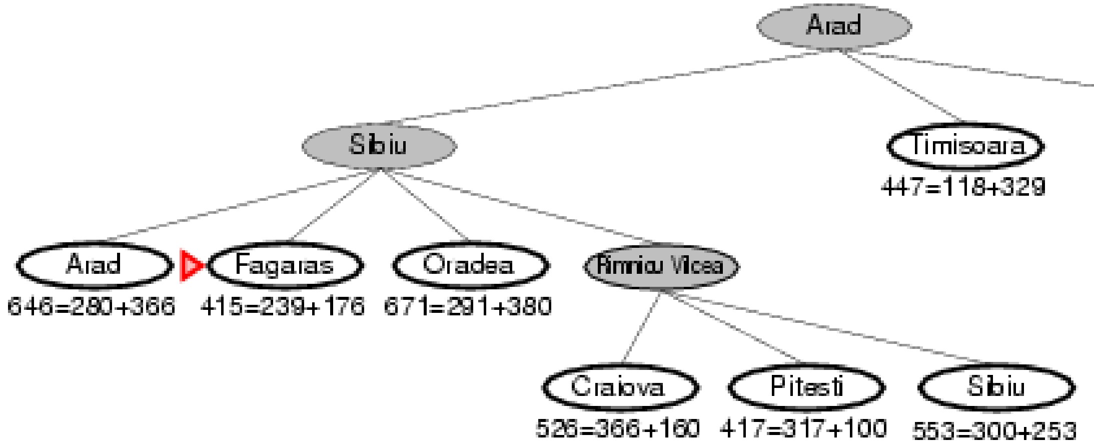


f(n) = g(n) + h(n)

g(n) = distance from Arad to n
h(n) = straight-line distance from n to Bucharest

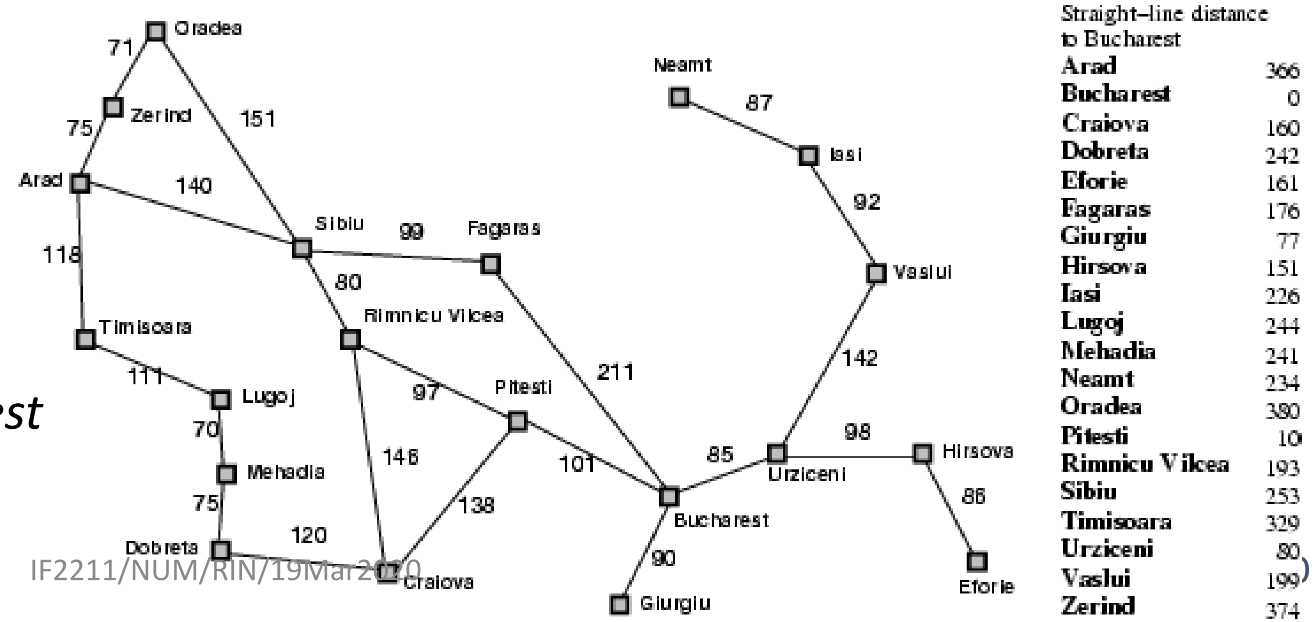


A* search example

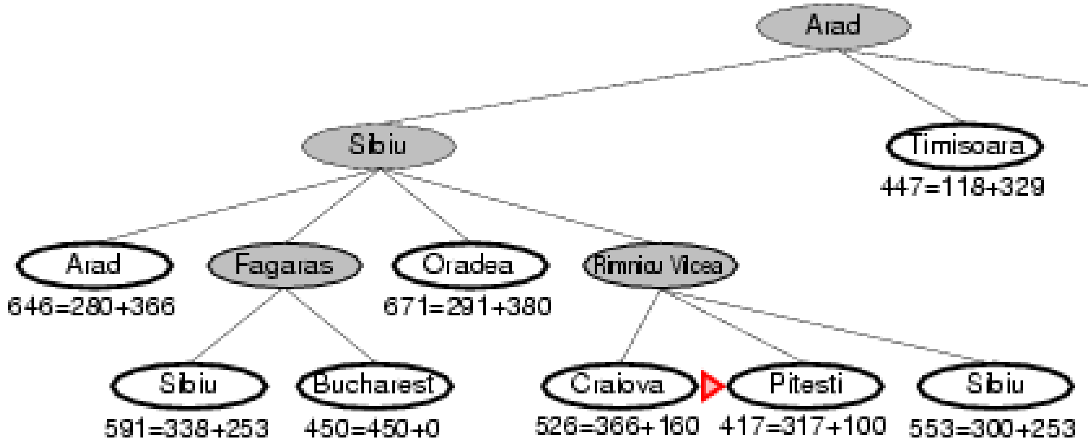


f(n) = g(n) + h(n)

g(n) = distance from Arad to n
h(n) = straight-line distance from n to Bucharest

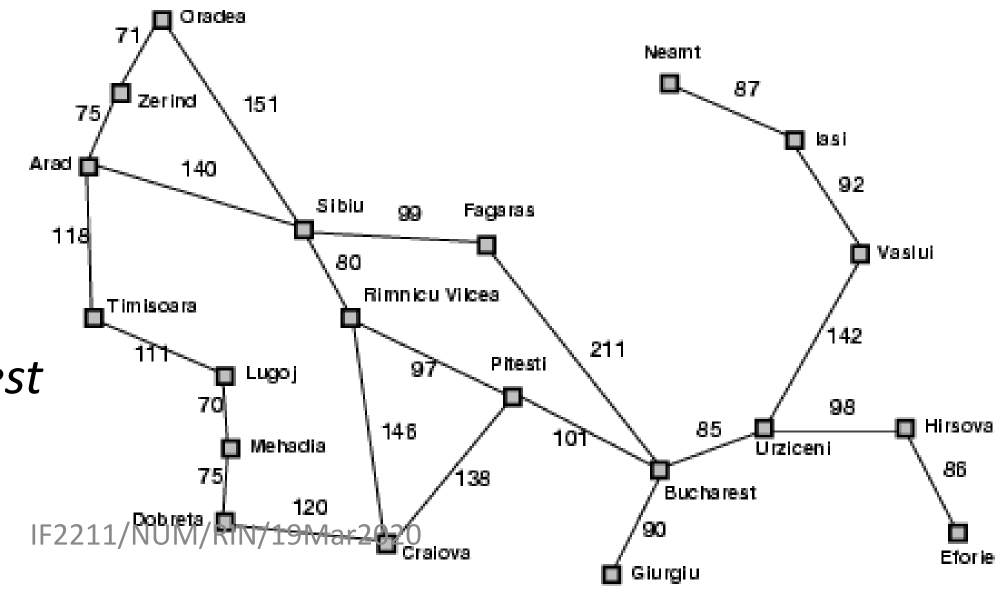


A^* search example



$f(n) = g(n) + h(n)$

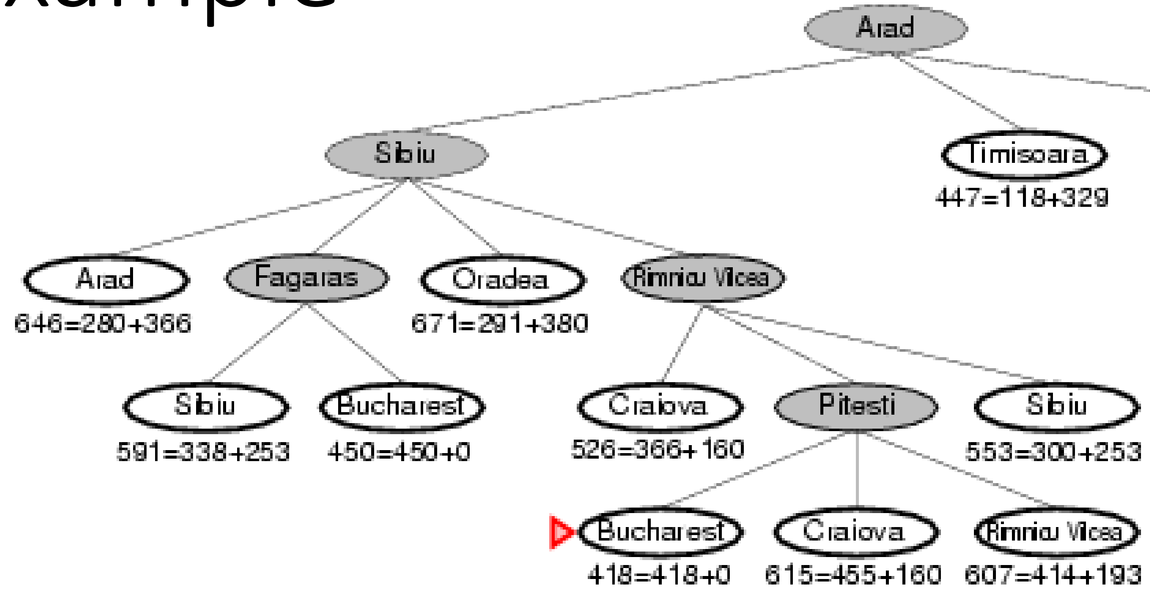
$g(n)$ = distance from Arad to n
$h(n)$ = straight-line distance from n to Bucharest

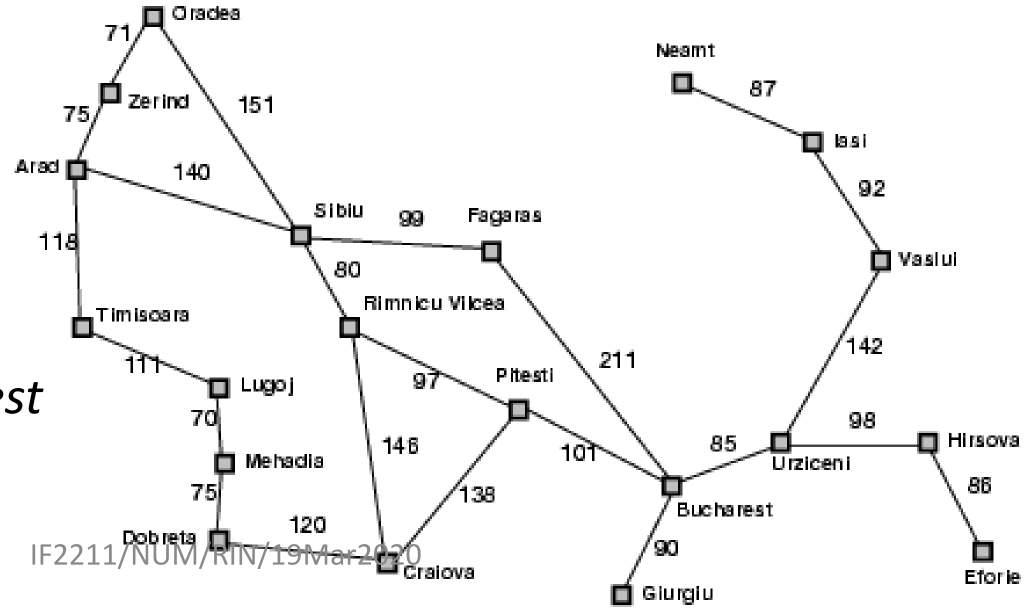


Straight–line distance to Bucharest

Arad	366
Bucharest	0
Craiova	160
Dobreta	242
Eforie	161
Fagaras	176
Giurgiu	77
Hirsova	151
Iasi	226
Lugoj	244
Mehadia	241
Neamt	234
Oradea	380
Pitesti	10
Rimnicu Vilcea	193
Sibiu	253
Timisoara	329
Urziceni	80
Vaslui	199
Zerind	374

A^* search example





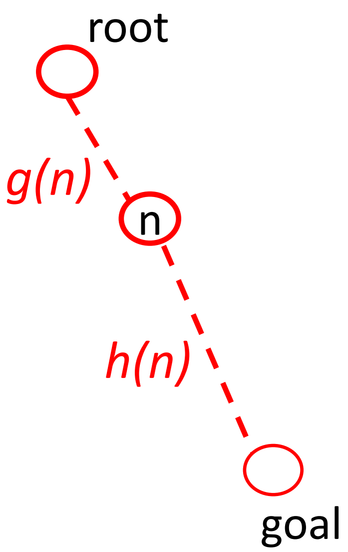
Straight–line distance
to Bucharest

Arad	366
Bucharest	0
Craiova	160
Dobreta	242
Eforie	161
Fagaras	176
Giurgiu	77
Hirsova	151
Iasi	226
Lugoj	244
Mehadia	241
Neamt	234
Oradea	380
Pitesti	10
Rimnicu Vilcea	193
Sibiu	253
Timisoara	329
Urziceni	80
Vaslui	199
Zerind	374

f(n) = g(n) + h(n)

g(n) = distance from Arad to n
h(n) = straight-line distance from n to Bucharest

A* Special



- Goal: find a minimum sum-cost path
- Notation:

$f(n) = g(n) + h(n)$

  - c(n,n') - cost of arc (n,n')
  - g(n) = cost of current path from start to node n in the search tree.
  - h(n) = estimate of the cheapest cost of a path from n to a goal.
  - Special evaluation function: f = g+h
- f(n) estimates the cheapest cost solution path that goes through n.
  - h*(n) is the true cheapest cost from n to a goal.
  - g*(n) is the true shortest path from the start s, to n.
- If the heuristic function, h always underestimate the true cost (h(n) is smaller than h*(n)), then A* is guaranteed to find an optimal solution → admissible; and also has to be consistent

Properties of A*

- Complete? Yes, unless there are infinitely many nodes with $f \leq f(G)$
- Time? Exponential: $O(b^m)$
- Space? Keep all the nodes in memory: $O(b^m)$
- Optimal? Yes

Branch-and-Bound vs A*

- As in A*, look for a ***bound*** which is guaranteed lower than the true cost
- Search the branching tree in any way you like
  - e.g. depth first (no guarantee), best first
- Cut off search if cost + bound > best solution found
- If heuristic is cost + bound, search = best first
  - then B&B = A*
- Bounds often much more sophisticated
  - e.g. using mathematical programming optimisations

Admissible heuristics

- A heuristic $h(n)$ is admissible if for every node n, $h(n) \leq h^*(n)$, where $h^*(n)$ is the true cost to reach the goal state from n.

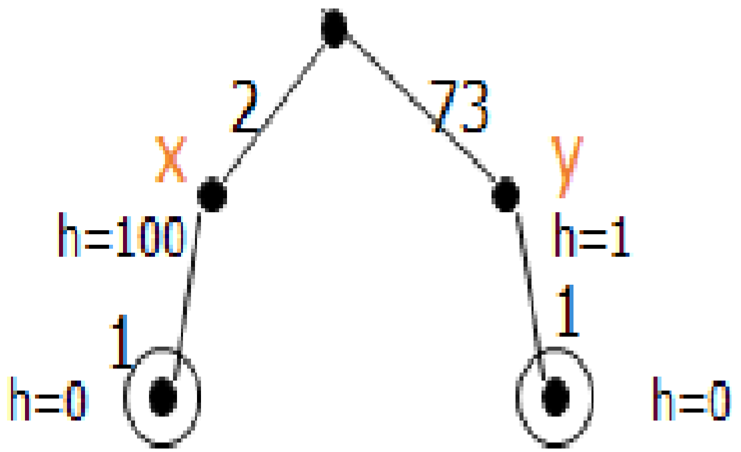
- An admissible heuristic never overestimates the cost to reach the goal, i.e., it is optimistic

- Example: $h_{SLD}(n)$ (never overestimates the actual road distance)

Theorem: If $h(n)$ is admissible, A^* using `TREE-SEARCH` is optimal

Admissibility

- What must be true about h for A* to find optimal path?
- A* finds optimal path if h is admissable; h is admissible when it never overestimates.
- In this example, h is not admissible.
- In route finding problems, straight-line distance to goal is admissible heuristic.



g(X)+h(X)=2+100=102

G(Y)+h(Y)=73+1=74

Optimal path is not found!

Because we choose Y, rather than X which is in the optimal path.

Latihan soal A*

Terdapat persoalan 8-puzzle seperti pada Gambar 1. Gambar 1(a) adalah *start state* persoalan, dan gambar 1(b) adalah *goal state*. Ubin yang dapat bergerak adalah ubin kosong, dengan urutan pergerakan (jika diperlukan) adalah kiri, kanan, atas, bawah. Selesaikan persoalan 8-puzzle tersebut dengan teknik A*. Biaya suatu *state* dari *start state* adalah banyaknya langkah ubin kosong yang telah dilakukan. Jika diperlukan heuristik, maka digunakan jumlah *manhattan distance* semua ubin **selain** ubin kosong. *Manhattan distance* sebuah ubin adalah banyaknya ubin secara horisontal dan vertikal terdekat untuk menuju posisi ubin yang sesuai

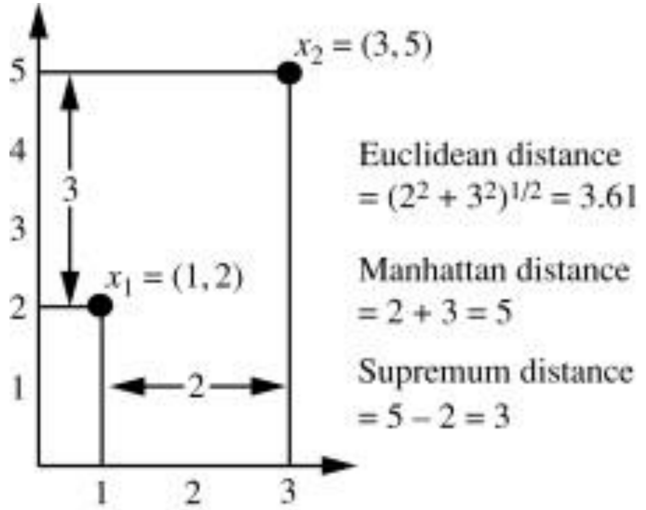
2	4	3
1	7	5
6		8

(a)

1	2	3
4		5
6	7	8

(b)

Gambar 1. *Start State* dan *Goal State* Persoalan 8-Puzzle



Tuliskan langkah-langkah penyelesaiannya dengan menggambarkan pohon ruang pencarian, dan di setiap simpul pohon tunjukkan perhitungan nilai fungsi evaluasinya (yang menjadi penentu *state* berikutnya yang harus diperiksa).

State yang sudah pernah dilalui tidak perlu diperiksa kembali. Setelah pencarian selesai, tuliskan langkah pergeseran ubin kosong dari *start state* hingga *goal state*.

Asumsi: status tujuan selalu dapat dicapai dari status awal.

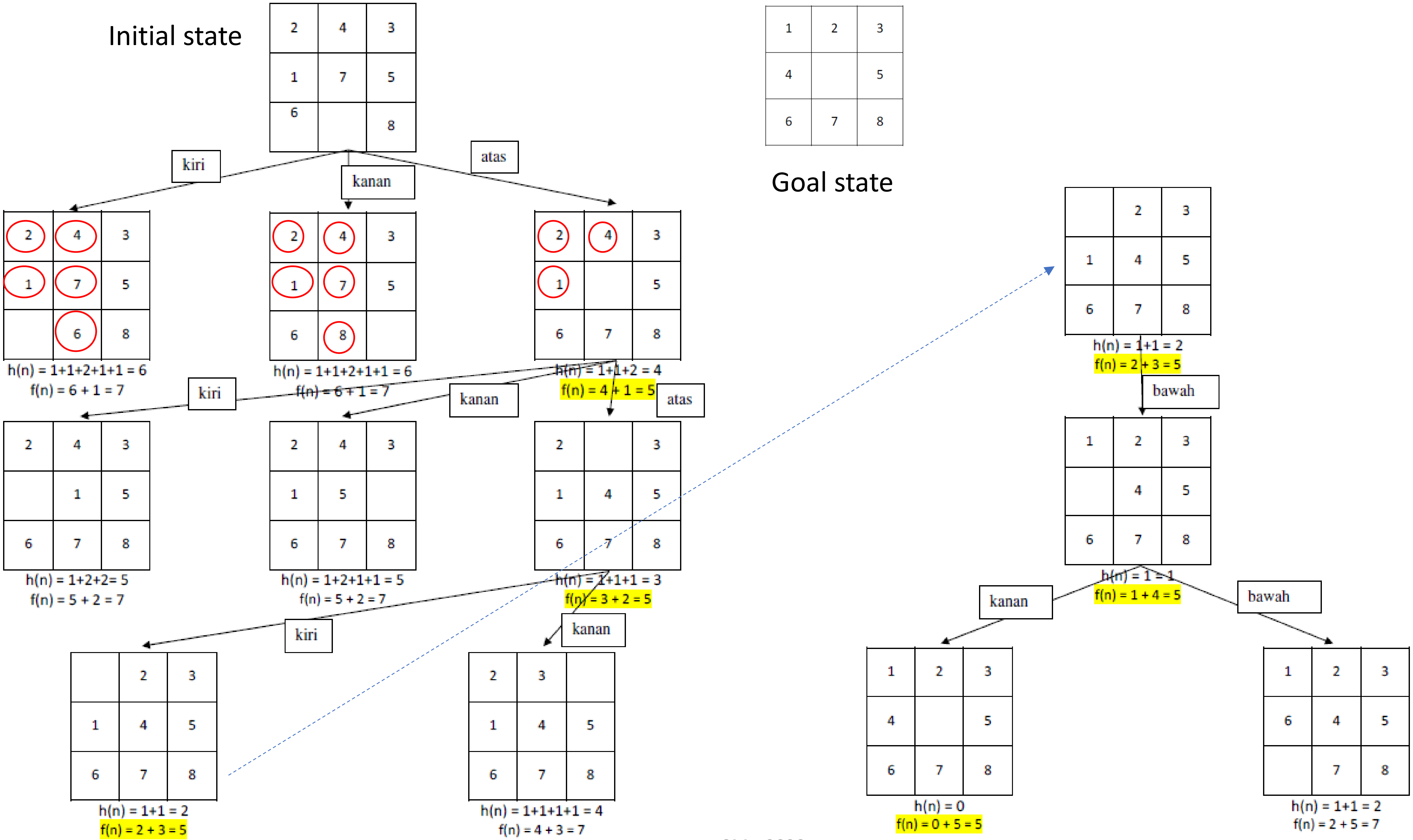
Penyelesaian:

f(n) = g(n) + h(n)

g(n) = jumlah langkah ubin kosong yang telah dilakukan

h(n) = jumlah *manhattan distance* semua ubin **selain** ubin kosong.

(*Manhattan distance* sebuah ubin adalah banyaknya ubin secara horisontal dan vertikal terdekat untuk menuju posisi ubin yang sesuai)


Initial state
Goal state
kiri
kanan
atas
h(n) = 1+1+2+1+1 = 6
f(n) = 6 + 1 = 7
h(n) = 1+1+2+1+1 = 6
f(n) = 6 + 1 = 7
h(n) = 1+1+2 = 4
f(n) = 4 + 1 = 5
kiri
kanan
atas
h(n) = 1+2+2= 5
f(n) = 5 + 2 = 7
h(n) = 1+2+1+1 = 5
f(n) = 5 + 2 = 7
h(n) = 1+1+1 = 3
f(n) = 3 + 2 = 5
kiri
kanan
h(n) = 1+1 = 2
f(n) = 2 + 3 = 5
h(n) = 1+1+1+1 = 4
f(n) = 4 + 3 = 7
h(n) = 1+1 = 2
f(n) = 2 + 3 = 5
bawah
h(n) = 1 = 1
f(n) = 1 + 4 = 5
kanan
bawah
h(n) = 0
f(n) = 0 + 5 = 5
h(n) = 1+1 = 2
f(n) = 2 + 5 = 7

Contoh Soal UAS Sem 2 2014/2015

Dalam permainan *video game*, adakalanya entitas bergerak dalam *video game* perlu berpindah dari satu posisi ke posisi lain. Seringkali proses perpindahan perlu mengutamakan jalur terdekat atau biaya minimal karena berhubungan dengan poin yang diperoleh. Gambar di bawah ini menunjukkan contoh jalur yang mungkin dilewati oleh entitas bergerak dalam suatu *video game*. Suatu entitas akan berpindah dari posisi titik A menuju ke posisi titik F. Jika diperlukan informasi heuristik, nilai heuristik dari suatu simpul adalah **banyaknya busur minimal** yang menghubungkan titik tersebut ke titik tujuan.

Contoh Soal UAS Sem 2 2014/2015 (2)

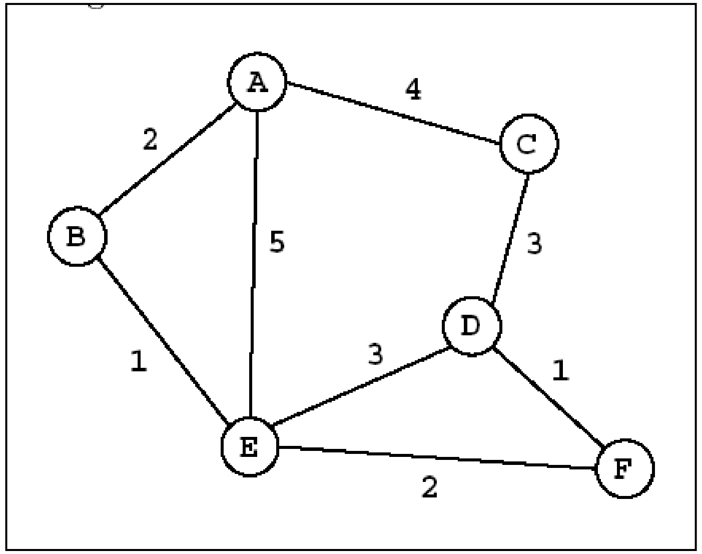
Pencarian solusi dengan:

a. UCS
b. Greedy Best First
c. A Star

Untuk masing-masing pendekatan tuliskan:

- Formula
- Iterasi
    - Simpul yang diekspansi
    - Simpul hidup & nilai f(n)

Urut abjad, simpul ekspan tidak mengulang, tidak membentuk sirkuit, berhenti saat satu solusi ditemukan



Solusi:

Jawaban:

Iterasi	UCS		Greedy Best First Search		A Star	
	Formula: f(n) = g(n)		Formula: f(n) = h(n)		Formula: f(n) = g(n) + h(n)	
	Simpul - Ekspan	Simpu lHidup	Simpul–Ekspan	Simpul Hidup	Simpul - Ekspan	Simpul Hidup
1	A	Ba f(B) = 2	A	Ea f(Ea) = 1	A	Ba f(Ba) = 2+2 = 4
		Ca f(C) = 4		Ba f(Ba) = 2		Ca f(Ca) = 4 + 2 = 6
		Ea f(E) = 5		Ca f(Ca) = 2		Ea f(Ea) = 5 + 1 = 6
2	Ba	Eba f(Eba) = 3	Ea	De f(De) = 1	Ba	Eba f(Eba) = 3 + 1 = 4
		Ca f(C) = 4		Fea f(Fea) = 0		Ca f(Ca) = 4 + 2 = 6
		Ea f(E) = 5		Bea f(Bea) = 2		Ea f(Ea) = 5 + 1 = 6
				Ba f(Ba) = 2		
				Ca f(Ca) = 2		

3	Eba	Ca f(C) = 4	Fea	Sudah sampai solusi	Eba	Feba f(Feba) = 5 + 0 = 5
		Ea f(E) = 5				Ca f(Ca) = 4 + 2 = 6
		Feba f(Feba) = 5				Ea f(Ea) = 5 + 1 = 6
		Deba f(Deba) = 6				Deba f(Deba) = 6+1 = 7
4	Ca	Ea f(E) = 5			Feba	Sudah sampai solusi
		Feba f(Feba) = 5				
		Deba f(Deba) = 6				
		Dca f(Dca) = 7				
5	Ea	Feba f(Feba) = 5				
		Deba f(Deba) = 6				
		Dca f(Dca) = 7				
		Fea f(Fea) = 7				
		Dea f(Dea) = 8				

6	Feba	Solusi sudah ditemukan				
	Jalur: A-B-E-F		Jalur: A-E-F		Jalur: A-B-E-F	
	Jarak: 5		Jarak: 7		Jarak: 5	
Hasil	Banyaknya iterasi: 6		Banyaknya iterasi: 3		Banyaknya iterasi: 4	

TAMAT
-Selamat Belajar-